INVESTIGATING SCHOOL MATHEMATICS

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INVESTIGATION DISCUSSION UTILIZATION

Investigating
School Mathematics

Investigating School Mathematics

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Contents

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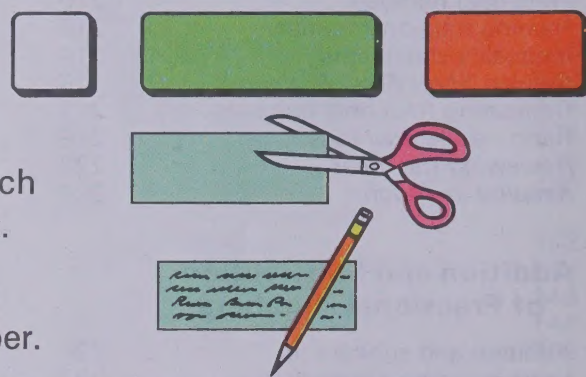
ABCDEFGHIJKL 7987654

Let's explore your mathematics book.

Investigating the Ideas

This is a sample lesson to help you understand how to use your book. In this part of a lesson you will find things to **investigate** and discover.

> **?** Can you find Investigations in your book like the ones below?



A Find an "Investigating the Ideas" section that uses colored strips.

B Find an Investigation in which you must cut something out.

C Find an Investigation which requires only pencil and paper.

Discussing the Ideas

In this part of a lesson you will **discuss the ideas** of the lesson with your classmates and teacher. You will share your ideas with others. You will be getting ready to use the ideas.

1. Does each "Investigating the Ideas" section contain a key question for you to answer?

2. A What color do you find on the border of the page beside each Investigation section?
   B What color is the strip next to the "Discussing the Ideas" section?

3. Is there always a "Discussing the Ideas" section following an "Investigating the Ideas" section?

In this part of a lesson you will be **using the ideas** that you investigated and discussed on the opposite page. You will work problems to improve your understanding of those ideas. Try the problems below.

1. What color is the strip beside the "Using the Ideas" problems?

2. How many "Investigating the Ideas" sections are in Chapter 3?

3. Look up the word **angles** in the index. What page numbers are given?

4. How many special flags referring you to additional exercises can you find in Chapter 5?

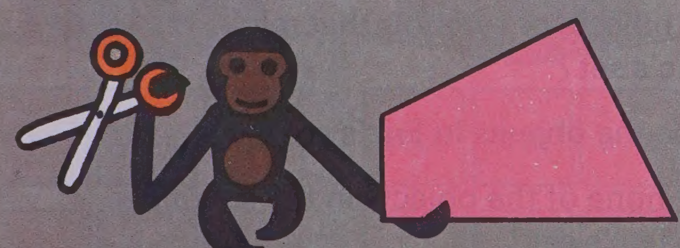
5. On page 67 you are invited to explore one of the "Mathematical Activities." Find the activity.

6. Is there a **glossary** in your book? Find it and look up the meanings of some words you choose.

Problems in these boxes are special challenge problems for you. Be sure to try some of them. See if you can do this one.

think

Make four copies of this region and cut them out.



Can you fit the four pieces together to form a square?

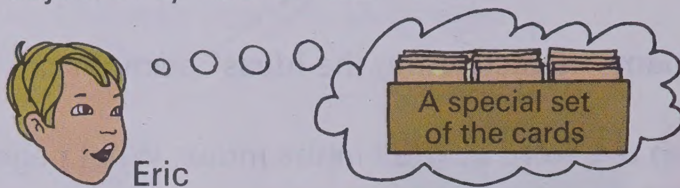
1 Sets, Logic, and Patterns

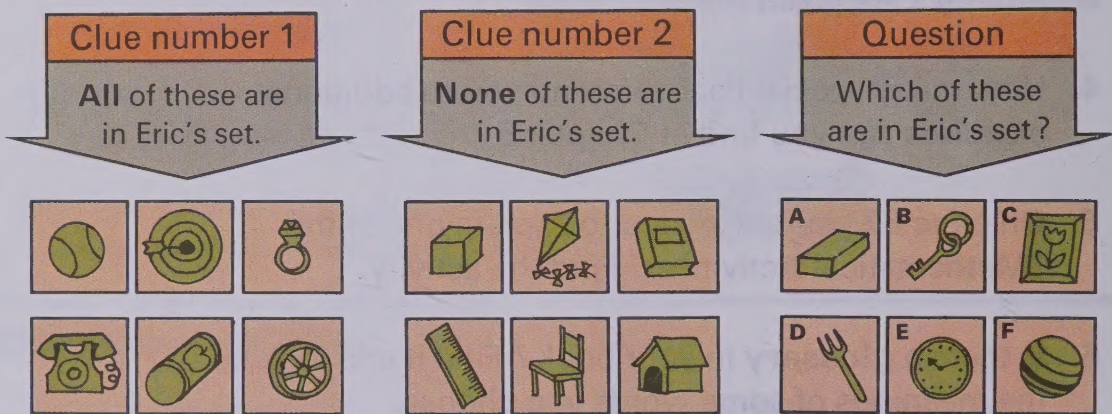
Let's explore sets.

Investigating the Ideas

Imagine that there is a giant stack of cards with a different object or symbol on each one.

Eric is thinking of a special set of the cards.





? Can you use the clues above to tell which cards are in Eric's set?

Discussing the Ideas

1. What are some other objects or symbols that might be on the cards in Eric's set?
2. How can you describe the objects in Eric's set?
3. Can you explain why none of the objects in the set under Clue number 2 are in Eric's set?
4. Choose a set of objects and describe it to your classmates.

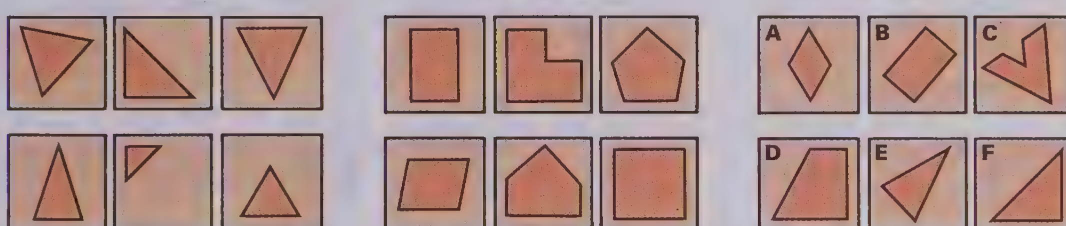
In each exercise you are given clues to help you figure out what special set of cards is involved. Study the clues and answer the question.

1.

All of these are in the set.			None of these are in the set.			Which of these are in the set?		
A	e	I	r	P	n	A b	B E	C O
U	a	o	S	t	j	D i	E Z	F x

2.

All of these are in the set.	None of these are in the set.	Which of these are in the set?



★ **3.**

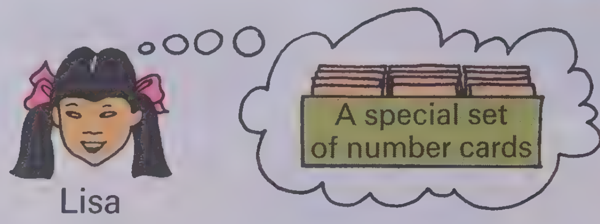
All of these are in the set.			None of these are in the set.			Which of these are in the set?		
Ten	Wow	Jan	Four	Sixty	Born	A Yes	B Nice	C Two
Man	Six	Pop	Golly	No	Sandra	D Cat	E Kitten	F To

* *Let's explore sets of numbers.*

Investigating the Ideas

Suppose the numerals 1 through 1000 were printed on cards.

Lisa is thinking of a special set of these number cards.



? Can you use the clues below to tell which cards are in Lisa's set? | List 5 more numbers in her set.

Clue number 1	Clue number 2	Question
All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
5, 10, 15, 25, 20, 30	2, 28, 6, 14, 1, 23	4, 35, 18, 27, 45, 39, 51, 50

Discussing the Ideas

1. How can you describe the numbers in Lisa's set?

2. All these number cards are in the same set. 232 32 2 23 323

A What are some clues for cards that are not in the set?

B Give some more number cards that would be in the set.

3. Think of a special set of number cards.
Then give clues like those above and see if your classmates can tell what numbers are in your set.

Each exercise gives you clues to help you figure out what special set of number cards is involved. Study the clues and answer the question.

1.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
2, 4, 6	1, 3, 5	A 8, B 9, C 21
16, 28, 40	15, 27, 39	D 32, E 17, F 10

2.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
10, 30, 70	11, 53, 45	A 25, B 20, C 50
90, 100, 420	115, 96, 1	D 105, E 238, F 150

3.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
3, 6, 9	2, 4, 5	A 12, B 21, C 33
18, 27, 30	20, 31, 100	D 25, E 44, F 99

Investigating the Ideas

A **figure card** must have one colored figure on it.

To make each figure card you may use:

- one of the 2 **colors**, red  or blue 
- one of the 3 **figures**, square □, circle ○, or triangle △
- one of the 2 **sizes**, large ▲ or small ▲

? How many different figure cards can you make?

Discussing the Ideas

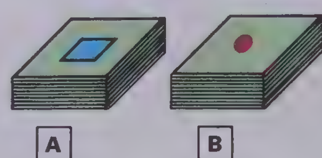
1. Use your cards to form the set of circles. How many cards are in this set? What other sets can you form and name?

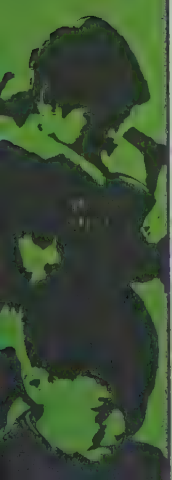
2. How many cards have figures which are
   A red? B red circles? C small red circles?

3. Mix up a complete set of cards and have someone secretly take one of them. How can you find which one he took?

4. Suppose one of the cards is turned facedown. Someone who knows tells you the figure on the card is **not** blue. What do you then know for sure about the figure?

5. If you have one set of cards, can you put all the cards with blue figures in pile A **and** all the cards with circles in pile B? Explain.





1. Use the cards you made in the Investigation.
How many are in each set?

A Squares	E Large figures	I Small squares
B Triangles	F Blue figures	J Large triangles
C Circles	G Red figures	K Large red figures
D Small figures	H Blue circles	L Small blue triangles

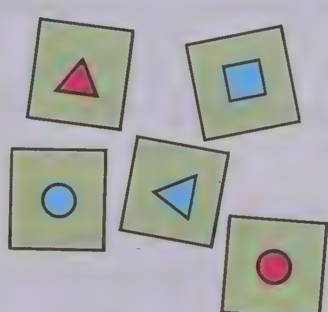
2. One of the cards is shown facedown in each exercise.
Some clues have been written on the back.
Which figure is on the card?

A
It is small.
It is a square.
It is **not** red.
What is it?

B
It is **not** small.
It is **not** blue.
It is a circle.
What is it?

C
It is **not** large.
It is **not** red.
It is **not** a circle.
It is **not** a square.
What is it?

3. This is part of a special set of the cards. Which card is missing?



★ 4. Make up a different set of cards and write some problems about them.

think

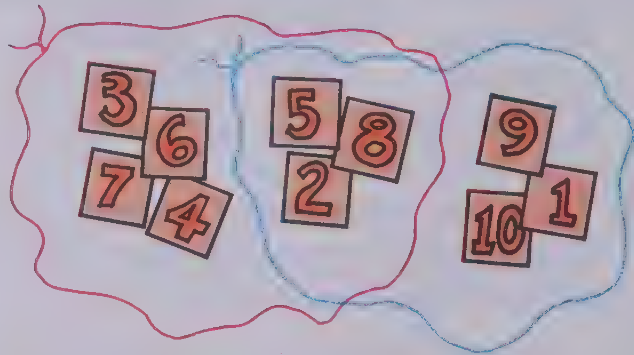
After the 3rd inning the baseball scoreboard looked like this.

Mets	2
Expos	4

The final score was
7 to 3
Who won the game?
How do you know?

Investigating the Ideas

Make cards for the numbers 1 through 10. Make two 1-metre loops from red and blue yarn.



CLUE CARD
It is inside the red loop **and** it is inside the blue loop.
It is odd.
Which number is it?

Use the clues to find the number.

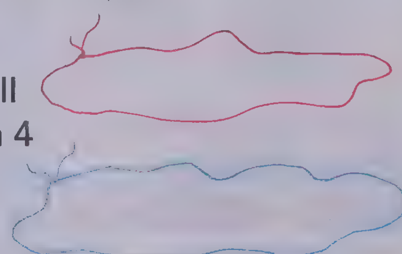
?	
Can you put the cards into the loops in a different way and make a new clue card?	Challenge a classmate to find **your** number.

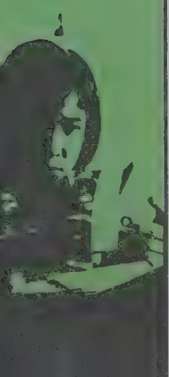
Discussing the Ideas

1. Which cards are inside the red loop **and** also inside the blue loop in the Investigation?

2. A student is thinking about a card that is **not** inside the red loop. What do you know about his card?

3. A student placed his loops like this. Can he lay the cards down so that all the cards with numbers greater than 4 are inside the red loop and all the cards with numbers less than 7 are inside the blue loop? Explain.





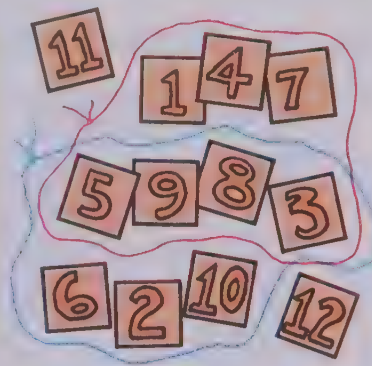
1. Use the clues and give the number for each clue card.

A

CLUE CARD
It is not
inside either
loop.
It is not 11.

B

CLUE CARD
It is not
inside the
blue loop.
It is inside
the red loop.
It is even.



C

CLUE CARD
It is not
inside
the red loop.
It is inside
the blue loop.
It is greater
than 6.

D

CLUE CARD
It is inside
the red loop
and
it is inside
the blue loop.
It is even.

E

CLUE CARD
It is not
inside the
blue loop.
It is not even.
It is less
than 7.

2. Use the set of cards for the numbers 1 through 10. How many cards have numbers that are

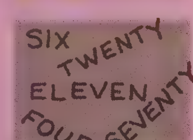
- **A** not even?
- **B** not odd?
- **C** less than 5?
- **D** more than 5?
- **E** more than 4 and less than 7?
- **F** even and more than 6?

★ **3.** Put the cards with odd numbers inside the red loop. Put the cards with multiples of 3 inside the blue loop. Which numbers are inside both loops?

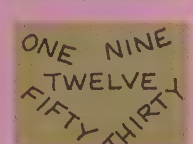
think

These are in a set of "special" words.

These are not in the set.

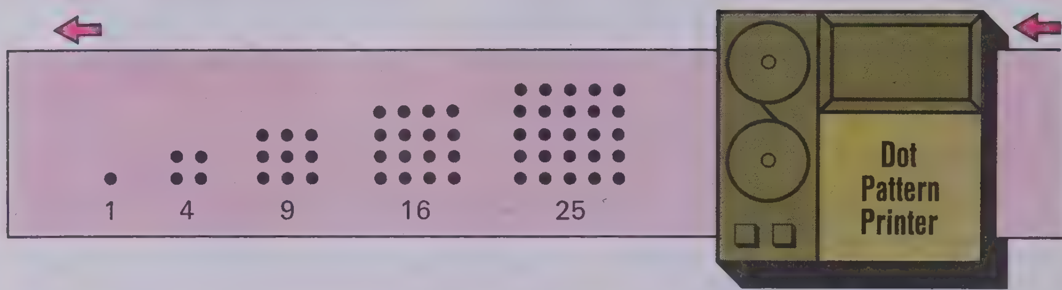


Which of these is in the set?



Investigating the Ideas

The **Dot Pattern Printer** printed patterns for the number sequence 1, 4, 9, 16, 25, ...

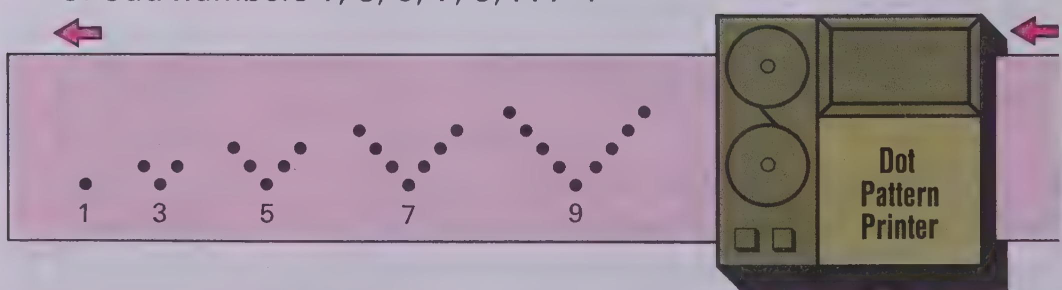


Pretend that the dot patterns can go on and on.

> **?** Can you draw the next two patterns the printer will make and give the next two numbers in the sequence?

Discussing the Ideas

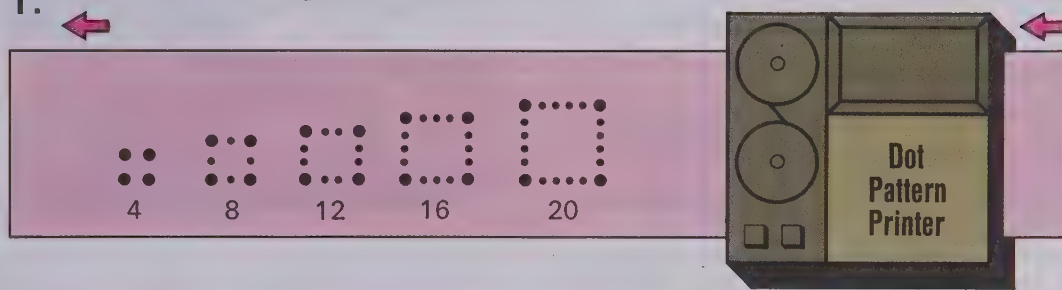
1. Why do you think the numbers in the sequence above are called square numbers? What are some other square numbers?
2. What pattern might the machine print for the sequence of even numbers 2, 4, 6, 8, 10, 12, 14, . . . ?
3. What pattern might the machine print for the sequence of odd numbers 1, 3, 5, 7, 9, . . . ?



4. Can you invent another sequence of numbers and draw the dot patterns that go with the sequence?

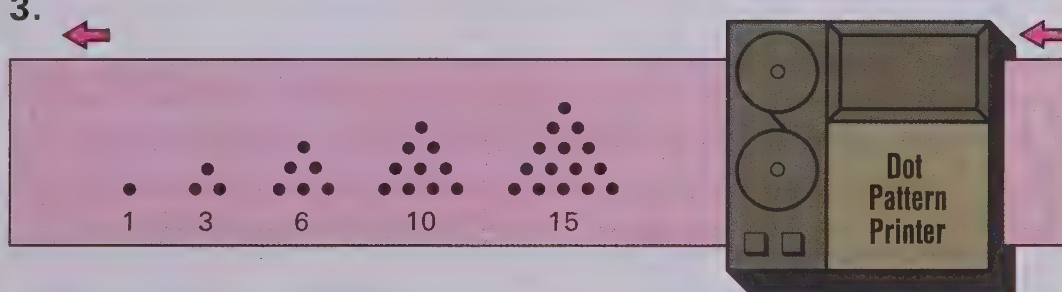
Draw the next dot pattern the "printer" will make and give the next two numbers in each sequence.

1.



2.

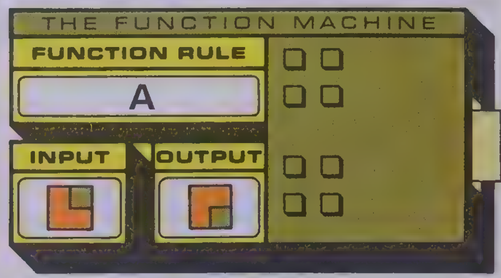
3.

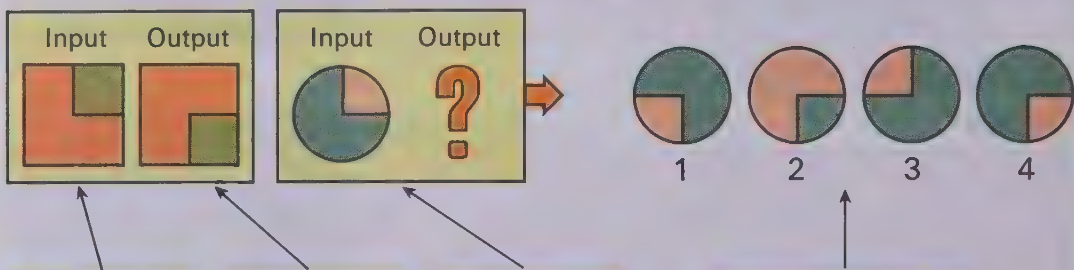


4.

Investigating the Ideas

When you put a figure in this function machine, the machine changes it in a certain way. The machine prints yellow input-output cards to show what happened.





When this figure is the input

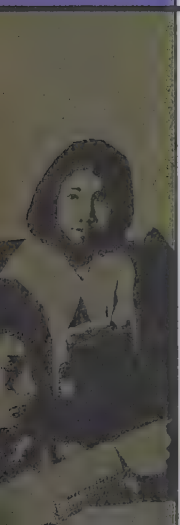
this figure is the output.

When this figure is the input

which of these is the output?

Answer the question above.

> **?** Can you invent your own function rule like the one above and make some input-output cards to show your rule?

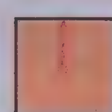


Discussing the Ideas

1. Explain function rule **A** in your own words.

2. Which figure did you select as the output in the Investigation? Why?

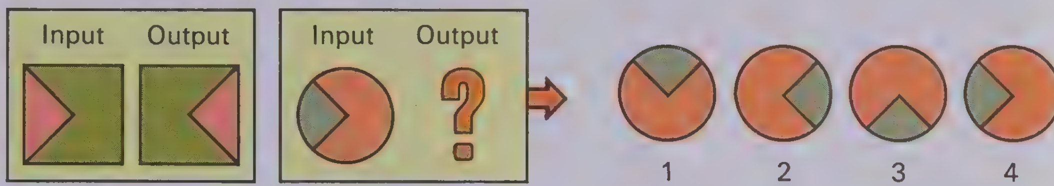
3. Can you draw the output that the machine above will produce if this figure is the input?



Using the Ideas

The first card in each exercise shows what the machine does.
Give the number of the output for the second card.

1.



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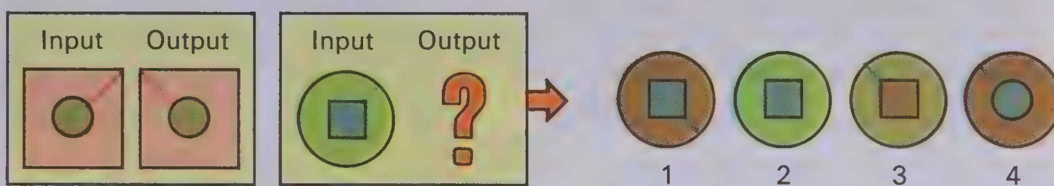

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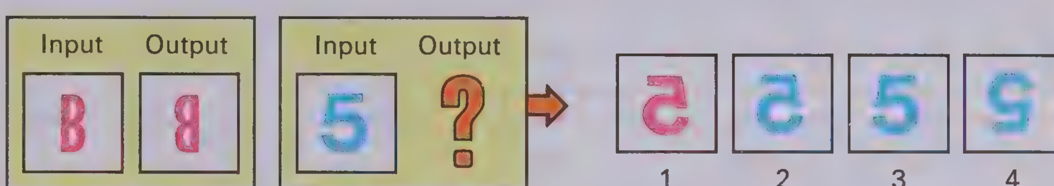

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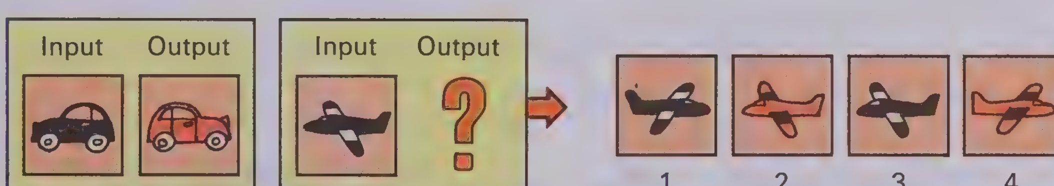

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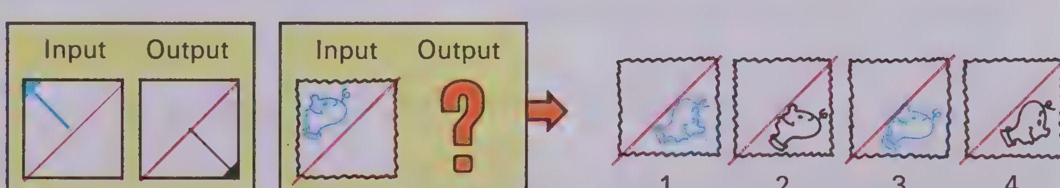


Investigating the Ideas

Study the first 4 equations on each poster.

$(1 \times 9) - 1 = 8$

$(21 \times 9) - 1 = 188$

$(321 \times 9) - 1 = 2888$

$(4321 \times 9) - 1 = 38888$

$(\_\_\_\_ \times 9) - 1 = \_\_\_\_$

$1 + 3 = 2 \times 2$

$1 + 3 + 5 = 3 \times 3$

$1 + 3 + 5 + 7 = 4 \times 4$

$1 + 3 + 5 + 7 + 9 = 5 \times 5$

$1 + 3 + \_\_ + \_\_ + \_\_ + \_\_ = \_\_\_\_$

? Can you copy and complete the last equation on each poster?

Discussing the Ideas

1. Use the pattern on the first poster to guess the number for n in this equation.

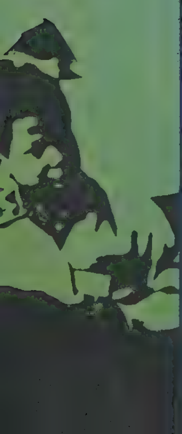
$$(987654321 \times 9) - 1 = n.$$

Check your guess by computing.

2. A certain number multiplied by itself gives a product equal to this sum: $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$. Explain how to find the number.

3. Can you find the pattern and give three more numbers in this sequence?

$$2,\ 3,\ 5,\ 9,\ 17,\ 33,\ \_\_,\ \_\_,\ \_\_\ \ldots$$



1. Study the patterns in the first four equations on each poster. Copy and complete the next equation in each part.

A

0 + 1 + 2 = 3
1 + 2 + 3 = 6
2 + 3 + 4 = 9
3 + 4 + 5 = 12
4 + __ + __ = __

B

143 × 7 = 1001
143 × 14 = 2002
143 × 21 = 3003
143 × 28 = 4004
143 × __ = ____

C

(1 × 9) + 2 = 11
(12 × 9) + 3 = 111
(123 × 9) + 4 = 1111
(1234 × 9) + 5 = 11 111
(____ × 9) + __ = ____

D

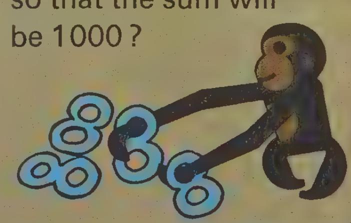
(1 × 8) + 1 = 9
(12 × 8) + 2 = 98
(123 × 8) + 3 = 987
(1234 × 8) + 4 = 9876
(____ × 8) + __ = ____

2. Check each equation in exercise 1. Is each "answer" correct?

3. Give the next three numbers in each sequence.
   A 2, 4, 6, 8, 10, 12, 14, ?, ?, ?
   B 1, 2, 4, 8, 16, 32, ?, ?, ?
   C 10, 11, 20, 21, 30, 31, 40, 41, ?, ?, ?

think

Can you arrange eight "eights" in an addition problem so that the sum will be 1000?

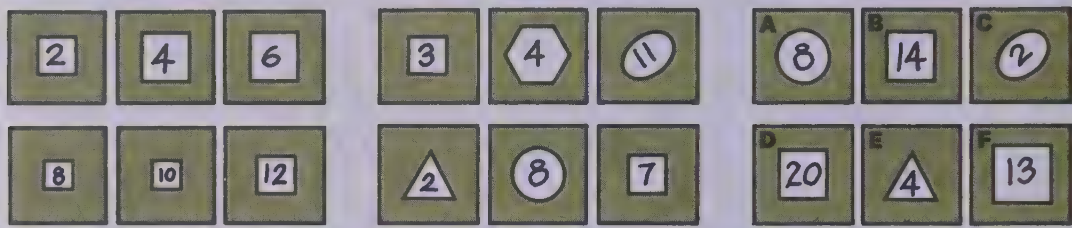


1.

All of these are in the set.

None of these are in the set.

Which of these are in the set?



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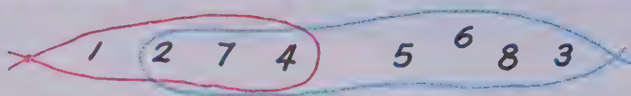


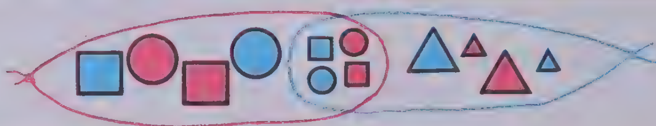
A
It is inside the red loop.
It is not inside the blue loop.

Which number is it?

B
It is inside the red loop
and
it is inside the blue loop.
It is odd.

Which number is it?

3.

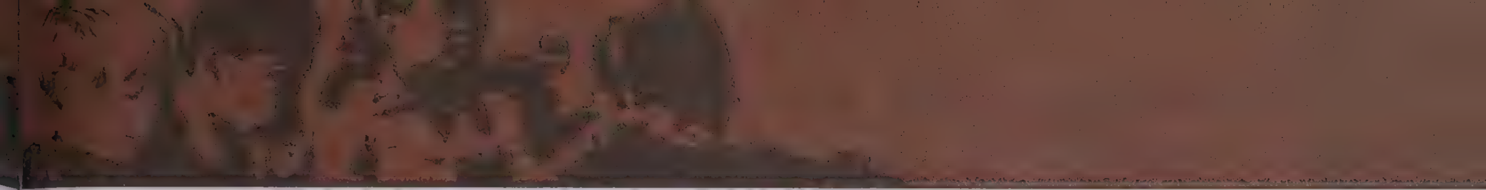


A
It is inside the red loop.
It is large.
It is not a circle.
It is blue.
What is it?

B
It is blue.
It is large.
It is not inside the red loop.
What is it?

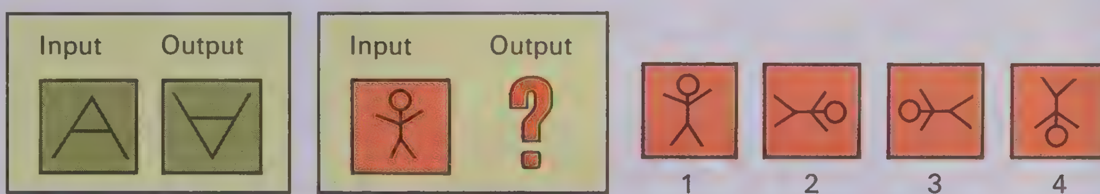
C
It is inside both loops.
It is not a circle.
It is not red.
What is it?

D
It is small.
It is red.
It is not in the red loop.
What is it?



4. Give the next two patterns and numbers.

5. The first card shows what the function machine does.
Can you give the number for the output for the second card?



6. Study the first three equations on each poster.
Then copy and complete the next equation.

A

$0+2+4=6$

$2+4+6=12$

$4+6+8=18$

$\_\_+\_\_+\_\_=\_\_$

B

$15\times15=(1\times2\times100)+25=225$

$25\times25=(2\times3\times100)+25=625$

$35\times35=(3\times4\times100)+25=1225$

$45\times45=(\_\_\_\_\_\_)+25=\_\_\_$

7. Give the next three numbers in each sequence.

A 1, 6, 11, 16, 21, __, __, __ **B** 1, 3, 13, 131, 1313, __, __, __

You are invited to explore

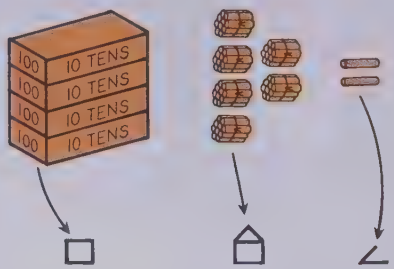
ACTIVITY CARD 1
Page 333

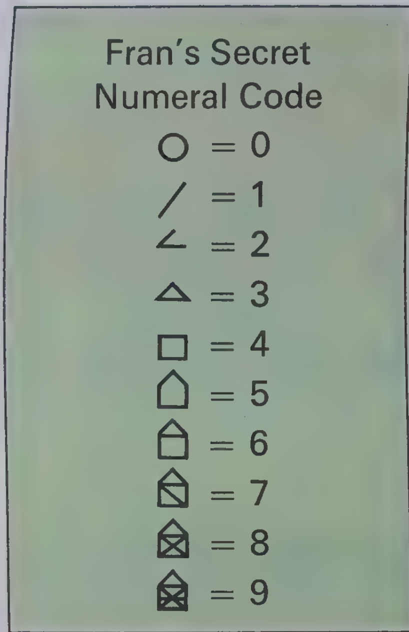
2 Numbers and Numerals

Let's explore symbols for numbers.

Investigating the Ideas

Fran decided to invent new numerals for numbers. She used them like this:





Here is Fran's date of birth.

June ∠⌂, / ⌂ □.

? Can you use Fran's numerals to show your date of birth and some other interesting numbers?

Discussing the Ideas

1. How many sticks are shown in the Investigation above?

2. When was Fran born?

3. Once Fran decided to use ⌂ for 5, why do you think she used the numerals , , , for 6, 7, 8, and 9?

4. In the numeral △ ○ , the △ indicates which of these numbers: 3, 30, 300, or 3000?

1. Change these to "ordinary" numerals.

2. Use Fran's numerals to rewrite each of these.

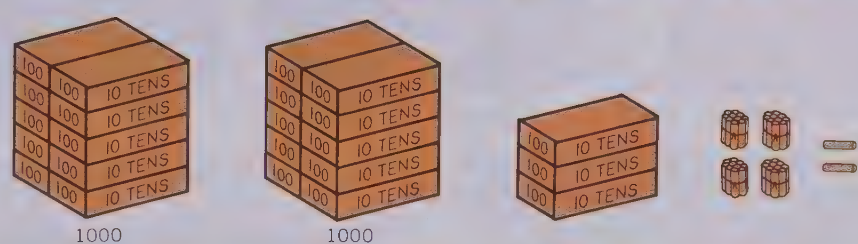
A 36	D 136	G 5265	J 4000
B 423	E 200	H 7068	K 1007
C 58	F 405	I 9000	L 6528

3. Use "ordinary" numerals and then Fran's numerals to tell how many are in each set.

A

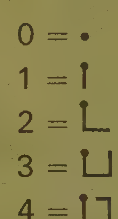
B

C



think

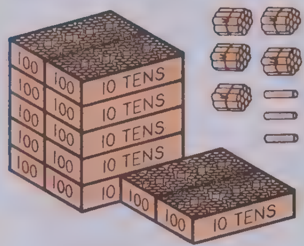
Here is the beginning of a code. Can you complete it to 9 and then write some numerals with it?



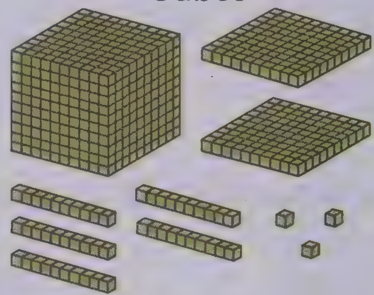
Investigating the Ideas

Each of these figures shows 1253.

Sticks

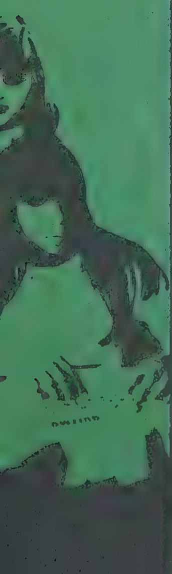


Cubes



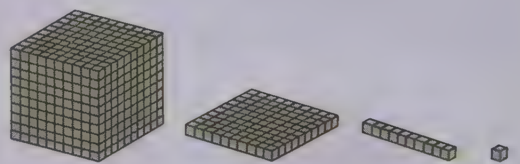
Abacus

? Can you draw an abacus that shows 3426 ?



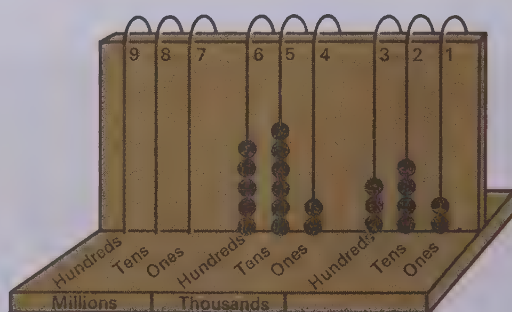
Discussing the Ideas

1. We name the parts in the cube figure like this.
   - A How many units in a rod ?
   - B How many rods in a layer ?
   - C How many units in a layer ?
   - D How many units in a block ?



Block Layer Rod Unit

2. It is difficult to show thousands and millions with sets of objects, but we can show larger numbers easily on the abacus. Write the number shown on this abacus.

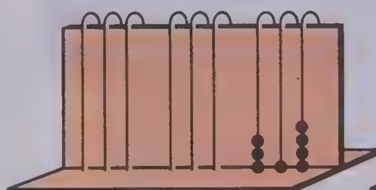


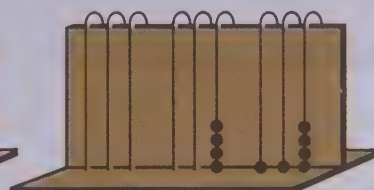
3. Explain how you would show these numbers on the abacus.

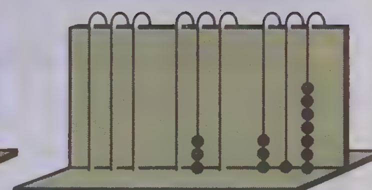
A 26 714	C 46 205	E 2 615 284
B 132 476	D 813 495	F 39 700 408

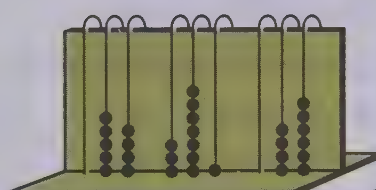
1. Give the number for each of these.
   - A 5 blocks, 3 layers, 2 rods, 4 units
   - B 7 units, 4 layers, 9 blocks, 3 rods
   - C 6 rods, 1 block, 7 units, 5 layers
   - D 12 layers

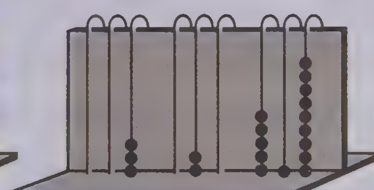
2. Give the number shown by each abacus.

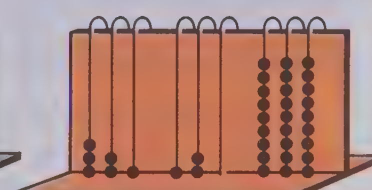
A


B


C


D


E


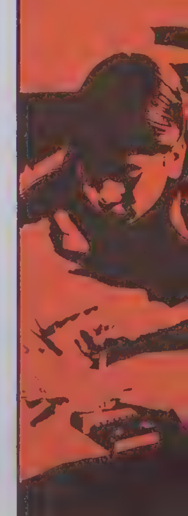
F


3. Solve the equations.
   - A $2756 = 2000 + 700 + \mathbf{v} + 6$
   - B $3428 = 3000 + 400 + 20 + \mathbf{n}$
   - C $1206 = 1000 + \mathbf{a} + 6$
   - D $9257 = \mathbf{m} + 200 + 50 + 7$
   - E $6328 = 6000 + 300 + \mathbf{s} + 8$
   - F $4005 = \mathbf{t} + 5$
   - G $7065 = 7000 + \mathbf{y} + 5$
   - H $8920 = 8000 + \mathbf{b} + 20$

think

My name is shown
By one abacus bead.
But to write my name,
Six digits you will need.

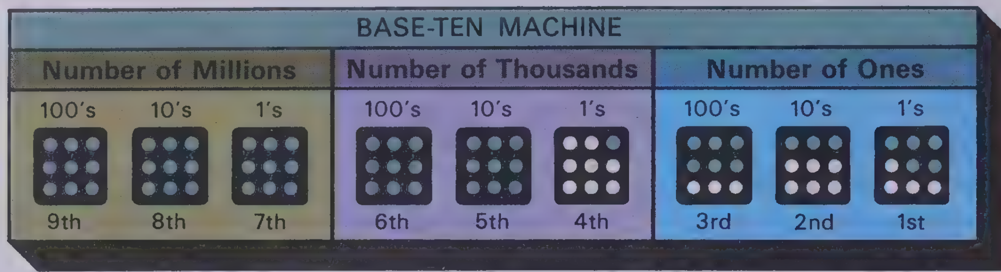
WHO AM I?



More practice, page A-1, Set 1

Investigating the Ideas

A card placed in the Base-Ten Machine below makes the lights show a certain number.



? Can you tell how the Base-Ten Machine works? | Record the numeral that it shows.

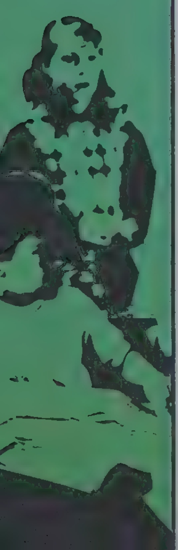
Discussing the Ideas

1. We use **place value** when we write symbols for whole numbers. Counting from right to left on the Base-Ten Machine, we call the 1st place the **ones'** place, the 2nd place the **tens'** place, and the 3rd place the **hundreds'** place. What is the 4th place called?

2. The 5th place is called the **ten thousands'** place. Think carefully, then decide what the 6th, 7th, 8th, and 9th places are called.

3. In what way are the abacus and the Base-Ten Machine alike?

4. Each sentence below tells something about the numeral 35 746. Read and complete each sentence.
   A The 3 is in the **ten thousands'** place.
   B The 5 is in the ______ ? ______ place.
   C The 7 is in the ______ ? ______ place.
   D The 4 is in the ______ ? ______ place.
   E The 6 is in the ______ ? ______ place.



1. Give the number the Base-Ten Machine was signalled to remember.

A

B

C

D

2. For 683 547 201, tell what digit is in each of these places.
   A thousands'
   B millions'
   C ten millions'
   D hundred thousands'
   E hundred millions'
   F ten thousands'

3. Write a numeral that has 7 in the ten thousands' place.

4. Write a numeral that has 6 in the millions' place, 4 in the hundred thousands' place, 7 in the ten thousands' place, and zero in all other places.

5. Write a numeral that has 328 thousands, 496 millions, and 507. Now list the names of the first 9 places and give the digit that is in each place.

think

I could mean five million,
Or even fifty billion.
My value isn't known
Until my place is shown.

WHO AM I?

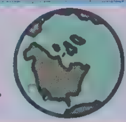
More practice, page A-1, Set 2

Do you know number names for larger numbers?

Discussing the Ideas

1. Can you read the number in the sentence below?

> The oceans of the earth cover
> 356 877 107 square kilometres.



Follow the directions below to read the number.

First read: **356** 877 107 — 356 million

Then read: 356 **877** 107 — 877 thousand

Then read: 356 877 **107**

2. Read each of these numbers.

A	426 753 127	E	127 240 316	I	17 286 000
B	306 526 319	F	5 103 286	J	80 000 275
C	345 281 407	G	175 024 156	K	100 000 000
D	36 258 342	H	100 300 346	L	65 070 002

3. We read the number in example A as:
   six hundred fifty-three billion,
   two hundred sixty-one million,
   five hundred four thousand,
   eight hundred ninety-two.
   Read the number in example B.

A: **653** 261 504 892 — billion

B: **492** 765 931 407 682 — trillion

4. Read the numbers in these statements.
   - A Estimated world population in 2000 A.D.: 6 000 000 000.
   - B Recent estimate of earth's age: 4 950 000 000 years.
   - C Distance light travels in one year: 9 405 594 460 800 km.

1. For large numbers, digits are grouped by threes. Each group is called a **period.** The names of some of the periods are given below. Give the missing word for each sentence.

Quintillions	Quadrillions	Trillions	Billions	Millions	Thousands	
6 4 3	5 0 1	0 7 6	8 3 0	2 7 5	1 6 3	7 8 5

A The 830 tells how many __?__. Answer: billions
B The 643 tells how many __?__.
C The 275 tells how many __?__.
D The 501 tells how many __?__.
E The 076 tells how many __?__.

2. For each number write an equation as in the example.
Example: 3654 = 3000 + 600 + 50 + 4

A 3653
B 4821
C 7260
D 26 385
E 90 371
F 84 027
G 163 827
H 704 346
I 762 005
J 1 265 837
K 6 860 831
L 23 456 029

3. Sometimes the multiplication sign is used to show place value. For each number write an equation as in the example.
Example: $3654 = (3 \times 1000) + (6 \times 100) + (5 \times 10) + 4$

A 2845
B 6734
C 9258
D 27 643
E 148 296
F 7 268 489

think

Count my digits.
You'll find eleven.
All are zeros
Except one seven.

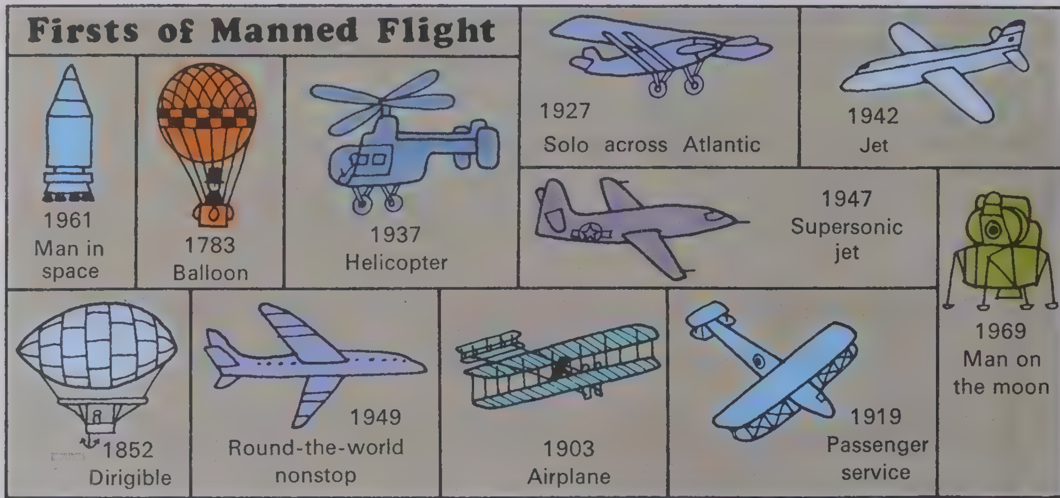
WHO AM I?

4. Write the ordinary numeral for each part.

A $300 + 50 + 7$
B $5000 + 400 + 30 + 8$
C $(4 \times 100) + (6 \times 10) + 3$
D $(7 \times 1000) + (6 \times 100) + (5 \times 10) + 2$

More practice, page A-2, Set 3

Investigating the Ideas

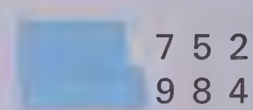


Look up at least one more date that interests you.

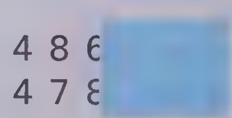
> **?** Can you make a chart that lists your dates and those above in order?

Discussing the Ideas

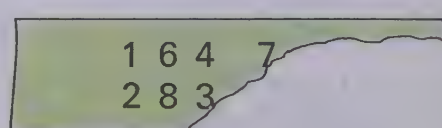
1. Which date used in the Investigation was longest ago?
2. Which date is most recent?
3. Part of these two numerals is covered. Can you tell which one names the greater number? Explain.



4. Can you tell which of these is greater if you know they have the same number of digits?



5. Why can't you be sure which of these named the greater number before the paper was torn?



1. Write the word (greater or less) that should go in each blank.
   - A Since 95 is __?__ than 52, we write $95 > 52$.
   - B Since 48 is __?__ than 79, we write $48 < 79$.
   - C Since 256 is __?__ than 254, we write $256 > 254$.
   - D Since 714 is __?__ than 734, we write $714 < 734$.

2. Give the correct sign ($<$ or $>$) for each ◍.
   - A 36 ◍ 26
   - B 78 ◍ 74
   - C 326 ◍ 356
   - D 785 ◍ 985
   - E 653 ◍ 597
   - F 6328 ◍ 6348
   - G 6428 ◍ 6228
   - H 9761 ◍ 9716
   - I 15 286 ◍ 15 317
   - J 162 834 ◍ 162 097
   - K 9 762 483 ◍ 7 651 642
   - L 5 400 122 ◍ 5 399 876

3. Give the correct sign ($=$ or $\neq$) for each ◍.
   - A 3624 ◍ 3000 + 600 + 20 + 4
   - B 72 437 ◍ 70 000 + 2000 + 400 + 40 + 7
   - C 632 847 ◍ 600 000 + 30 000 + 2800 + 47
   - D 56 482 ◍ 50 000 + 6000 + 410 + 70 + 2
   - E 734 ◍ $(7 \times 100) + (3 \times 10) + 4$
   - F 9284 ◍ $(9 \times 1000) + (8 \times 100) + (2 \times 10) + 4$

4. Give the number 100 000 greater than 54.

5. Give the number 10 000 000 greater than 54 365 847.

6. Give the number 1 000 000 less than
   - A 1 000 038
   - B 1 467 896

7.
   - A Give the largest 3-digit number that has the digits 4 and 7.
   - B Give the smallest 4-digit number that has only one 0 digit.
   - C Give the largest 6-digit number with no two digits alike.

think

Give the largest number that uses all the digits once.

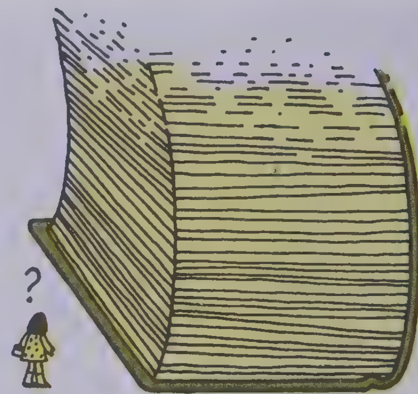
Give the smallest such number.

More practice, page A-2, Set 4

Investigating the Ideas

A book with 1000 pages would be about 5 centimetres thick.

Guess how tall your mathematics book would be if it had 1 000 000 pages.

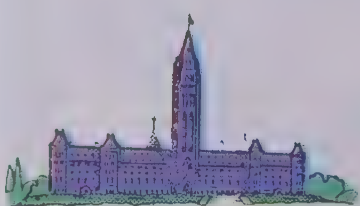


Can you **figure out** about how tall your book would be if it had 1 000 000 pages?

Discussing the Ideas

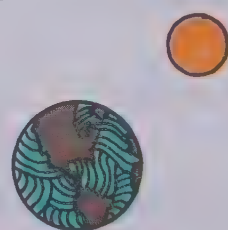
1. How large is each of these numbers? First guess the number of digits. Then check your guess.

A



Population of the capital of your province

B



Distance in kilometres to the sun

C

Speed of light in kilometres per second

2. Find a use for at least one number that has more than 4 digits. Report your findings to the class.

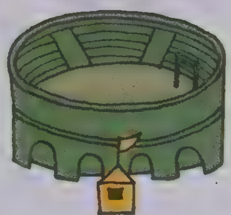
1. Match each picture with one of these large numbers.

1 000

100 000

1 000 000

A



B

C

Number of seats in a large stadium

Number of pages in a thick telephone book

Number of grains of sand in a cup

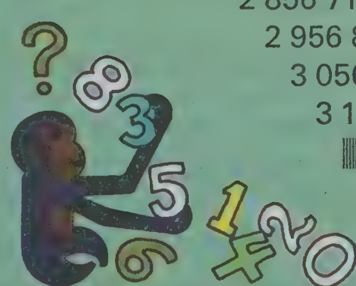
2. Which is more,
   A 100 thousands or a million?
   B a thousand thousands or 10 millions?
   C a thousand hundreds or a million?
   D a billion or 100 millions?

3. Write the number that is
   A a thousand thousands.
   B a thousand hundreds.
   C ten millions.
   D a thousand more than a million.
   E a hundred millions.
   F a thousand millions.
   G a million more than a million.
   H a million more than a billion.

think

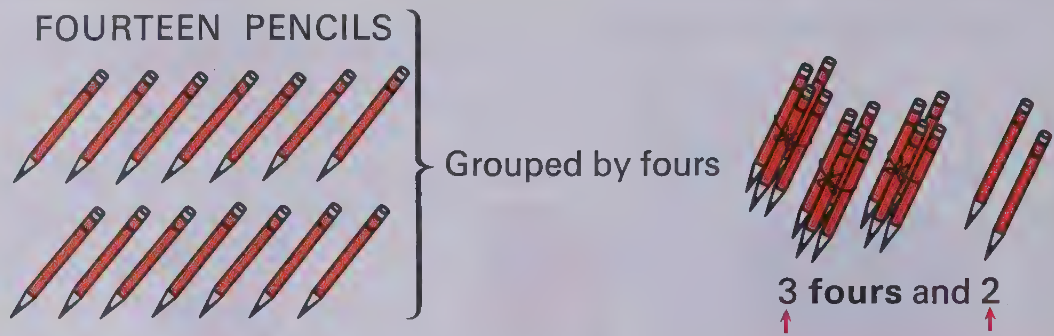
Find the next number in this sequence.

2 856 719
2 956 819
3 056 919
3 157 019



Let's explore different bases for numerals.

Investigating the Ideas



Grouping Rule These numbers are less than the group size.

?	How many other ways can you find to group fourteen pencils?	Record only the groupings that follow the grouping rule.

Discussing the Ideas

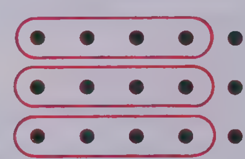
1. Study the chart below. Then explain how to write a numeral for some of the grouping you did in the Investigation.

NUMBER	GROUPING	NUMERAL
fourteen	3 fours and 2 (base four)	$32_{(4)}$

2. A How many groups of four dots are in this set?

   B How many dots are left over?

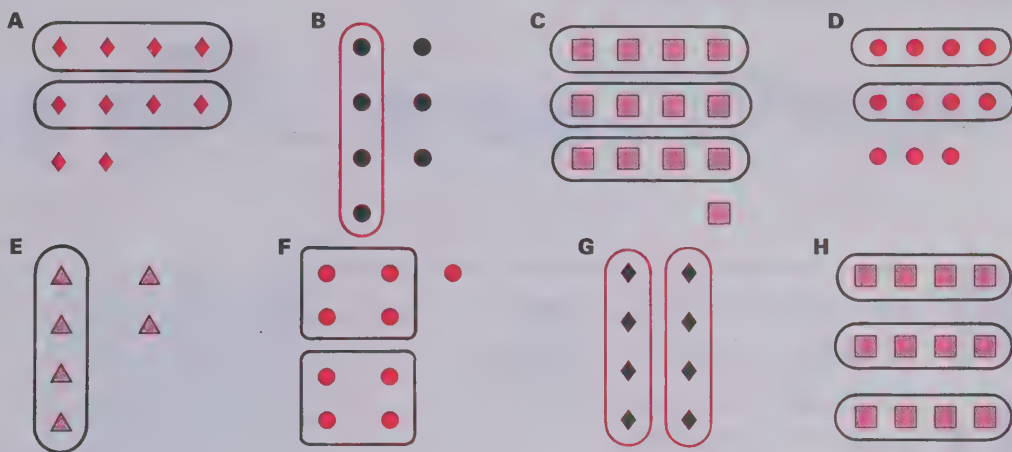
   C What numeral in base four tells how many dots are in the set?



Grouping by fours

3. Explain how to write the base-five and base-six numerals for the set of dots.

1. Write the **base-four** numeral for the number of each set. (Example: For 2 fours and 3, write $23_{(4)}$.)



2. Write a base-four numeral for the number eleven. It may help to draw and group eleven dots.

3. Continue the counting in base four.

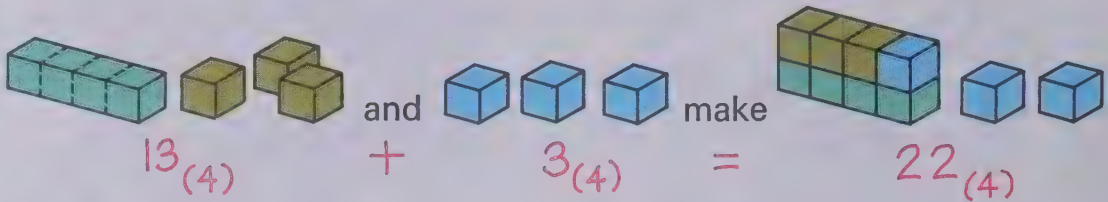
$0_{(4)}$	$1_{(4)}$	$2_{(4)}$	$3_{(4)}$	$10_{(4)}$	$11_{(4)}$	$12_{(4)}$	▥	▥	▥	▥	▥	▥	▥	▥

★ 4. Study the example. Then complete each sentence. (Example: $32_{(4)}$ means 3 fours and 2 or 14 in base ten.)

A $23_{(4)}$ means 2 fours and 3 or __?__ in base ten.
B $12_{(4)}$ means 1 four and ▥ or __?__ in base ten.
C $31_{(4)}$ means ▥ fours and 1 or __?__ in base ten.
D $30_{(4)}$ means ▥ fours and ▥ or __?__ in base ten.
E $21_{(4)}$ means ▥ fours and ▥ or __?__ in base ten.
F $20_{(4)}$ means ▥ fours and ▥ or __?__ in base ten.
G $10_{(4)}$ means ▥ fours and ▥ or __?__ in base ten.
H $13_{(4)}$ means ▥ fours and ▥ or __?__ in base ten.

Investigating the Ideas

Study the diagram below.

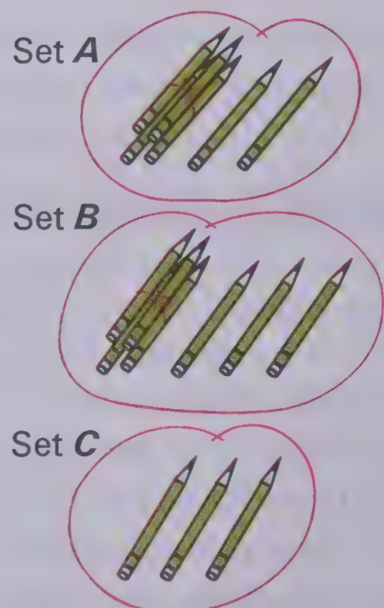


? Can you use a set of fifteen or fewer counters to help you write and solve some more base-four problems?

$3_{(4)} + 3_{(4)} = n_{(4)}$
$10_{(4)} + 11_{(4)} = n_{(4)}$
$11_{(4)} + 2_{(4)} = n_{(4)}$

Discussing the Ideas

1. A Explain how you can use sets *A* and *B* to "prove" this equation is correct.
$12_{(4)} + 13_{(4)} = 31_{(4)}$



B Use sets *A* and *C* to help you "prove" this equation is correct.
$12_{(4)} - 3_{(4)} = 3_{(4)}$

2. How could you think of Set *B* twice to "prove" this equation is correct?

$$2_{(4)} \times 13_{(4)} = 32_{(4)}$$

1. Copy and complete the addition and multiplication tables for base four.

$+$	$0_{(4)}$	$1_{(4)}$	$2_{(4)}$	$3_{(4)}$
$0_{(4)}$	$0_{(4)}$	$1_{(4)}$		
$1_{(4)}$			$3_{(4)}$	
$2_{(4)}$				$11_{(4)}$
$3_{(4)}$		$10_{(4)}$		

$\times$	$0_{(4)}$	$1_{(4)}$	$2_{(4)}$	$3_{(4)}$
$0_{(4)}$	$0_{(4)}$	$0_{(4)}$	$0_{(4)}$	
$1_{(4)}$				
$2_{(4)}$		$2_{(4)}$	$10_{(4)}$	
$3_{(4)}$			$12_{(4)}$	

2. Find the sums.

A $\begin{array}{r} 12_{(4)} \\ +1_{(4)} \\ \hline \end{array}$ **B** $\begin{array}{r} 12_{(4)} \\ +2_{(4)} \\ \hline \end{array}$ **C** $\begin{array}{r} 22_{(4)} \\ +1_{(4)} \\ \hline \end{array}$ **D** $\begin{array}{r} 22_{(4)} \\ +2_{(4)} \\ \hline \end{array}$ **E** $\begin{array}{r} 21_{(4)} \\ +3_{(4)} \\ \hline \end{array}$

3. Can you find and correct the mistake on Jim's paper?

Multiplication
Base Four
Jim

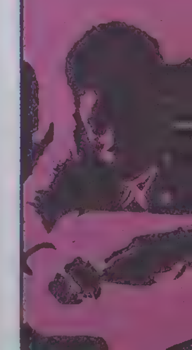
1. $\begin{array}{r} 2_{(4)} \\ \times 2_{(4)} \\ \hline 10_{(4)} \end{array}$ 2. $\begin{array}{r} 3_{(4)} \\ \times 2_{(4)} \\ \hline 12_{(4)} \end{array}$ 3. $\begin{array}{r} 3_{(4)} \\ \times 3_{(4)} \\ \hline 21_{(4)} \end{array}$

4. $\begin{array}{r} 12_{(4)} \\ \times 2_{(4)} \\ \hline 30_{(4)} \end{array}$ 5. $\begin{array}{r} 10_{(4)} \\ \times 3_{(4)} \\ \hline 30_{(4)} \end{array}$ 6. $\begin{array}{r} 13_{(4)} \\ \times 2_{(4)} \\ \hline 26_{(4)} \end{array}$

4. Find these products.

A $\begin{array}{r} 3_{(4)} \\ \times 2_{(4)} \\ \hline \end{array}$ **B** $\begin{array}{r} 11_{(4)} \\ \times 3_{(4)} \\ \hline \end{array}$ **C** $\begin{array}{r} 12_{(4)} \\ \times 2_{(4)} \\ \hline \end{array}$

D $\begin{array}{r} 11_{(4)} \\ \times 2_{(4)} \\ \hline \end{array}$ **E** $\begin{array}{r} 23_{(4)} \\ \times 1_{(4)} \\ \hline \end{array}$ **F** $\begin{array}{r} 13_{(4)} \\ \times 2_{(4)} \\ \hline \end{array}$



5. Thinking about addition will help you find these differences.

A $\begin{array}{r} 3_{(4)} \\ -2_{(4)} \\ \hline \end{array}$ **B** $\begin{array}{r} 10_{(4)} \\ -2_{(4)} \\ \hline \end{array}$ **C** $\begin{array}{r} 11_{(4)} \\ -2_{(4)} \\ \hline \end{array}$ **D** $\begin{array}{r} 12_{(4)} \\ -2_{(4)} \\ \hline \end{array}$ **E** $\begin{array}{r} 20_{(4)} \\ -2_{(4)} \\ \hline \end{array}$ **F** $\begin{array}{r} 31_{(4)} \\ -2_{(4)} \\ \hline \end{array}$

Let's explore some old numerals.

Investigating the Ideas

This table shows three kinds of numerals used long ago. Roman numerals are still used today. Study the table and the examples.

EGYPTIAN	GREEK			ROMAN
1 = 𓏺	1 = α	7 = ζ	40 = μ	1 = I
10 = 𓎆	2 = β	8 = η	50 = ν	5 = V
100 = 𓍢	3 = γ	9 = θ	60 = ξ	10 = X
1000 = 𓆼	4 = δ	10 = ι	70 = ο	50 = L
1,000,000 = 𓁨	5 = ε	20 = κ	80 = π	100 = C
	6 = ϛ	30 = λ	90 = ϙ	500 = D
		100 = ρ		1000 = M
126 = 𓍢𓎆𓎆𓏺𓏺𓏺𓏺𓏺𓏺		126 = ρκϛ		126 = CXXVI

? Can you choose another number and write the numeral for it in each of the three systems?

Discussing the Ideas

1. Which numeration system would be harder to learn, the Egyptian or the Greek system? Why?

2. Does the Egyptian system have place value like our system?

3. The Roman system **does not** use place value. When two Roman symbols are placed side by side, we **add** or **subtract** to determine the number represented. For example, when the smaller number is represented on the right, we add.

   VI means 5 + 1, or 6.

   When the smaller number is represented on the left, we subtract.

   IV means 5 − 1, or 4.

   Explain what number each of these Roman numerals shows.

A IX	C XL	E CD	G MC	I DIX
B XI	D LX	F DC	H CM	J MCM

1. Write the Egyptian numeral for each of the following.
   A 14 B 23 C 172 D 1974 E 1 002 653

2. Give the ordinary numeral for each Egyptian numeral.

3. Write the Greek numeral for each of the following.
   A 32 B 58 C 181 D 95 E 199

4. Give the ordinary numeral for each Greek numeral.
   A πγ B ρλδ C ρπε D ϙη E ρξϛ

5. Write a Roman numeral for each of the following.
   A 12 B 24 C 29 D 58 E 149 F 3624

6. Give the ordinary numeral for each Roman numeral.
   A XIII B IX C XIX D LIV E CDIX

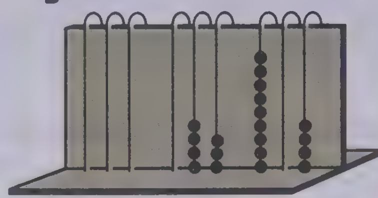
7. Give the Roman numerals for the numbers
   A 1 through 20.
   B 10 through 100 (counting by tens).
   C 100 through 1000 (counting by hundreds).

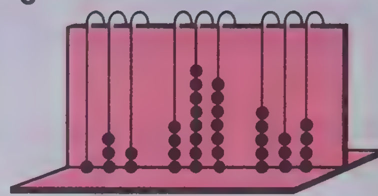
think

With ten toothpicks, "write" as many Roman-numeral equations as you can.

1. Give the numeral for each picture.

A


B


C


2. Solve these equations.

A $3458 = 3000 + 400 + t + 8$

B $7391 = 7000 + 300 + 90 + s$

C $1509 = 1000 + y + 9$

D $6854 = z + 800 + 50 + 4$

E $8006 = m + 6$

F $4472 = 4000 + x + 70 + 2$

3. For each number write an equation as in the example.
(Example: $2734 = 2000 + 700 + 30 + 4$)

A 6218
B 7466
C 3975
D 9218
E 84 721
F 76 097
G 2 796 458
H 34 681 075

4. Give the correct sign ($<$ or $>$) for each ●.

A 57 ● 47
B 74 ● 75
C 342 ● 362
D 324 ● 326
E 236 ● 234
F 623 ● 432
G 651 ● 703
H 8426 ● 8326
I 7521 ● 7512
J 9236 ● 8236
K 6421 ● 6399
L 7846 ● 5399
M 9696 ● 9710
N 646 387 ● 645 999
O 6 287 512 ● 6 300 000

5. Give the correct sign ($<$, $=$, or $>$) for each ●.

A 28 260 ● 20 000 + 8000 + 200 + 60

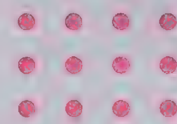
B 45 024 ● 40 000 + 5000 + 200 + 40

C 675 800 ● 600 000 + 70 000 + 5000 + 80

6. Give the base-four numeral for each set.

A


B


C


D


7. Give the base-ten numeral for each exercise.
A $21_{(4)}$ B $31_{(4)}$ C $13_{(4)}$ D $22_{(4)}$ E $10_{(4)}$ F $2_{(4)}$

8. Write the Roman numeral for each of the following.
A 1000 B 500 C 90 D 3

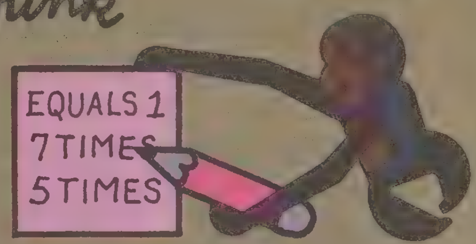
9. Combine the Roman numerals from exercise 8 to write 1593.

10. Write a Roman numeral for each of these.
A 243 C 1256 E 1974
B 594 D 2349 F 1492

11. Give the Roman numeral for each number in exercise 7.

12. What is the largest 4-digit number that uses only the digits 3, 5, and 7?

think



Find a four-digit number so that the number of hundreds plus the number of thousands is one, the number of tens is seven times the number of thousands, and the number of ones is five times the number of hundreds.

You are invited to explore

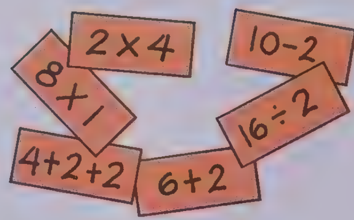
ACTIVITY CARD 2
Page 334

3 Equations and Operations

Let's explore number symbols.

Investigating the Ideas

Each of these slips of paper shows a symbol for the number 8.



Use these digits ⟶ 1 4 2 6 3 9 8

and these signs. ⟶ + − × ÷

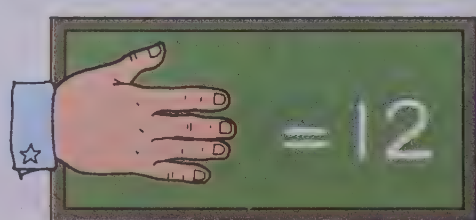
? How many slips of paper can you make that show symbols for the number 6?

Discussing the Ideas

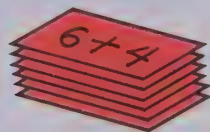
1. Study the chart below. Then give some equations by using the symbols you made that show 6.

We think:	13 is a symbol for the number thirteen. 8 + 5 is a symbol for the number thirteen.
We write an equation:	$8 + 5 = 13$

2. Give some symbols that might be covered in the figure.



1. Each stack of six cards has symbols for just one number. Write what might be on the other 5 cards.

A 

B 

C 

2. An equation for the top card in exercise 1A is $6 + 4 = 10$. Write an equation for each of your answers in exercise 1.

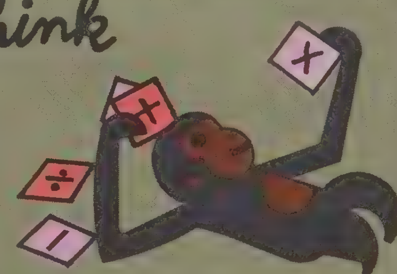
3. Mark **T** (True) or **F** (False) for each statement.

A $4 + 3 = 12$
B $3 \times 4 = 12$
C $3 + 3 + 3 = 12$
D $6 \times 6 = 12$
E $6 \times 2 = 12$
F $12 \div 1 = 12$
G $5 + 7 = 7 + 5$
H $5 \times 7 = 7 \times 5$
I $8 - 3 = 3 - 8$
J $8 + 9 = 9 \times 8$
K $7 \times 0 = 7 + 0$
L $23 \div 1 = 23 \times 1$
M $(5 + 9) + 7 = 5 + (9 + 7)$
N $(8 - 3) - 2 = 8 - (3 - 2)$
O $(4 \times 3) \times 2 = 4 \times (3 \times 2)$
P $3 \times 3 = (2 \times 4) + 1$
Q $5 \times 5 = (4 \times 6) + 1$
R $87 + 5 = 85 + 7$

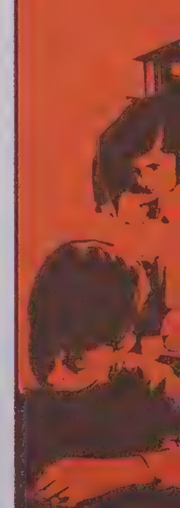
★ **4.** Give symbols for ten different numbers. In each symbol, use each of the three digits "3," "6," and "9" only once. You may use parentheses and one or more of the signs, +, ×, −, or ÷.

Examples: $(3 \times 6) \div 9$
$(9 + 3) - 6$

think

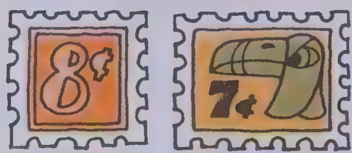


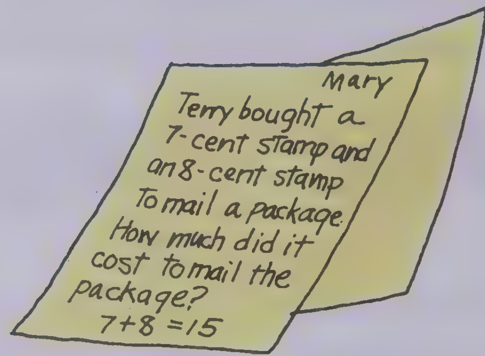
Give two different symbols for ninety. For each symbol use the digit "9" three times. You may use parentheses and any of the signs, +, ×, −, ÷, that you need.



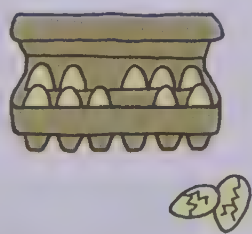
Investigating the Ideas

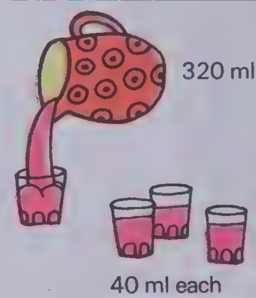
Mary wrote and solved this problem for the stamp picture below.





? Can you write and solve a problem for each picture below?





Discussing the Ideas

1. Make up a problem using one of the operations (+, −, ×, ÷) and tell it to the class.

2. Suppose each ||||| covers a numeral. Which operation (+, −, ×, ÷) would you use to solve this problem?

   ||||| boy scouts now.
   ||||| boy scouts needed for a full troop.
   How many more needed?

3. Make up a problem that uses two operations and tell it to the class.

Use **A, S, M,** or **D** to tell which operation or operations (**A**ddition, **S**ubtraction, **M**ultiplication, **D**ivision) you would use to find the answers if numbers were given.

1. ||||| girls' bikes. ||||| boys' bikes. How many bikes?

2. ||||| girls in the choir. ||||| moved away. How many girls in the choir now?

3. Bob, ||||| years old. Don, ||||| years old. How much older is Bob than Don?

4. ||||| chocolate cookies in each box. ||||| boxes. How many cookies?

5. Coin collection. ||||| coins in all. ||||| on each page. How many pages?

6. Hiked ||||| km in the morning, ||||| km in the afternoon. How many km?

7. ||||| skirts. ||||| sweaters. How many different outfits?

8. Butterfly collection. ||||| butterflies. ||||| boxes. How many in each box?

9. Had ||||| hockey cards. Gave ||||| away. Bought |||||. How many cards now?

10. ||||| nickels. How many cents?

11. ||||| nickels. ||||| dimes. How many cents?

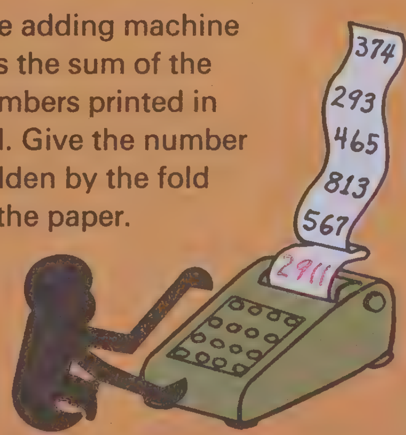
12. ||||| large bottles of pop. ||||| ml in each. ||||| small bottles of pop. ||||| ml in each. How many ml in all?

13. Have ||||| cents. How many |||||-cent stamps can you buy?

14. ||||| green marbles. ||||| red marbles. Each boy gets ||||| marbles. How many boys?

think

The adding machine has the sum of the numbers printed in red. Give the number hidden by the fold in the paper.



More practice, page A-3, Set 5

Investigating the Ideas

You might think of this picture as showing

$$3 \times 4 = 12$$

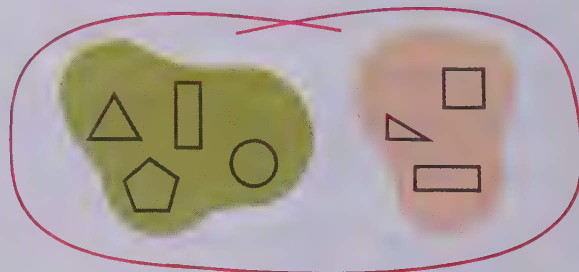
or

$$4 + 4 + 4 = 12.$$

3 sets of 4

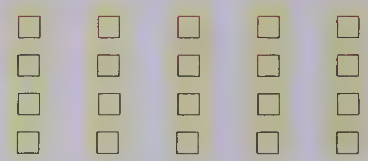
Can you use a set of 12 objects and show some other arrangements?	Write an addition, subtraction, multiplication, or division equation for each of your arrangements.

Discussing the Ideas



1. Give the two addition and two subtraction equations suggested by these sets of figures.

2. Give one addition, one multiplication, and one division equation for these sets.



5 sets of 4

3. Rewrite each equation. Use an operation different from the one given. (Answer to A: $18 \div 6 = 3$)

   A $3 \times 6 = 18$

   B $7 \times 3 = 21$

   C $2 + 2 + 2 + 2 = 8$

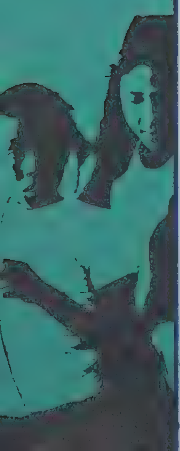
   D $6 \div 2 = 3$

   E $5 \times 6 = 30$

   F $8 + 8 + 8 = 24$

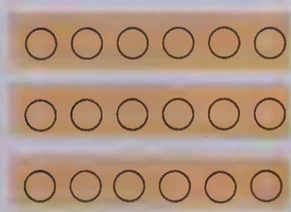
   G $\begin{array}{r} 24 \\ -6 \\ \hline 18 \\ -6 \\ \hline 12 \\ -6 \\ \hline 6 \\ -6 \\ \hline 0 \end{array}$

   H $\begin{array}{r} 15 \\ -5 \\ \hline 10 \\ -5 \\ \hline 5 \\ -5 \\ \hline 0 \end{array}$

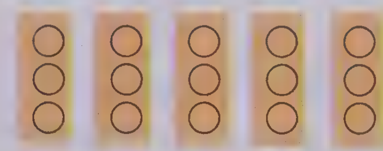


1. Write an addition and a multiplication equation for each set.

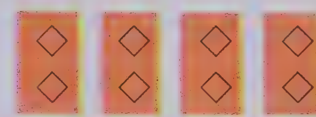
A



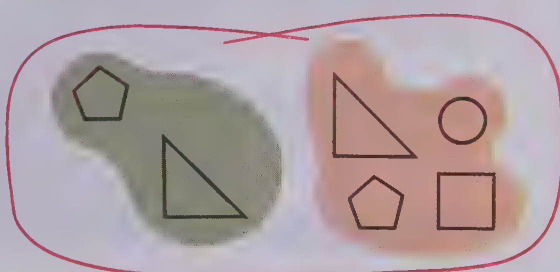
B



2. Write one multiplication and one division equation for this set.



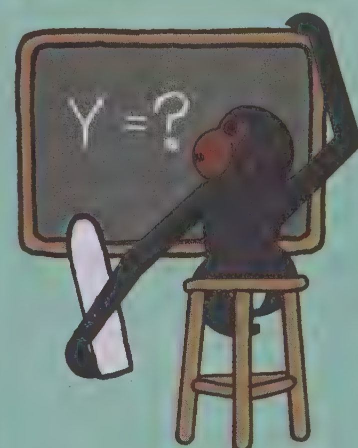
3. Write two addition and two subtraction equations for this set.



4. Match each equation in the first column with an equation having the same solution in the second column.

A $7 - 3 = n$	1 $18 \div 9 = n$
B $8 + 8 = n$	2 $n + 3 = 7$
C $n \times 9 = 18$	3 $13 - 7 = n$
D $15 - 8 = n$	4 $25 \div 5 = n$
E $n \times 7 = 21$	5 $3 \times 3 = n$
F $n + 7 = 13$	6 $n + 8 = 15$
G $3 + 3 + 3 = n$	7 $2 \times 8 = n$
H $n \times 5 = 25$	8 $21 \div 7 = n$

think

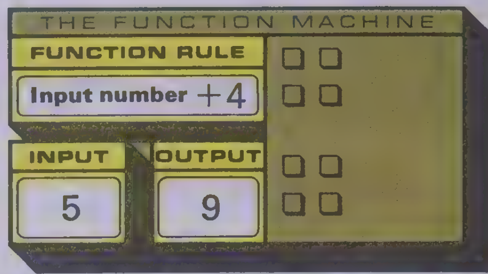


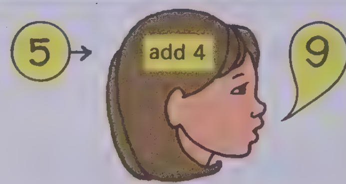
Give a number for ***y*** so that the equation $y \times y = y + y$ will be true.

Find another such number.

More practice, page A-3, Set 6

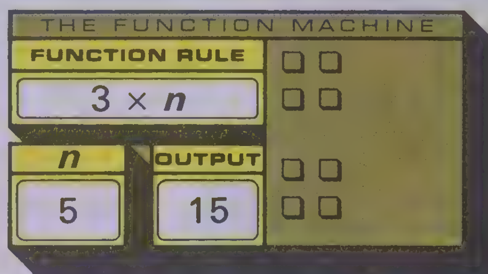
Discussing the Ideas





1. You can think of yourself as a function machine. Study the picture. Then use the rule to give the output for each of these input numbers.

   A 3 E 0
   B 6 F 9
   C 8 G 12
   D 10 H 18



2. In how many ways is this function machine different from the one above? Use this new function rule to give output numbers for each of these inputs.

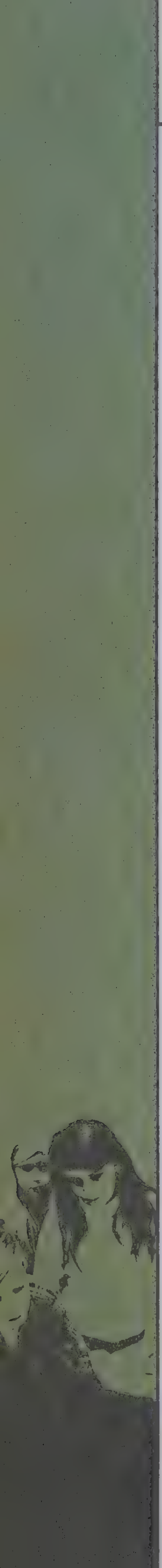
   A 2 E 0
   B 4 F 6
   C 7 G 1
   D 3 H 10

3. Sometimes we write a function rule like this:

   Output number $= n - 2$

   We can use a table to record input and output numbers. Give the missing numbers.

Function Rule	
$n - 2$	
n Input number	Output number
12	10
5	3
7	A
4	B
10	C



Using the Ideas

For exercises 1 through 6, think about the function machine and complete the function tables.

1. Function Rule: $n + 6$

Input number	Output number
4	A
6	B
8	C
18	D
9	E

2. Function Rule: $n - 1$

Input number	Output number
3	A
8	B
9	C
19	D
29	E

3. Function Rule: $8 + n$

Input number	Output number
2	A
5	B
7	C
27	D
47	E

4. Function Rule: $2 \times n$

n	Output
3	A
B	14
6	C
10	D
E	12
F	40

5. Function Rule: $n + n + 1$

n	Output
4	A
7	B
9	C
D	17
21	E
F	101

6. Function Rule: A

n	Output
5	15
0	10
10	20
50	B
C	40
37	D

For exercises 7, 8, and 9, give the output for each input number, A, B, C, and D.

★ **7.** output $= \begin{cases} n + 10, \text{ if } n \text{ is even} \\ n + 5, \text{ if } n \text{ is odd} \end{cases}$ A 2 B 13 C 0 D 20

★ **8.** output $= \begin{cases} 3 \times n, \text{ if } n < 10 \\ n + 100, \text{ otherwise} \end{cases}$ A 6 B 30 C 7 D 100

★ **9.** output $= \begin{cases} n \times 10, \text{ if } n < 10 \\ n - 10, \text{ if } n > 9 \end{cases}$ A 7 B 18 C 4 D 36

Investigating the Ideas

AUTOMATIC 8 MULTIPLIER and DIVIDER

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +	8 +
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128

Use the Automatic 8 Multiplier and Divider to find these products and quotients.

A 9×8 B 13×8 C $104 \div 8$ D $120 \div 8$ E 16×8

? Can you make and use an automatic multiplier and divider for a number other than 8?

Discussing the Ideas

1. Solve these equations. Use the 8 Multiplier.

A $7 \times 8 = n$ D $8 \times 12 = t$ G $136 \div 8 = m$

B $17 \times 8 = y$ E $14 \times 8 = s$ H $128 \div 8 = b$

C $8 \times 15 = a$ F $96 \div 8 = q$ I $88 \div 8 = u$

2. How can you figure out the product 18×8 by looking at the 8 Multiplier?

3. The Automatic Multiplier uses **repeated addition** to find products and quotients. The example below uses **repeated subtraction** (subtracting one 6 at a time) to find quotients.

$42 \div 6 = n \longrightarrow 42 - 6) - 6) - 6) - 6) - 6) - 6) - 6 = 0$

A How many sixes were subtracted to get from 42 to 0?

B What is the quotient for $42 \div 6$?

C Subtract one 3 at a time to find $51 \div 3$.

1. Use this Automatic 13 Multiplier to find the products and quotients.

1	2	3	4	5	6	7	8	9	10	11	12	13
13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +	13 +
13	26	39	52	65	78	91	104	117	130	143	156	169

A $5 \times 13 = r$
B $8 \times 13 = f$
C $13 \times 4 = n$
D $12 \times 13 = a$
E $13 \times 9 = s$
F $13 \times 11 = d$
G $117 \div 13 = b$
H $52 \div 13 = m$
I $91 \div 13 = t$
J $143 \div 13 = y$
K $104 \div 13 = w$
L $156 \div 13 = c$

2. Find each sum and product.

A $6 + 6 + 6 + 6 = n$
$4 \times 6 = n$

B $4 + 4 + 4 + 4 + 4 = r$
$5 \times 4 = r$

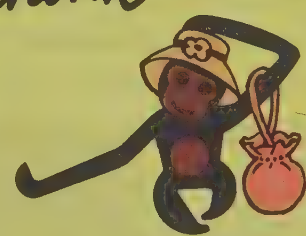
C $9 + 9 + 9 + 9 = t$
$4 \times 9 = t$

D $11 + 11 + 11 + 11 = m$
$4 \times 11 = m$

3. Write a division equation for each correct repeated subtraction. Two are incorrect.

A $30 - 6 - 6 - 6 - 6 - 6 = 0$
B $36 - 9 - 9 - 9 - 9 = 0$
C $48 - 8 - 8 - 8 - 8 - 8 = 0$
D $24 - 6 - 6 - 6 - 6 = 0$
E $28 - 7 - 7 - 7 - 7 - 7 = 0$
F $48 - 12 - 12 - 12 - 12 = 0$

think



Pam told Sam, "If you give me a nickel, we will each have the same amount of money."

1. What is the smallest amount of money they could have started with?
2. If they have 36¢ in all, how much money did Pam start with?

Investigating the Ideas

You can think of Addends and a Sum in both addition and subtraction.

$2 + 3 = 5$ | $5 - 3 = 2$

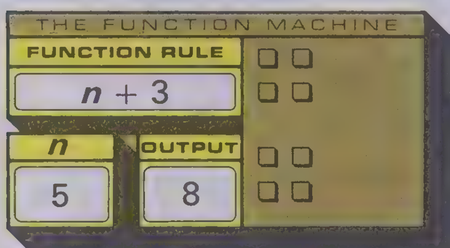
You can think of Factors and a Product in both multiplication and division.

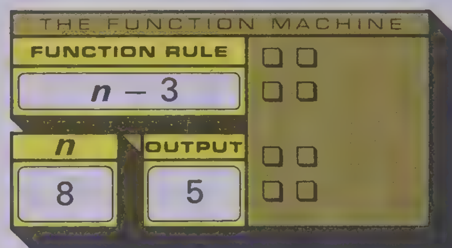
$2 \times 3 = 6$ | $6 \div 3 = 2$

? How many different equations can you write using 6 and 4 when both are addends **or** both are factors?

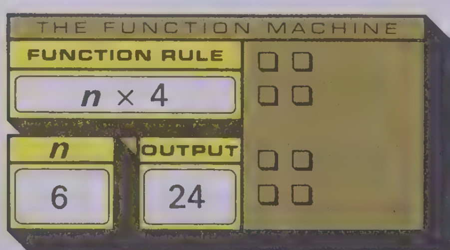
Discussing the Ideas

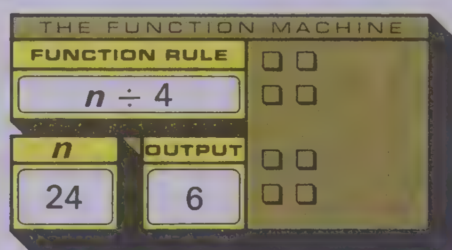
1. The two function machines help you see how addition and subtraction are related. Which numbers are addends and which is the sum?





2. These two function machines help you see how multiplication and division are related. Which numbers are factors and which is the product?





1. Find the missing addend in the addition equation. Then write the subtraction equation with the correct difference.

A $n + 7 = 12$ $12 - 7 = n$	C $r + 8 = 15$ $15 - 8 = r$	E $b + 5 = 13$ $13 - 5 = b$	G $t + 18 = 24$ $24 - 18 = t$
B $a + 6 = 13$ $13 - 6 = a$	D $s + 9 = 14$ $14 - 9 = s$	F $n + 9 = 17$ $17 - 9 = n$	H $q + 27 = 36$ $36 - 27 = q$

2. Find the missing factor in the multiplication equation. Then write the division equation with the correct quotient.

A $n \times 4 = 32$ $32 \div 4 = n$	C $n \times 5 = 45$ $45 \div 5 = n$	E $n \times 3 = 27$ $27 \div 3 = n$	G $a \times 6 = 54$ $54 \div 6 = a$
B $t \times 6 = 42$ $42 \div 6 = t$	D $n \times 8 = 40$ $40 \div 8 = n$	F $p \times 8 = 8$ $8 \div 8 = p$	H $b \times 8 = 48$ $48 \div 8 = b$

3. In **multiplication** we find the product (P) of the factors (F). → $\overset{F}{6} \times \overset{F}{4} = \overset{P}{n}$

In **division** we find one of the factors of the product. → $\overset{P}{24} \div \overset{F}{4} = \overset{F}{n}$ $\quad \overset{P}{24} \div \overset{F}{6} = \overset{F}{n}$

Find the missing product or factor.

A $\overset{F}{6} \times \overset{F}{9} = \overset{P}{n}$

B $\overset{P}{54} \div \overset{F}{9} = \overset{F}{n}$

C $\overset{P}{54} \div \overset{F}{n} = \overset{F}{9}$

D $\overset{F}{n} \times \overset{F}{6} = \overset{P}{42}$

E $\overset{P}{42} \div \overset{F}{6} = \overset{F}{n}$

F $\overset{P}{42} \div \overset{F}{n} = \overset{F}{6}$

4. Which equation has no solution, which has many, and which has just one?

A $n \times 0 = 0$ C $n \times 0 = 7$

B $n \times 5 = 0$

5. Use exercise 4 to explain why we **do not divide by zero.**

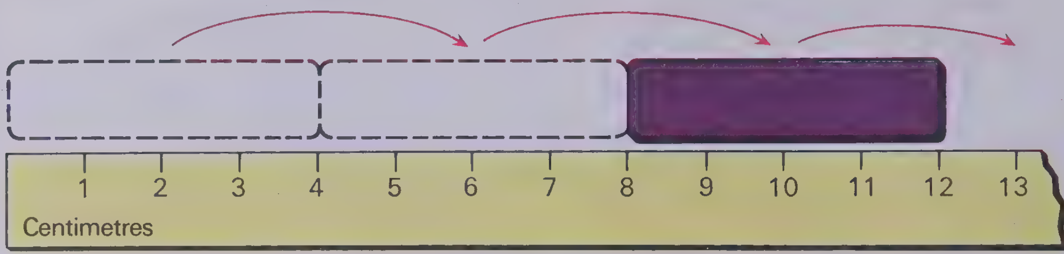
think

In a magic square the row, column, and diagonal sums are all the same. Give the missing numbers so that the square will be a magic square.

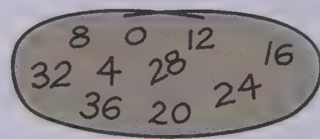
19	5	6	16
8	14		11
	10	9	
7		18	4

More practice, page A-4, Set 7

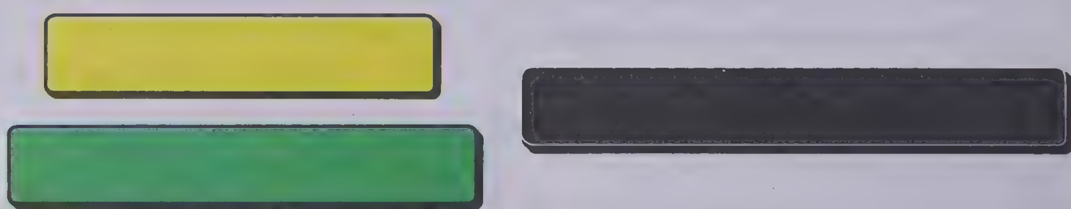
Investigating the Ideas



Can you tell what is "special" about this set of numbers?



Can you make a "special" set of numbers for each of these strips?



Discussing the Ideas

1. Give the "special" sets of numbers for 8 and 9.

2. It is important for you to be able to recall addition, subtraction, multiplication, and division facts quickly. Try these.

A $6 + 5$	G $15 - 7$	M 6×7	S $42 \div 6$
B $7 + 5$	H $14 - 7$	N 5×8	T $54 \div 9$
C $9 + 8$	I $17 - 9$	O 6×9	U $48 \div 8$
D $8 + 7$	J $16 - 8$	P 8×6	V $36 \div 4$
E $8 + 8$	K $15 - 9$	Q 9×5	W $72 \div 8$
F $6 + 9$	L $13 - 6$	R 8×9	X $49 \div 7$

1. Can you find these sums in $1\frac{1}{2}$ minutes?

A $3 + 9$	G $4 + 6$	M $7 + 9$	S $7 + 4$
B $9 + 3$	H $9 + 5$	N $5 + 1$	T $2 + 3$
C $7 + 7$	I $8 + 2$	O $6 + 8$	U $9 + 8$
D $9 + 1$	J $4 + 0$	P $8 + 3$	V $8 + 5$
E $8 + 6$	K $5 + 9$	Q $5 + 4$	W $8 + 7$
F $2 + 8$	L $2 + 9$	R $5 + 7$	X $7 + 3$

2. Try these in 2 minutes.

A $12 - 7$	G $9 - 6$	M $15 - 8$	S $16 - 8$
B $11 - 6$	H $13 - 6$	N $16 - 7$	T $8 - 6$
C $12 - 5$	I $16 - 9$	O $15 - 6$	U $12 - 4$
D $7 - 2$	J $11 - 5$	P $9 - 8$	V $17 - 9$
E $13 - 7$	K $13 - 9$	Q $12 - 8$	W $14 - 8$
F $12 - 6$	L $6 - 4$	R $14 - 5$	X $5 - 3$

3. How many of these products can you find in 2 minutes?

A 3×4	G 0×3	M 9×4	S 9×8
B 6×6	H 7×9	N 8×8	T 8×5
C 3×2	I 4×9	O 9×6	U 6×3
D 7×6	J 1×5	P 7×4	V 3×5
E 2×9	K 5×6	Q 3×8	W 8×7
F 7×7	L 5×8	R 8×2	X 8×9

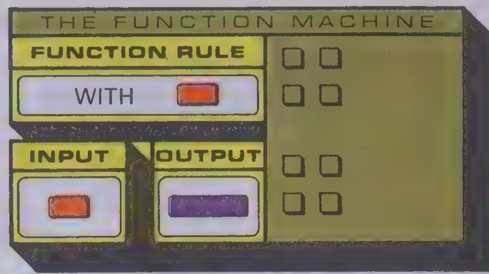
4. Find as many of these as you can in 3 minutes.

A $12 \div 3$	G $63 \div 9$	M $27 \div 9$	S $18 \div 6$
B $20 \div 4$	H $18 \div 2$	N $49 \div 7$	T $5 \div 5$
C $24 \div 4$	I $64 \div 8$	O $24 \div 3$	U $30 \div 5$
D $24 \div 8$	J $15 \div 5$	P $28 \div 4$	V $42 \div 6$
E $35 \div 5$	K $30 \div 6$	Q $21 \div 7$	W $48 \div 6$
F $9 \div 1$	L $36 \div 6$	R $56 \div 8$	X $32 \div 8$

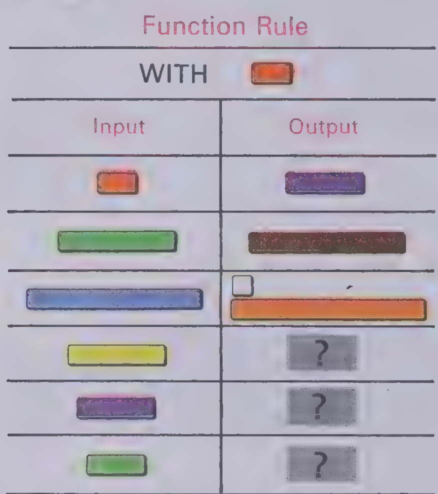
More practice, page A-4, Set 8

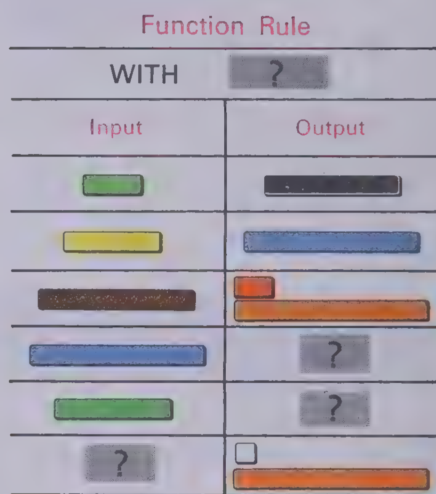
Investigating the Ideas

This is a special function machine that uses the strips instead of numbers.



? Can you use your strips to help you find the missing strips in the tables?





Discussing the Ideas

1. Can you find the function rule for each of these tables?

2. Invent your own function rule. List a set of input and output numbers and see if some of your classmates can find your rule.

Function Rule: ?

n	Output
256	2
1475	4
5000	0
12 763	7
46	0

Function Rule: ?

n	Output
2	4
4	16
1	1
7	49
5	25

Find the missing numbers and function rules.

1.

Function Rule	
$n + 7$	
n	Output
9	A
6	B
7	C
D	12
E	15

2.

Function Rule	
$n + 5$	
n	Output
4	A
6	B
8	C
D	25
E	45

3.

Function Rule	
$n \times 7$	
n	Output
8	A
6	B
5	C
D	28
E	63

4.

Function Rule	
A	
n	Output
5	30
1	6
4	24
6	B
C	48
D	54

5.

Function Rule	
A	
n	Output
6	32
3	17
9	47
5	27
8	B
4	C

6.

Function Rule	
A	
n	Output
2	7
3	10
5	16
1	4
6	B
10	C

7.

Function Rule	
A	
n	Output
2	3
4	15
1	0
7	48
5	B
6	C

★ **8.**

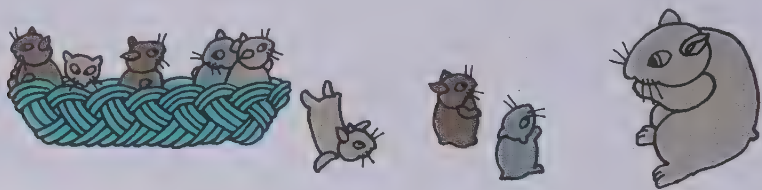
Function Rule	
A	
n	Output
7	10
12	20
39	40
24	30
16	B
87	C

★ **9.**

Function Rule	
A	
n	Output
🟢	🔵
🟠	🟡
🟡	🟤
🟢	🟢
🟣	B
⚫	C

Solving Equations

1. Write an equation for each problem. Solve the equation.
   - A Jeff scored 9 points in the first half of the game. He scored 7 points in the last half. How many points did he score?
   - B Sally bought 4 hamsters. Each of her hamsters had 8 baby hamsters. How many hamsters does Sally have now?



2. To give the number for n in an equation such as $n + 3 = 7$, we write $n = 4$. Solve the equations. The solution is given for exercise A.

A $n + 2 = 9$ $n = 7$	E $8 + 9 = n$	J $32 = n \times 4$	O $n + 9 = 16$
	F $8 \times 9 = n$	K $32 = n + 4$	P $n \times 7 = 63$
B $n \times 3 = 6$	G $n \times 6 = 24$	L $n \times 6 = 54$	Q $40 = 8 \times n$
C $18 = n + 9$	H $6 + n = 20$	M $n + 9 = 14$	R $49 = 7 \times n$
D $5 \times n = 20$	I $n + 8 = 24$	N $6 \times n = 48$	S $15 = 9 + n$

3. We often use other letters in equations in the same way we have been using the letter n. Solve the equations. The solution is given for exercise A.

A $a \times 4 = 36$ $a = 9$	E $t \times 7 = 42$	J $54 = s \times 9$	O $8 + q = 12$
	F $y + 7 = 16$	K $r + 6 = 15$	P $6 \times r = 6$
B $n + 7 = 13$	G $64 = 8 \times n$	L $9 \times 0 = n$	Q $n \times 6 = 0$
C $9 + c = 20$	H $14 = 6 + x$	M $0 \times 7 = n$	R $7 + s = 7$
D $8 \times 7 = s$	I $b + 7 = 13$	N $9 + 0 = y$	S $7 \times t = 28$

4. Solve the equations.

A $5 + (8 + 7) = n$	F $5 + (3 \times 4) = r$	K $(3 \times 4) + (5 \times 4) = q$
B $(5 + 8) + 7 = n$	G $(5 + 3) \times 4 = s$	L $8 \times 4 = s$
C $(9 + 7) + 8 = x$	H $(5 + 4) \times 3 = a$	M $(5 \times 6) + (4 \times 6) = n$
D $9 + (7 + 8) = x$	I $3 \times (5 + 2) = t$	N $9 \times 6 = t$
E $(9 + 8) + 7 = q$	J $(3 \times 5) + 2 = y$	O $5 \times 0 = a + 0$

5. In her rock collection, Sue had 6 boxes with 5 rocks in each box. She found 3 more rocks. How many rocks did she have in all? Write an equation and solve it.

6. Solve the equations.

A $(4 \times 6) + 7 = n$	H $(6 \times 2) + q = 20$	O $(n \times 4) + 3 = 23$
B $(3 \times 7) + 8 = y$	I $(3 \times 8) + t = 30$	P $(y \times 6) + 5 = 41$
C $(8 \times 3) + 9 = a$	J $(4 \times 7) + r = 34$	Q $(r \times 9) + 5 = 50$
D $(6 \times 3) + 6 = c$	K $(3 \times 9) + n = 35$	R $(t \times 8) + 3 = 51$
E $(9 \times 3) + 6 = d$	L $(6 \times 7) + b = 42$	S $(n \times 7) + 6 = 62$
F $(8 \times 2) + 8 = s$	M $(8 \times 9) + a = 81$	T $(b \times 6) + 5 = 59$
G $(7 \times 6) + 9 = t$	N $(9 \times 6) + x = 60$	U $(c \times 7) + 8 = 64$

7. The same letter is used more than once in the equations below. As in exercise A, give the number that will make the equation true.

A $n + n = 16$
  $n = 8$
B $a \times a = 16$
C $y + y = 18$
D $y \times y = 81$
E $t + t = 64$
F $5 + n + n = 9$
G $x + x + 7 = 15$
H $r \times r \times r = 27$
I $s + s + s = 12$

think

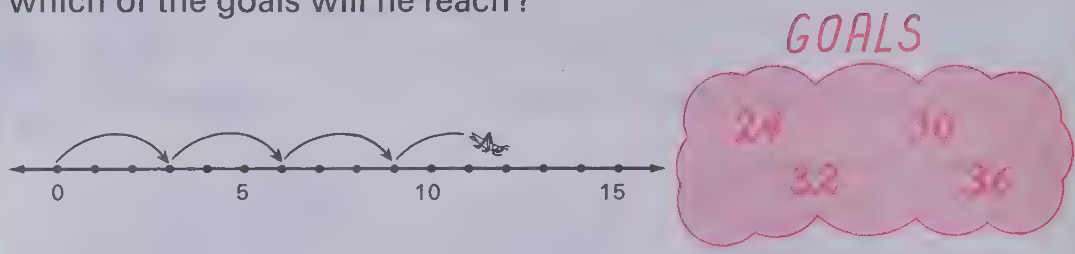
Give the number for *a* and the number for *b* in each exercise. The pair of numbers you give must serve as both addends and factors to give the correct sum and product.
For exercise A, you could write:

$a = 4, b = 2$

	Sum		$\times$ / $+$		Product
A	6	*a*	$+$ $\times$	*b*	8
B	9	*a*	$+$ $\times$	*b*	20
C	12	*a*	$+$ $\times$	*b*	36
D	13	*a*	$+$ $\times$	*b*	42
E	15	*a*	$+$ $\times$	*b*	56
F	17	*a*	$+$ $\times$	*b*	72

Investigating the Ideas

If the grasshopper keeps jumping by 3's, which of the goals will he reach?



? Can you select one of the goals and find **all** the ways the grasshopper can get there if, on any one trip, his jumps are all the same? (No fractions.)

Discussing the Ideas

1. Give a multiplication equation for each "trip" you found in the Investigation.

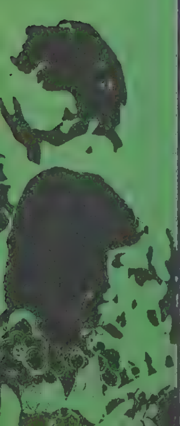
2. Study the new symbols. Then give the landing point for each part.

A 2 ③→ means: Start at 2. Jump 3 units to the **right.**

B 9 ←⑤ means: Start at 9. Jump 5 units to the **left.**

C 6 ③→ ←④ means: Start at 6. Jump 3 units to the **right**; then jump 4 units to the **left.**

D 12 ←⑥←⑥ means: Start at 12. Jump 6 units to the **left**; then jump 6 more units to the **left.**



1. Give the landing point for each exercise.

A 2 4→	F 5 6→	K 16 8→	P 6 ←6	U 48 5→
B 1 5→	G 7 6→	L 7 9→	Q 16 ←9	V 13 ←5
C 5 1→	H 8 7→	M 17 9→	R 26 ←9	W 37 6→
D 4 5→	I 18 7→	N 9 ←1	S 17 ←8	X 13 ←6
E 5 5→	J 6 8→	O 8 ←4	T 37 ←8	Y 53 ←6

2. Give the landing point for each exercise.

A 2 1→ 3→	E 6 4→ ←1	I 6 4→ 4→ ←8
B 7 3→ 5→	F 5 7→ ←6	J 8 ←8 8→ 8→
C 8 1→ ←1	G 12 ←9 3→	K 13 4→ 6→ ←9
D 9 ←3 3→	H 20 8→ 8→	L 17 8→ ←6 ←9

3. Give the landing point for each exercise.

A 0 4→ 4→ 4→ 4→ 4→	E 0 7→ 7→ 7→ 7→ 7→ 7→ 7→
B 0 3→ 3→ 3→ 3→ 3→ 3→ 3→	F 0 8→ 8→ 8→ 8→ 8→ 8→
C 0 5→ 5→ 5→ 5→ 5→ 5→ 5→ 5→	G 0 9→ 9→ 9→ 9→ 9→ 9→ 9→
D 0 6→ 6→ 6→ 6→ 6→ 6→ 6→ 6→ 6→	H 0 3→ 3→ 3→ 3→ 3→ 3→

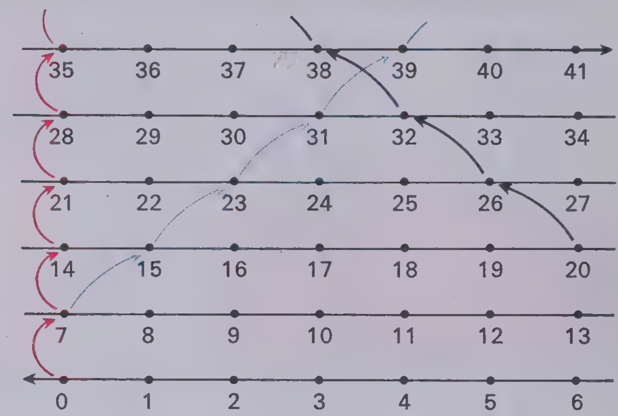
4. Give the missing numbers.

A ||||| 5→ : lands at 14	G ||||| ←7: lands at 6
B ||||| 8→ : lands at 15	H ||||| ←9: lands at 5
C ||||| 9→ : lands at 18	I ||||| 7→ 6→ : lands at 16
D ||||| 7→ : lands at 13	J ||||| 2→ 5→ : lands at 15
E ||||| 6→ : lands at 15	K ||||| ←7 7→ : lands at 54
F ||||| 9→ : lands at 16	L ||||| 9→ ←9 : lands at 73

Can jumps be made on a "number-line stack"?

Investigating the Ideas

Think of a number line that is stacked up in pieces. Imagine as many rows as you need.



? Can you figure out where the 6th jump will land for each color?

Discussing the Ideas

1. Can you find an easy rule to tell where you will be for each type of jump in the Investigation?

2. Can you find the missing landing points and explain those given?

	Symbol	Start	Land
A	4 ①→	4	
B	34 ←①	34	
C	0 ①↑	0	
D	8 ①↓	8	
E	10 ①↗	10	
F	30 ①↘	30	
G	20 ↙①	20	
H	4 ↖①	4	

	Symbol	Start	Land
	18 ②→	18	20
	0 ③↑	0	21
I	28 ③↓	28	
	0 ⑤↗	0	40
J	18 ②↘	18	
K	8 ②→ ③↑	8	
	20 ③→	20	23
★ L	14 ←②	14	
	13 ②↗	13	29
★ M	28 ↙①	28	

1. Using the number-line stack in the Investigation, give the landing point for each exercise.

A 0 ①↑	D 0 ⑤↑	G 0 ⑩↑	J 1 ⑤↑	M 4 ②↑	P 42 ②↓
B 0 ②↑	E 0 ⑥↑	H 0 ⑧↑	K 1 ⑨↑	N 4 ⑥↑	Q 42 ③↓
C 0 ④↑	F 0 ⑨↑	I 1 ②↑	L 1 ⑦↑	O 4 ⑨↑	R 42 ⑥↓

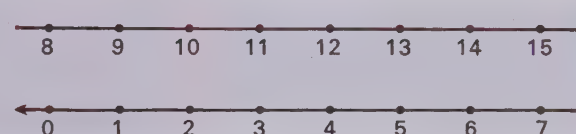
2. Give the landing point for each exercise.

A 24 ⑤↑	D 59 ②↓	G 3 ③→ ⑥↑	J 3 ②→ ②↑ ①↓
B 60 ⑤↑	E 28 ③↓	H 12 ④↑ ④↓	K 30 ←① ②↓
C 100 ⑧↑	F 63 ⑨↓	I 40 ⑤↓ ⑥↑	L 30 ②↓ ←①

★ **3.** Give the landing point for each exercise.

A 0 ⑤↗	D 20 ①↗	G 4 ⑤↗	J 6 ↖⑥	M 7 ⑥→
B 0 ⑥↗	E 4 ②↗	H 1 ⑧↗	K 7 ①↑ ←①	N 0 ↖②
C 27 ①→	F 4 ③↗	I 6 ↖⑤	L 7 ↖①	O 0 ↖⑧

Use a number-line stack like this for exercises 4 and 5.



★ **4.** Give the landing point for each exercise.

A 0 ①↑	C 0 ⑧↑	E 0 ①↗	G 12 ↖① ①↗ ①↘ ↙①
B 0 ⑥↑	D 24 ③↓	F 0 ⑦↗	H 12 ←① ②↑ ②→ ②↓ ←①

★ **5.** Give the missing numbers.

A 16 ◯↓ : lands at 0	D 72 ◯↓ : lands at 0	G 18 ↙◯ : lands at 0
B 32 ◯↓ : lands at 0	E 57 ◯↓ : lands at 1	H 63 ↙◯ : lands at 0
C 48 ◯↓ : lands at 0	F 67 ◯↓ : lands at 3	I 49 ◯↘ : lands at 7

Basic Principles

In this lesson you will review some of the basic principles for addition and multiplication. Special names for the principles are given. For each exercise, copy the equations and give the number for ***n***.

1. **0 principle**

When you choose a whole number and **add 0,** the sum is the number you chose.

A $37 + 0 = n$ B $n + 0 = 86$ C $13 + n = 13$ D $n + 0 = 0$

2. **1 principle**

When you choose a whole number and **multiply by 1,** the product is the number you chose.

A $53 \times 1 = n$ B $64 \times n = 64$ C $n \times 1 = 89$ D $n \times 1 = 0$

3. **Commutative principle, +**
(Order principle, +)

When you add, you can **change the order** of the addends and the sum is the same.

A $9 + 7 = n + 9$ B $n + 93 = 93 + 68$ C $537 + n = 86 + 537$

4. **Commutative principle, ×**
(Order principle, ×)

When you multiply, you can **change the order** of the factors and the product is the same.

A $5 \times n = 7 \times 5$ B $49 \times 51 = n \times 49$ C $90 \times 70 = 70 \times n$

5. **Associative principle, +**
(Grouping principle, +)

When you add, you can **change the grouping** and get the same sum.

A $(5 + 2) + 3 = 5 + (2 + n)$ B $87 + (n + 93) = (87 + 34) + 93$

6. **Associative principle, ×**
(Grouping principle, ×)

When you multiply, you can **change the grouping** and get the same product.

A $(n \times 3) \times 4 = 2 \times (3 \times 4)$ B $34 \times (8 \times 7) = (34 \times 8) \times n$

1. Use part I and one of the basic principles to help you complete part II. Then tell which principles you used.

		I		II
A	Since	$937+685=1622$,	we know that	$685+937=n$.
B	Since	$(18+36)+37=91$,	we know that	$18+(36+37)=n$.
C	Since	$6\times(9\times37)=1998$,	we know that	$(6\times9)\times37=n$.
D	Since	$39\times8=312$,	we know that	$n=8\times39$.
E	Since	$17\times1=17$,	we know that	$44\times1=n$.
F	Since	$43+(39+27)=109$,	we know that	$(43+n)+27=109$.
G	Since	$559+0=559$,	we know that	$n=747+0$.
H	Since	$534+876=1410$,	we know that	$(534+876)\times1=n$.

2. Think about part I. Then use part I and any principles you need to complete part II.

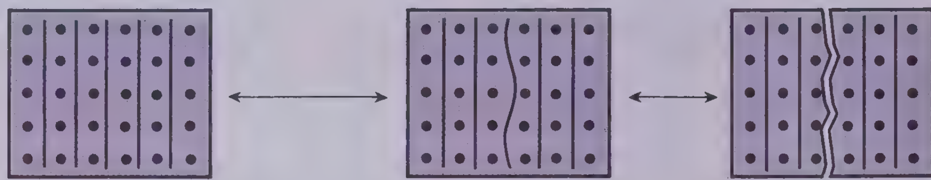
	I		II
A	$23+17=40$	→	$23+64+17=40+n$
B	$16+14=30$	→	$14+51+16=n+51$
C	$25+40=65$	→	$25+6+40=n$
D	$39+47+35=121$	→	$35+39+47=n$
E	$7\times5\times9\times2=630$	→	$5\times2\times7\times9=n$
F	$10\times6=60$ and $8\times5=40$	→	$10\times8\times6\times5=60\times n$
G	$18\times9=162$	→	$(18\times9)\times1=n$
H	$17+18=35$ and $24+36=60$	→	$18+36+17+24=n$
I	$4\times2\times10\times10=8\times100$	→	$4\times10\times2\times10=n$

Do you understand the multiplication-addition principle?

Investigating the Ideas

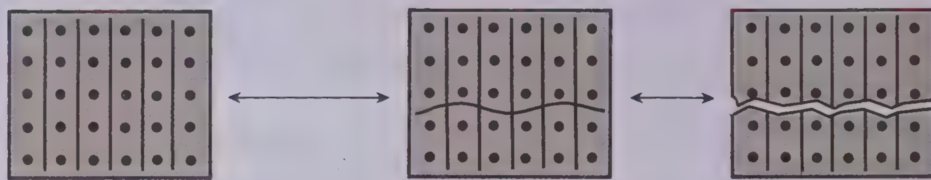
Example A shows how you can think about "breaking apart" the first factor before multiplying. Example B shows how you can think about "breaking apart" the second factor before multiplying.

A



$6 \times 5 = (3 + 3) \times 5 = (3 \times 5) + (3 \times 5)$

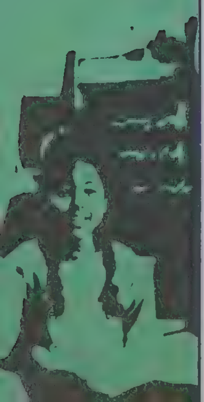
B



$6 \times 5 = 6 \times (3 + 2) = (6 \times 3) + (6 \times 2)$

? Can you draw a set of dots for a multiplication problem?	Show pictures and equations for your problem like the ones above.

Discussing the Ideas



1. The principle illustrated above is called the **multiplication-addition** principle or the distributive principle. Explain how you can use this principle to help you find the product 6×9 if you know 6×5 and 6×4.

2. How can you use the figures to help you solve the equations?

A

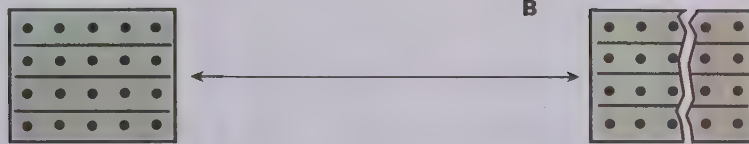
$8 \times (1 + 2) = (8 \times 1) + (8 \times n)$

B

$6 \times 4 = (n \times 4) + (4 \times 4)$

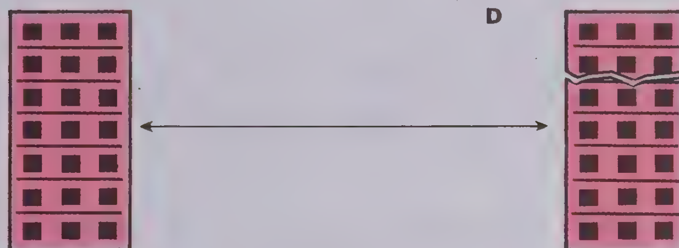
1. Solve the equations.

A | B



$4 \times 5 = a$ | $(4 \times 3) + (4 \times 2) = b$

C | D



$7 \times 3 = c$ | $(2 \times 3) + (5 \times 3) = d$

Solve the equations.

2. $5 \times 8 = (n + 3) \times 8$
3. $4 \times 7 = (2 + n) \times 7$
4. $6 \times 8 = (3 \times 8) + (n \times 8)$
5. $8 \times 7 = (n \times 7) + (3 \times 7)$
6. $6 \times 9 = (n \times 9) + (1 \times 9)$
7. $7 \times 7 = (5 \times 7) + (n \times 7)$
8. $9 \times 8 = (n \times 8) + (4 \times 8)$
9. $5 \times 12 = n \times (10 + 2)$
10. $3 \times 14 = 3 \times (n + 4)$
11. $2 \times 13 = 2 \times (10 + n)$
12. $6 \times 11 = (6 \times n) + (6 \times 1)$
13. $4 \times 16 = (n \times 10) + (4 \times 6)$
14. $7 \times 15 = (7 \times 10) + (7 \times n)$
15. $8 \times 17 = (8 \times n) + (8 \times 7)$
16. $7 \times 23 = (7 \times n) + (7 \times 3)$

think

A number pair are we.
Our product, 54.
We differ by just 3.
We'll tell you nothing more.

WHO ARE WE?

More practice, page A-5, Set 10

1. Solve the equations.

A $x + 9 = 16$	E $8 + n = 17$	I $5 \times b = 5 \times 3$	M $42 \div 7 = r$
B $16 - 9 = n$	F $17 - x = 8$	J $8 \times 7 = r \times 8$	N $8 \times 0 = q$
C $13 = 5 + p$	G $54 - 54 = n$	K $17 \times 1 = b$	O $t \times 9 = 0$
D $13 - 5 = a$	H $4 \times n = 0$	L $n \times 7 = 7$	P $0 \div 9 = s$

2. Solve the equations.

A $7 + 7 + 7 + 7 + 7 = n$
$5 \times 7 = n$

B $30 - 5 - 5 - 5 - 5 - 5 - 5 = n$
$30 \div 5 = n$

3. Solve.

A $8 + 7$	E $15 - 9$	I 5×9	M 7×8	Q 9×8	U $42 \div 7$
B $6 + 5$	F $14 - 6$	J 7×6	N 9×6	R 7×7	V $63 \div 9$
C $9 + 4$	G 7×4	K 9×9	O 8×8	S $40 \div 5$	W $48 \div 6$
D $12 - 8$	H 8×6	L 5×7	P 6×5	T $32 \div 8$	X $56 \div 8$

4. Find the missing output numbers.

Function Rule $(2 \times n) + 1$

n	Output
0	A
2	B
3	C
5	D

Function Rule $(n \times 8) + 5$

n	Output
3	E
5	F
7	G
9	H

Function Rule $(4 \times n) + (3 \times n)$

n	Output
5	I
0	J
8	K

Function Rule $(n \times n) - n$

n	Output
4	L
8	M
10	N

think

For this table, give four different function rules, each of which uses only one of the signs, +, −, ÷, ×.

Function Rule ?

n	Output
1	1
2	2
3	3
4	4
5	5
⋮	⋮

1. Solve the equations.

A $10 \times 10 = n$

B $(10 \times 10) \times 10 = n$

C $(10 \times 10 \times 10) \times 10 = n$

D $(10 \times 10) \times (10 \times 10) = n$

2. Find the products.

A $5 \times (10 \times 10) = n$

B $8 \times (10 \times 10 \times 10) = n$

C $7 \times (10 \times 10) = n$

D $6 \times (10 \times 10 \times 10) = n$

E $8 \times (10 \times 10 \times 10 \times 10) = n$

F $9 \times (10 \times 10 \times 10 \times 10) = n$

G $18 \times (10 \times 10) = n$

H $23 \times (10 \times 10 \times 10) = n$

3. Solve the equations.

A $638 = (6 \times n) + (3 \times 10) + 8$

B $375 = (3 \times 10 \times 10) + (n \times 10) + 5$

C $5671 = (5 \times n) + (6 \times 100) + (7 \times 10) + 1$

D $54\,965 = (n \times 1000) + (9 \times 100) + (6 \times 10) + 5$

E $39\,732 = (39 \times 1000) + (7 \times n) + (3 \times 10) + 2$

F $5472 = (5 \times 10 \times 10 \times 10) + (4 \times n) + (7 \times 10) + 2$

4. Write the numerals for the numbers.

A six thousand three hundred thirty-seven

B fifty-four thousand five hundred twenty

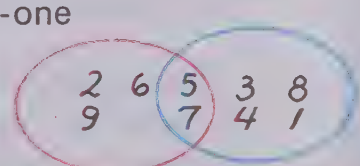
C seven thousand thirty-four

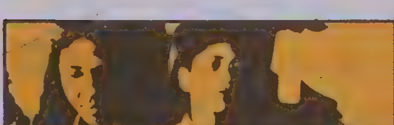
D eighty thousand ninety

E three million nine thousand five

F six billion two hundred thirty million seventy-one

5. Which number is inside the red loop, inside the blue loop, and larger than 6?





You are invited to explore

ACTIVITY CARD 3
Page 334

4 Geometry and Measurement I

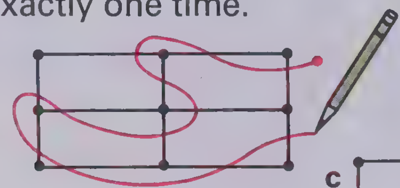
What are points, lines, and segments?

Investigating the Ideas

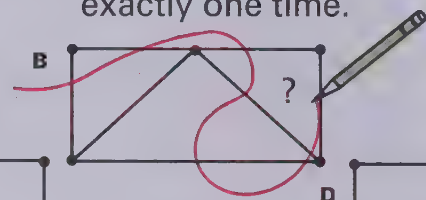
In figure A, a path can be drawn that passes through each segment of the figure exactly one time.

In figure B, **no** path can be found that will cross each segment exactly one time.

A

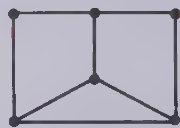


B



Copy figures C, D, and E on your paper.

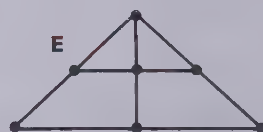
C



D

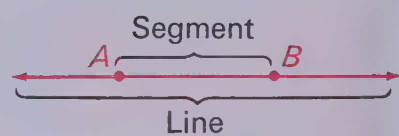
Can you find a path that will cross each segment of each figure exactly one time?

E

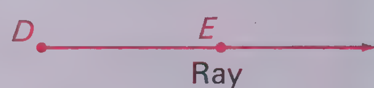


Discussing the Ideas

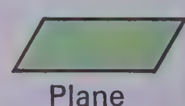
1. The figures in the Investigation show **points** and **segments**. Are points and segments parts of **lines**?

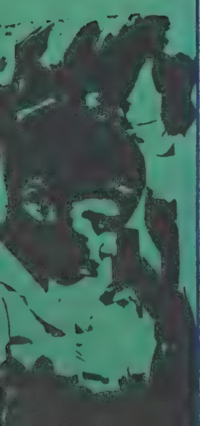


2. How would you describe a segment?
3. Can you name some physical objects that remind you
   A of points? B of segments? C of lines?
4. This is one way we picture rays.
   A How is a ray different from a line?
   B How is it different from a segment?
   C Can you name some things that remind you of rays?



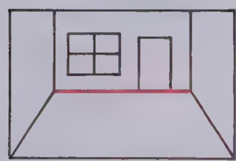
5. Any flat surface suggests a plane. What are some objects that remind you of a plane?



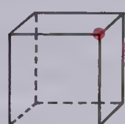


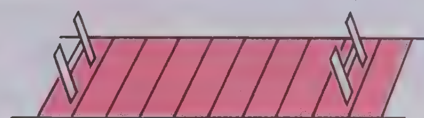
1. Give the geometric figure suggested by each picture.

A 

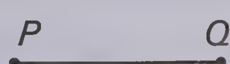
B 

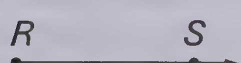
C 

D 

E 

2. Study each of the examples. Then give the symbol for each figure shown in exercises A through G.

A 

B 

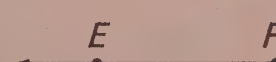
C 

D 

For segment *AB*, we write $\overline{AB}$.

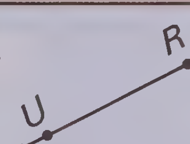
For line *RS*, we write $\overleftrightarrow{RS}$.

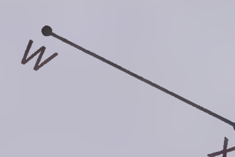
For ray *CD*, we write $\overrightarrow{CD}$.



For ray *FE*, we write $\overrightarrow{FE}$.

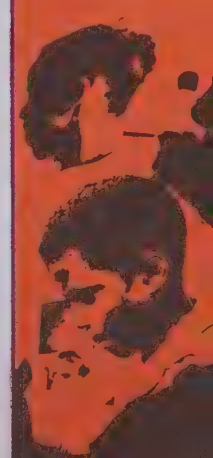
E 

F 

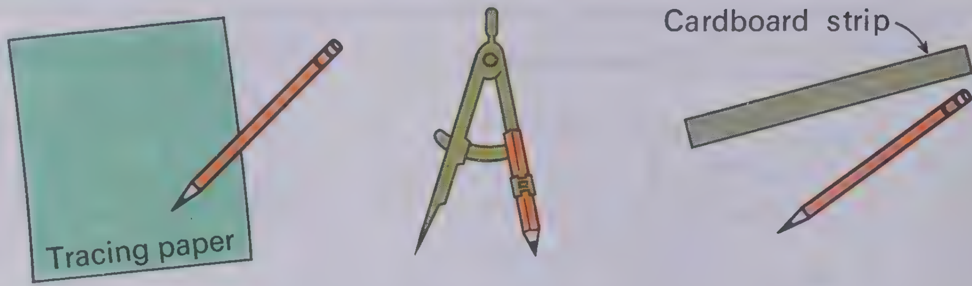
G 

3. Draw and label a picture for each symbol.

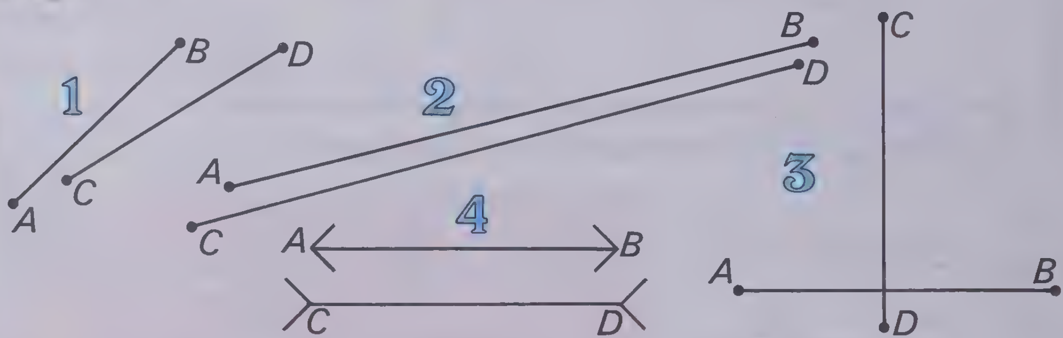
A $\overline{MN}$ B $\overrightarrow{RS}$ C $\overleftrightarrow{PQ}$ D $\overrightarrow{XY}$ E $\overline{FG}$



Investigating the Ideas



? Can you find a way to use each of the devices shown above to tell which segment in each pair is longer (if one is)?



Discussing the Ideas

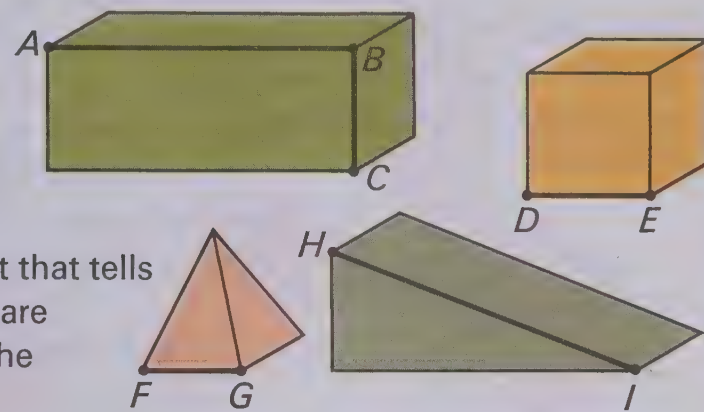
Two segments are congruent to each other if their ends are equally far apart.

If $\overline{AB}$ is congruent to $\overline{CD}$, we write $\overline{AB} \cong \overline{CD}$.

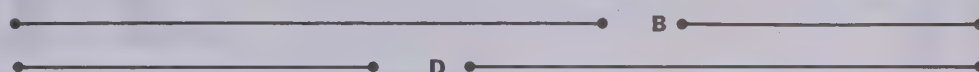
1. A Explain how you used the devices to decide whether the segments above are congruent.
   B In which case above can we write $\overline{AB} \cong \overline{CD}$? Why?

2. How could you use each of the devices above to help you draw $\overline{EF}$ so that $\overline{EF} \cong \overline{PQ}$? P Q

1. Each darkened segment is an edge of one of the figures below.
   - **A** Which darkened segment is longest?
   - **B** Which of these segments is shortest?
   - **C** Write a statement that tells which segments are congruent. Use the correct symbols.



2. Use one of the devices shown in the Investigation to draw a segment that is congruent to each segment given below.



3. Draw a segment on your paper. Label it *XY*. Draw another segment *PQ* so that $\overline{XY} \cong \overline{PQ}$.

★ 4. Write a story about the picture below. Be sure to include some of the mathematical ideas you learned in this lesson.

think

1. Start inside a triangle and draw a path to cross each side exactly once. Where does your path end, inside or outside?

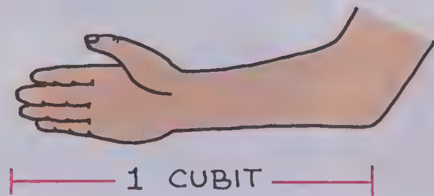
   Try this with a figure of
   A 4 sides. **B** 5 sides. **C** 6 sides.

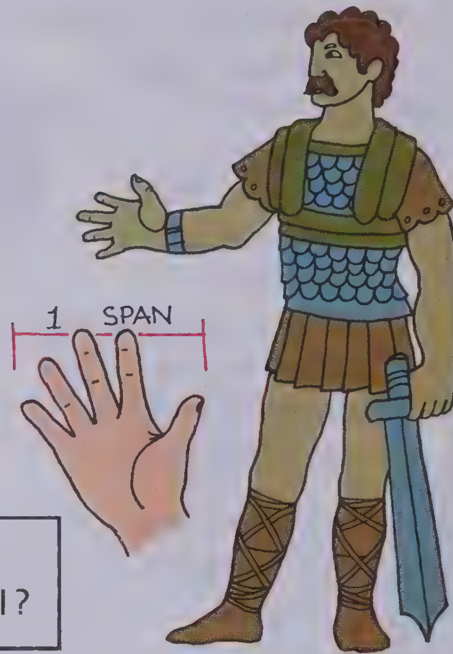
2. Would you be inside or outside if you did this with a figure of
   A 2384 sides? **B** 6749 sides?

How do you find the length of a segment?

Investigating the Ideas

Goliath, the Philistine giant, was reported to have had a measured height of 6 cubits and 1 span.

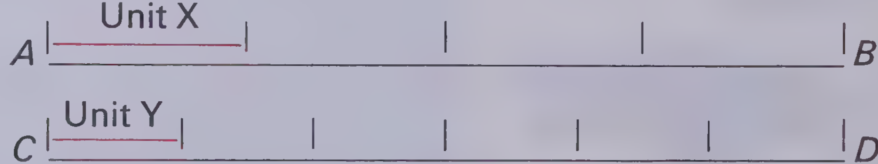




Can you cut a segment of string that is as long as Goliath was tall?

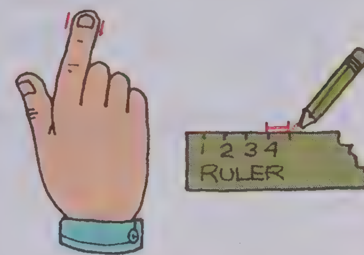
Discussing the Ideas

1. Is each classmate's string the same length? Why?
2. Why is it difficult to find out how tall Goliath was?
3. If an adult completed the Investigation, would his string probably be longer or shorter than yours? Why?
4. We find the length of a segment or object by counting the number of unit segments it takes to "fill" the segment.
   - **A** Using unit X, what is the length of segment *AB*?
   - **B** Using unit Y, what is the length of segment *CD*?

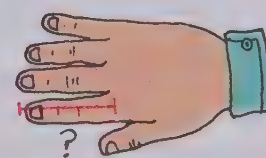


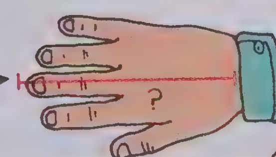
   - **C** The segments *AB* and *CD* are congruent. Explain why you got a larger number for the length of $\overline{CD}$ than for the length of $\overline{AB}$.

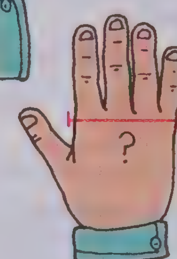
1. Mark a segment that is the width of your index finger. Using this segment as a unit, make a ruler along the edge of a sheet of paper that is at least 16 units long. Use this ruler to complete exercises 1A through 1E. Measure to the nearest unit.



   A Find the length of your index finger.
   B Find the length and width of your mathematics book.
   C Find the length of your desk.
   D Find the length of your pencil.
   E Measure, on your hand, the distance shown in the picture.
   F Mark half units on the ruler you made. Measure to the nearest half unit, on your hand, the distances shown in the picture. Which distance is greater?





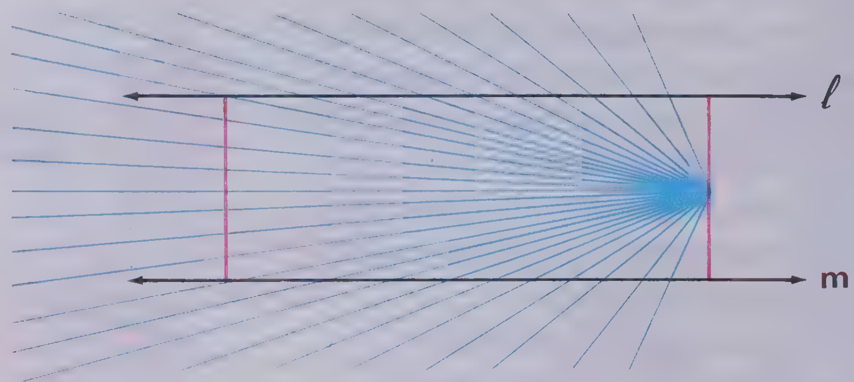


2. You can choose any unit you wish for measuring. You have used **centimetres** and **metres.**
   The red mark shows the length of a centimetre unit.

centimetre (cm)

   A Is the centimetre a longer or shorter unit than the one you used in exercise 1?
   B Without measuring, tell whether the length of your mathematics book, using the centimetre unit, would be more or less than the length you found in exercise 1B.
   C Use a centimetre ruler to check your answer to exercise 2B.
   D Give the length of your shoe to the nearest centimetre.

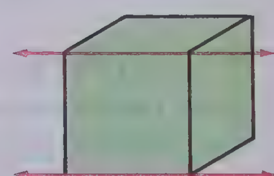
Investigating the Ideas



Do lines ℓ and m intersect? (Would the black lines meet on one side if extended?)

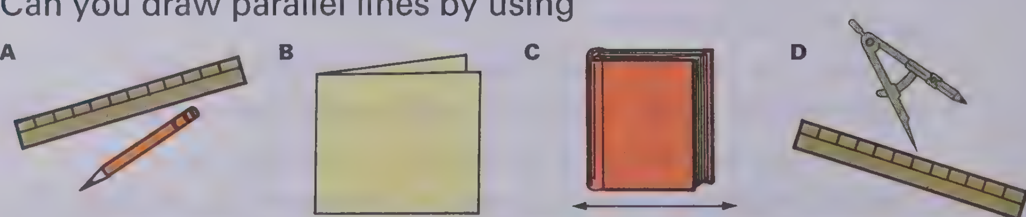
> **?** Can you figure out a way to use tracing paper to help you decide if the lines would meet?

Discussing the Ideas



1. Two lines that are in the same plane and do not intersect are parallel to each other.
   - A Ann said railroad tracks remind her of parallel lines. What objects remind you of parallel lines?
   - B Explain what Jeff meant when he said, "Parallel lines are everywhere the same distance apart."

2. Can you draw parallel lines by using



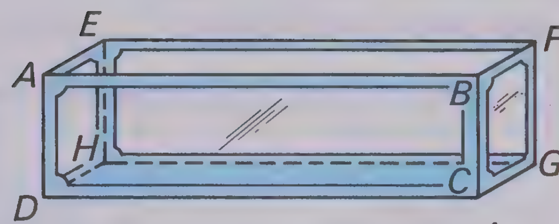
A a ruler and a pencil?

B paper folding?

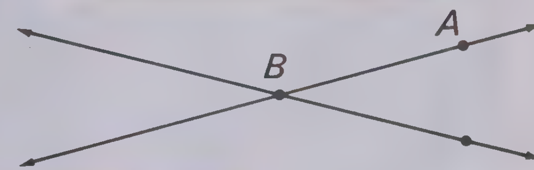
C a line and a book?

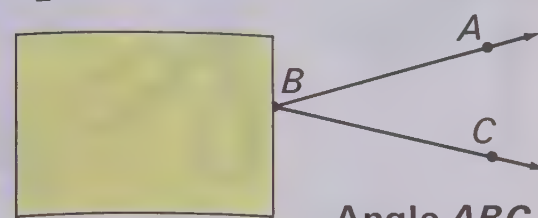
D a compass and a ruler?

1. Name as many pairs of parallel edges as you can find in this picture of an aquarium.



2. If we start with two intersecting lines and cover up "half" of the picture, an **angle** is formed by the two rays from the point.





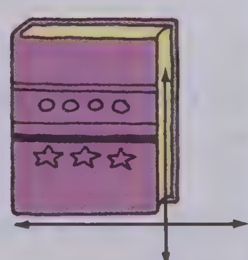
Angle *ABC*

   A Draw 5 different-sized angles.

   B Label one of your angles so that it could be called angle *RST* ($\angle RST$).

   C Label and name the other angles.

3. If two lines intersect like this,

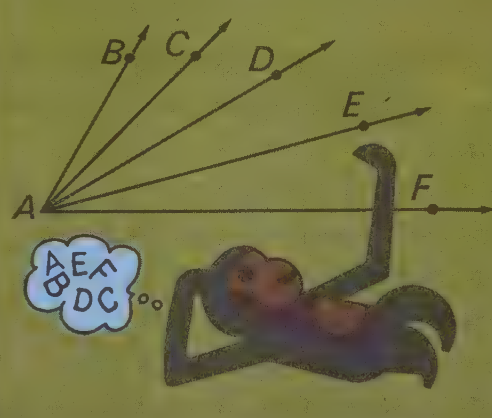


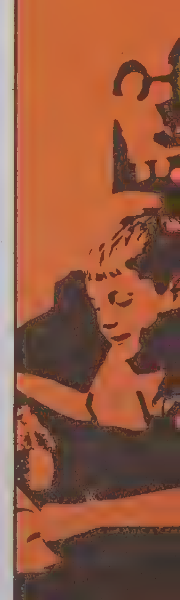
they are said to be perpendicular to each other. Draw a pair of perpendicular lines.

★ 4. Find out what a parallelogram is and draw a picture of one.

think

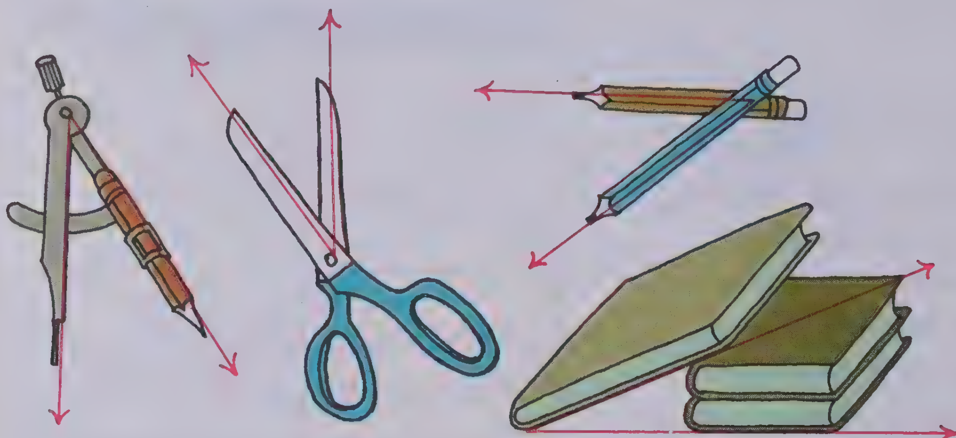
There are 10 different angles in the figure below. See how many of them you can name.





Investigating the Ideas

Each of these objects could remind you of an angle.



? Can you find a way to decide which "angle" is "open widest"?

Discussing the Ideas

1. Explain how you decided which angle in the Investigation was "open widest" (largest) and which was smallest.

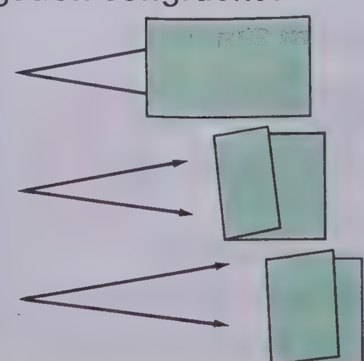
2. Two angles are **congruent** if neither is larger than the other. Are any of the angles in the Investigation congruent?

3. Think about the angle in A. We might uncover the drawing and find B, or we might find C. Is the angle in C larger than the angle in B? Why or why not? Explain how you think we compare angles.

A

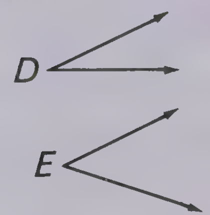
B

C

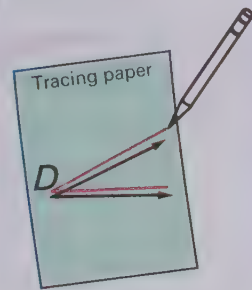


1. Study this method of comparing two angles.

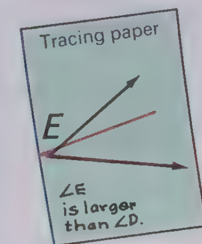
To compare $\angle D$ with $\angle E$



trace $\angle D$ and

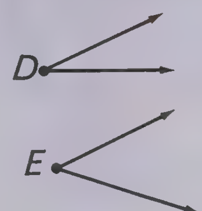


place the tracing over $\angle E$.

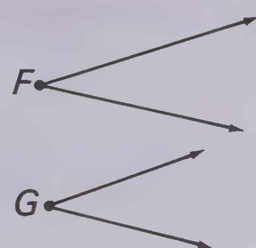


Use the method above to tell which angle is larger.

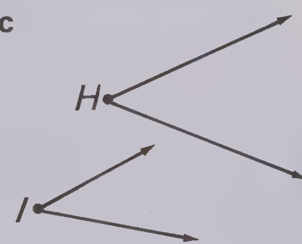
A



B

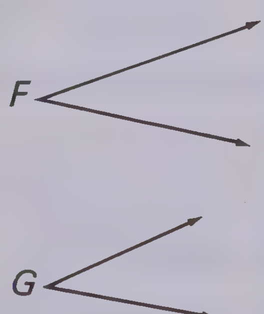


C

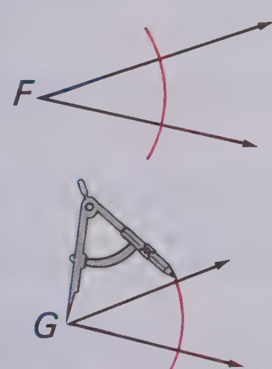


2. Study this method of comparing two angles.

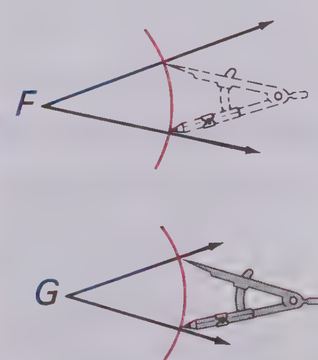
To compare $\angle F$ with $\angle G$



mark two arcs with the same compass opening, and



check these distances.

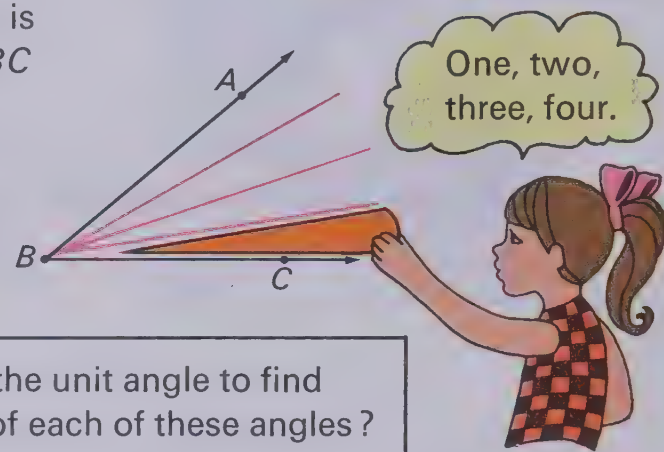


Draw two angles on your paper. Use your compass to decide whether or not the two angles are congruent.

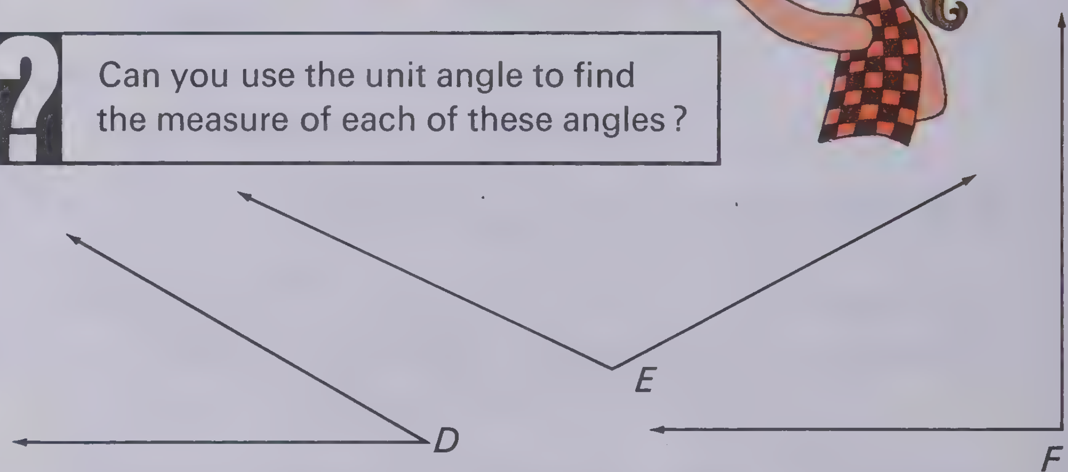
Investigating the Ideas

Ann measured ∠*ABC* using the **unit angle** shown. The red marks show where Ann placed the unit as she measured. What is the measure of ∠*ABC* using Ann's unit?

Trace this unit angle and cut it out.



? Can you use the unit angle to find the measure of each of these angles?



Discussing the Ideas

1. Explain how you measured the angles above.
2. A Could you use unit angles of other sizes to measure the angles above?
   B How would the measures of the angles above differ if you used a smaller unit? a larger unit?
3. How is measuring an angle like measuring a segment?

Using the Ideas

1. Use the unit you cut out on page 78 and draw an angle with measure: A 4 B 7 C 2 D 15 E 5 F 10 G 18

2. The corner of your book suggests a **right angle**. Draw a right angle on your paper and measure the angle. Use the unit you cut out on page 78.

3. An **acute angle** is an angle that is less than a right angle. An **obtuse angle** is an angle that is greater than a right angle. Which of the angles you drew for exercise 1 are acute? obtuse?

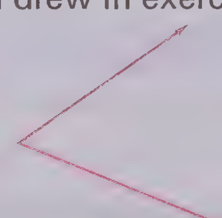
4. Here is an angle unit that was chosen by the Sumerians over 4000 years ago. It is one of the units used most often today.

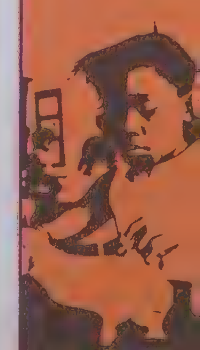
This angle is called a degree. The measure of a right angle, using the degree, is 90. We say the measure of the right angle is 90 degrees. We write **90°** for **90 degrees**.

A Trace the angle below. Give its degree measure by placing it over the unit above.

★ B If the degree measure of the unit used in exercise 1 is 10, give the degree measure of each angle you drew in exercise 1.

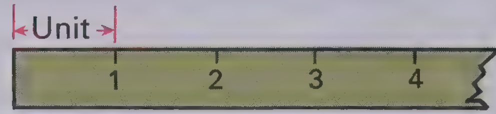
★ **5.** Here is one special angle unit often used. It is called a radian. On tracing paper, draw an angle with radian measure 3.



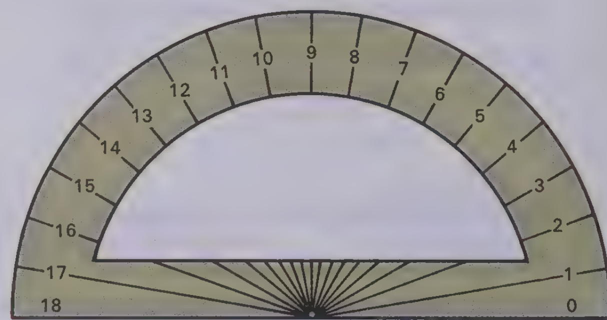


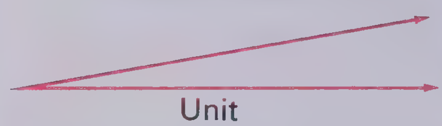
Let's make and use a protractor.

Investigating the Ideas



A **ruler** has several **segment units** placed end-to-end to make it easier for you to find length. A **protractor** has several **angle units** placed side-by-side to make it easier for you to measure angles. The numerals on the ruler and protractor make it easier for you to count units. Use the "unit" from the last lesson and a compass to make a protractor.

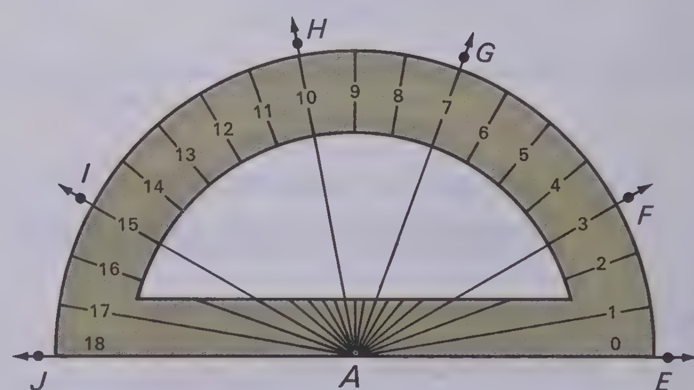




Can you use your protractor to measure the angles in the Investigation on page 78?

Discussing the Ideas

This picture of a protractor is placed over some rays from point A. Compare this protractor with the one you made.

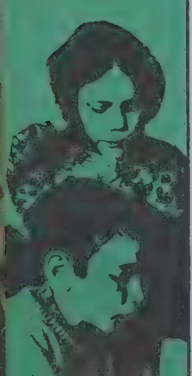


1. What is the measure of $\angle FAE$?

2. Explain how to tell that $\angle GAF$ is 4 units.

3. What is the measure of $\angle IAH$?

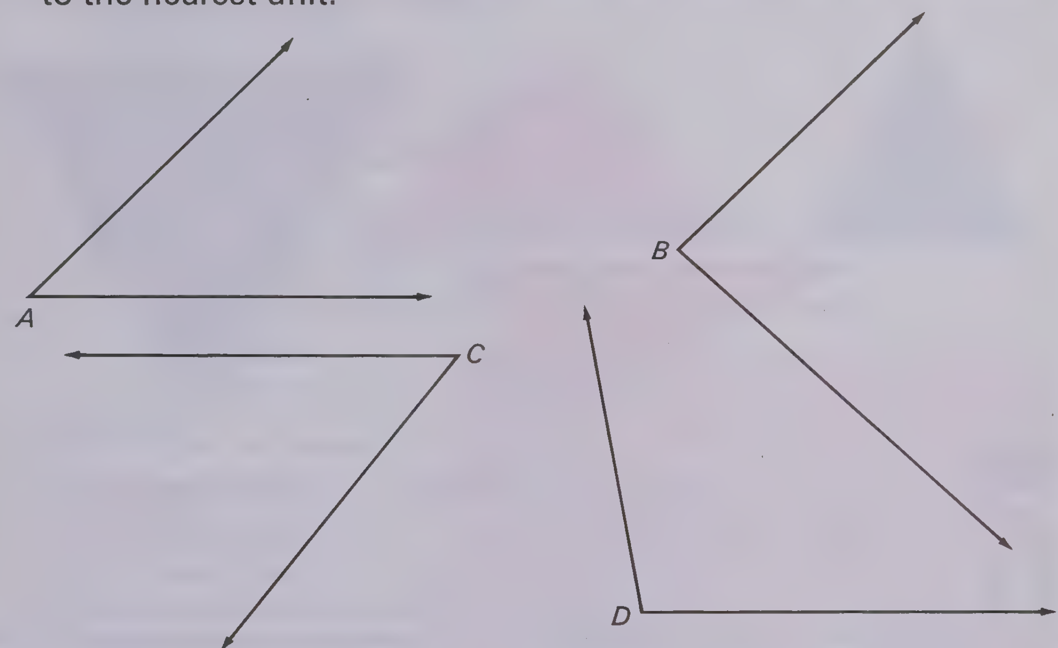
4. Explain how to find the measure of $\angle IAF$ quickly.



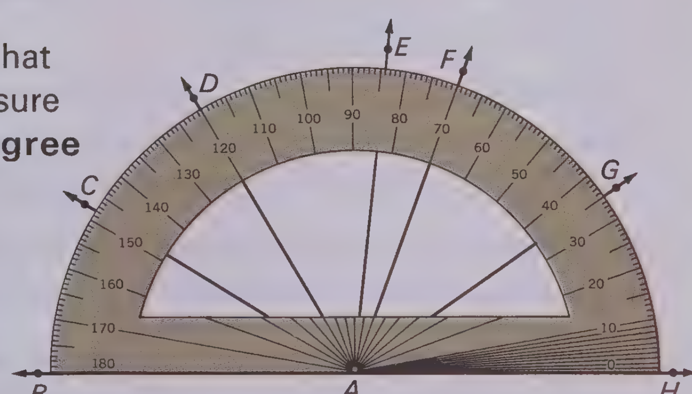
1. Use the figure in the Discussion to give the measure of the following angles.

A $\angle GAE$	C $\angle EAI$	E $\angle JAG$	G $\angle GAH$
B $\angle EAH$	D $\angle JAI$	F $\angle FAJ$	H $\angle IAG$

2. Use the protractor you made to measure these angles to the nearest unit.



3. This is a protractor that can be used to measure angles when the **degree** is the unit. It is placed over some rays from point *A*. Give the degree measure of these angles.



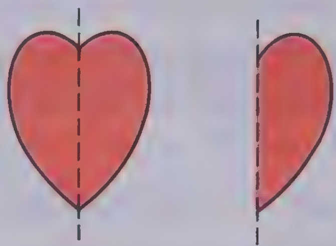
A *HAF* B *DAH* C *HAG* D *HAE* E *HAC* F *BAC*

4. Use a degree protractor to measure the angles in exercise 2.

Let's explore triangles and symmetry.

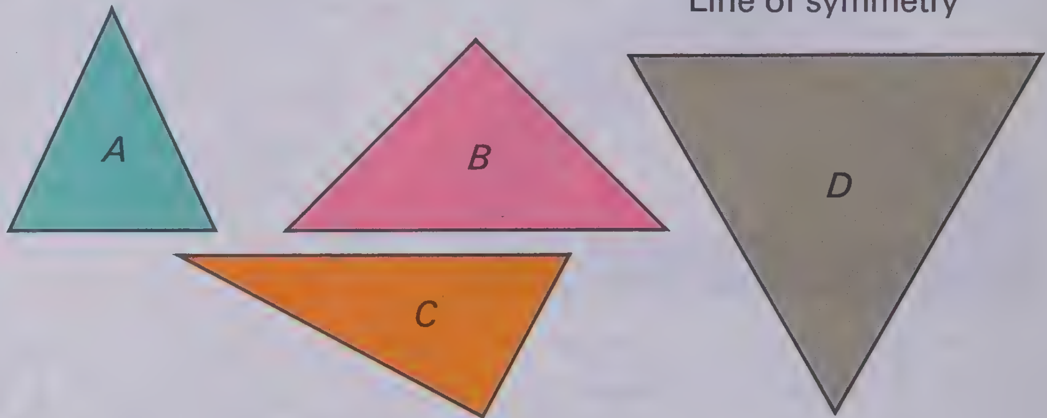
Investigating the Ideas

A figure has a **line of symmetry** if a fold on that line makes two halves of the figure that match exactly.

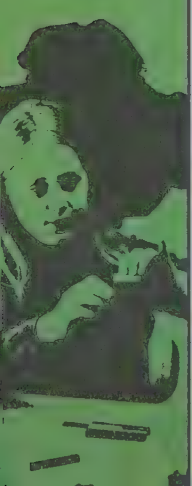


Line of symmetry

Trace and cut out these triangular shapes.



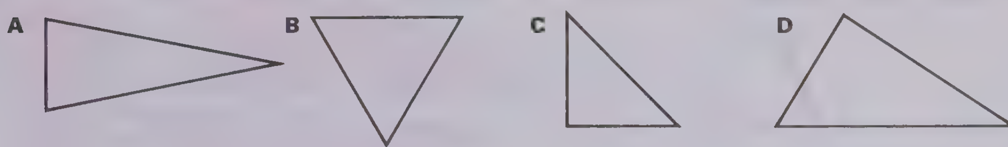
? Which of the shapes has 3 different lines of symmetry? only one line of symmetry? no lines of symmetry?	Show the lines by folding the figures.



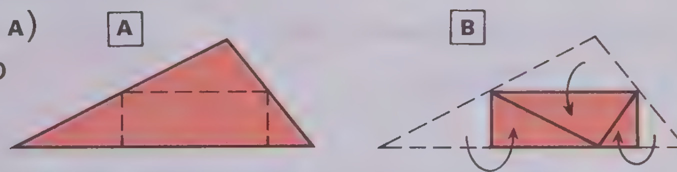
Discussing the Ideas

1. An **isosceles triangle** has 1 line of symmetry. Choose an isosceles triangle above and show, by folding, that it has 2 congruent sides and 2 congruent angles.
2. An **equilateral triangle** has 3 lines of symmetry. Show, by folding, that it has 3 congruent sides and 3 congruent angles.
3. A **scalene triangle** has no lines of symmetry and no congruent sides. Which triangle above is scalene?
4. Which two triangles above do you think are **right triangles**? Why?

1. Describe each triangle by writing **isosceles**, **equilateral**, **scalene**, **right**, or a combination of these.



2. A A student drew fold lines on a triangle (picture A) and then folded to form a rectangle (picture B).



The "angle flaps" fit together exactly with no overlap. Can each of the four triangles you cut out in the Investigation be folded to exactly form a rectangle?

B Another student placed a protractor like this:

What does this suggest about the sum of the angles of a triangle?

★ 3. Make an equilateral triangle by using

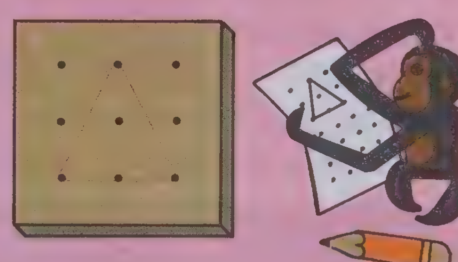
A a paper clip, 2 pencils, and a ruler. 

B a compass and a ruler.

C paper (for folding) and pencil.

think

Suppose you have a 3-nail by 3-nail geoboard.

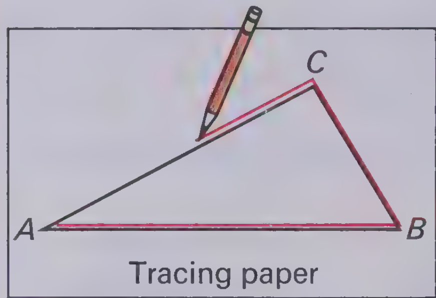


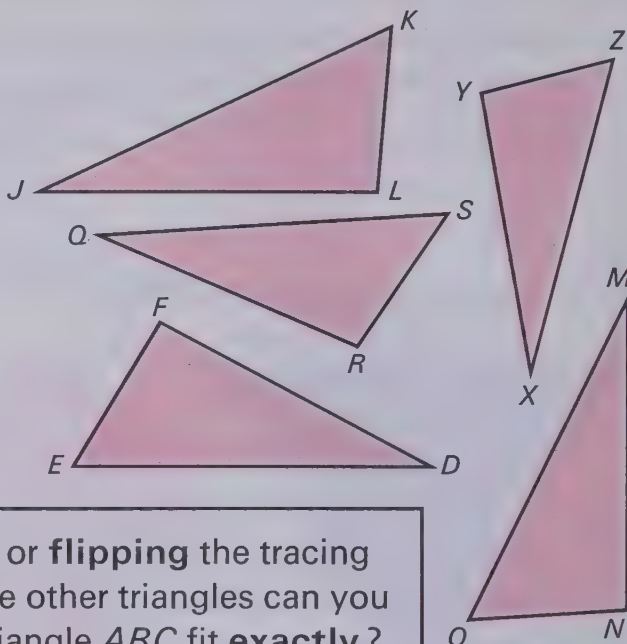
Can you show on dot paper 4 differently shaped isosceles triangles that can be formed with rubber bands on the board?

When are two triangles congruent?

Investigating the Ideas

Trace triangle *ABC*.





? By **sliding**, **turning**, or **flipping** the tracing paper, on which of the other triangles can you make the tracing of triangle *ABC* fit **exactly** ?

Discussing the Ideas

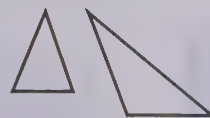
> Two triangles are congruent if the parts (angles and segments) of one can be matched with the parts of the other so that the pairs of angles and segments are congruent.

In the Investigation you found triangles that are **congruent** to triangle *ABC* ($\triangle ABC$). Explain this definition. Then use triangles *ABC* and *DEF* above and give the missing segment or angle for each ▥.

This part of $\triangle ABC$ ↓	is congruent to ↓	this part of $\triangle DEF$. ↓	This part of $\triangle ABC$ ↓	is congruent to ↓	this part of $\triangle DEF$. ↓
$\overline{AB}$	$\cong$	▥	$\angle CAB$	$\cong$	▥
$\overline{AC}$	$\cong$	▥	$\angle CBA$	$\cong$	▥
$\overline{BC}$	$\cong$	▥	$\angle ACB$	$\cong$	▥

1. For each exercise, tell (just by looking) which pairs of triangles are not congruent.

A


B


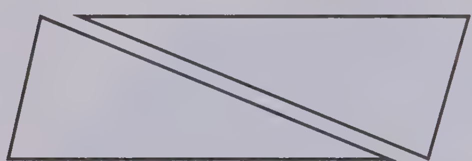
C


D

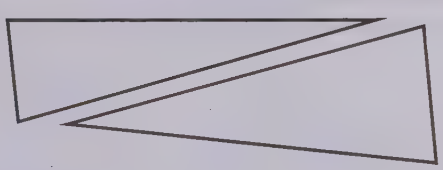

E


F

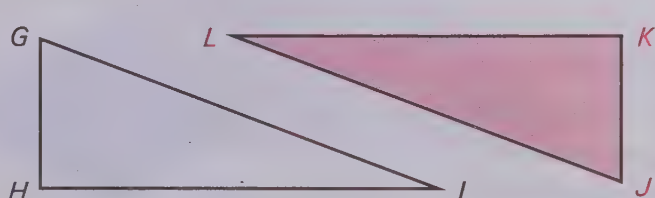

2. For each exercise, trace one of the triangles on a thin sheet of paper. Use this tracing to tell whether or not the two triangles are congruent.

A


B


C


3. The pair of right triangles (each has one right angle) below is congruent. Give the missing angles or segments in the table.

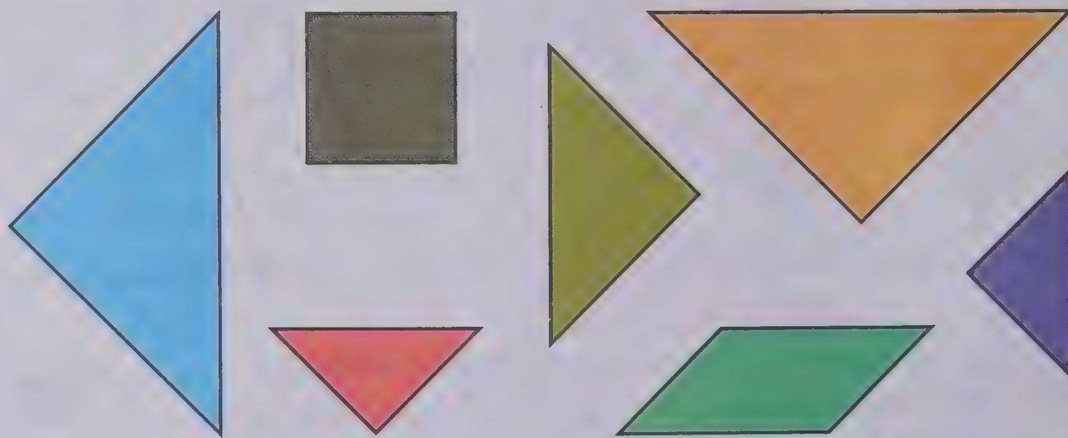


$\triangle GHI$	$\triangle JKL$
$\overline{GH} \cong$	$\overline{JK}$
$\overline{HI} \cong$	
$\overline{GI} \cong$	
$\angle GHI \cong$	
$\angle HIG \cong$	
$\angle IGH \cong$	

● *Can you use tangram pieces to form polygon shapes?*

Investigating the Ideas

Here are the seven pieces of the tangram puzzle.



Trace these seven shapes and cut them out.
Use the two large triangles to form a square.
Use the two small triangles to form a square.

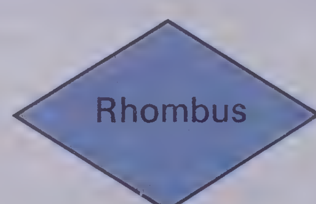
? Can you place the seven pieces together to form a square?

Discussing the Ideas

1. The green tangram piece is a **parallelogram**. Which two pieces will exactly cover it?

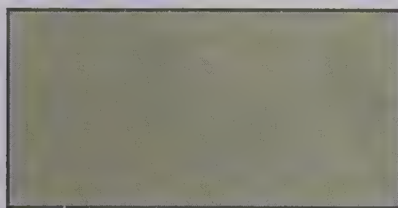
2. Can you find three pieces that will exactly cover one of the large triangles?

3. A **rhombus** is a parallelogram which has 4 congruent sides. Do you think you can place some of the tangram pieces together to form a rhombus?



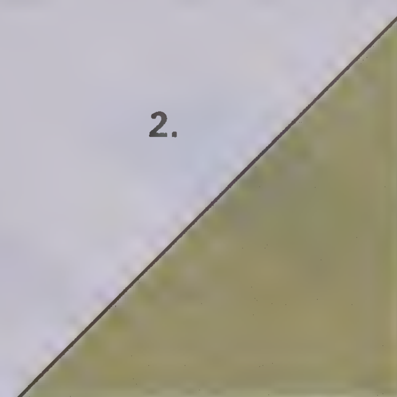
Use all 7 pieces of the tangram puzzle to make each of these shapes. Your shapes will be larger than the ones shown.

1.



Rectangle

2.

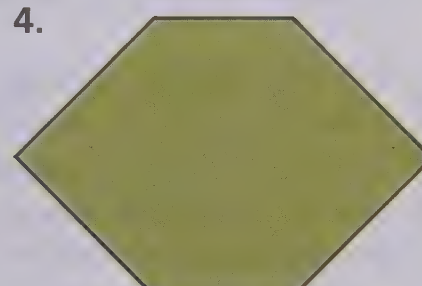


Triangle

3.

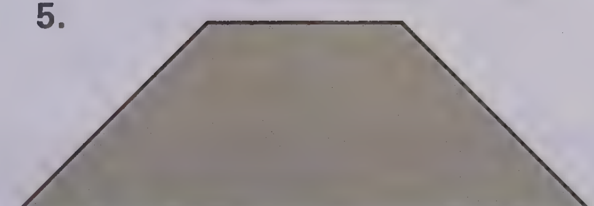
Parallelogram

4.



Hexagon

5.

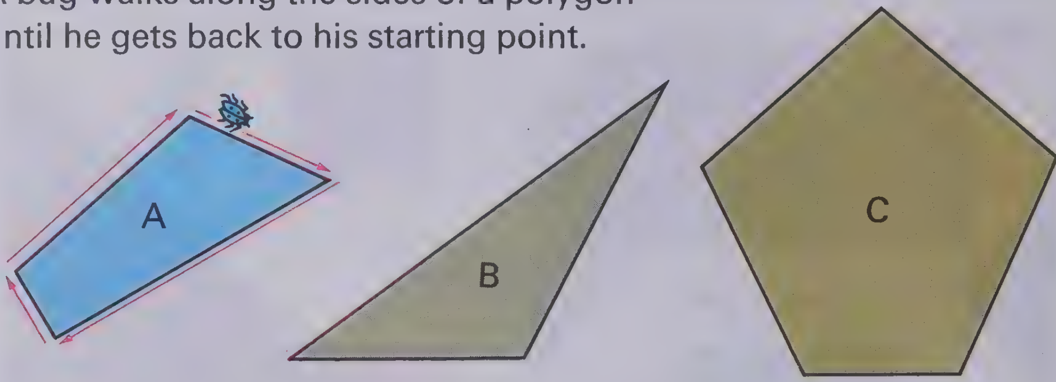


Trapezoid

How do you find the perimeter of a polygon?

Investigating the Ideas

A bug walks along the sides of a polygon until he gets back to his starting point.

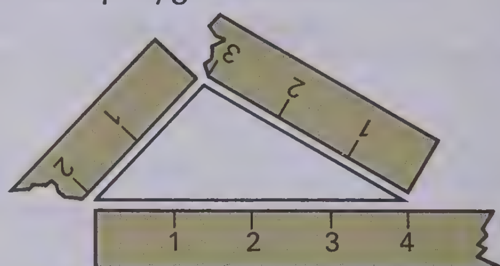


? Can you use your centimetre ruler to find how far the bug has to travel for each polygon above?

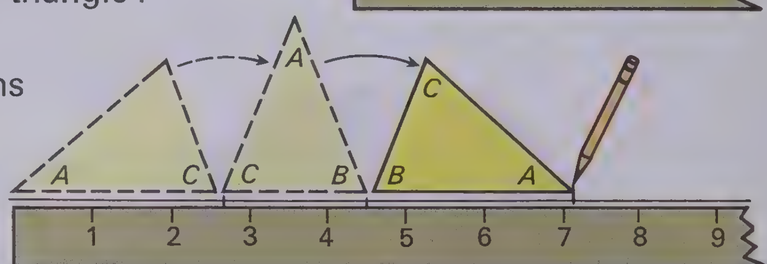
Discussing the Ideas

1. The **perimeter** of a polygon is the sum of the lengths of its sides. What is the perimeter of each polygon above?

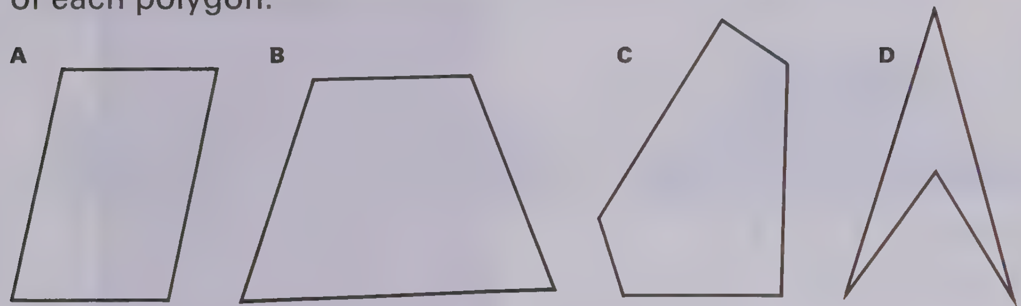
2. If the lengths of the sides of a polygon are whole numbers, it is easy to find the perimeter with a ruler. What is the perimeter of this triangle?



3. When the lengths of the sides of a polygon are not whole numbers, you can find the perimeter by "rolling" the polygon along your ruler. Explain how to find the perimeter of triangle *ABC*.

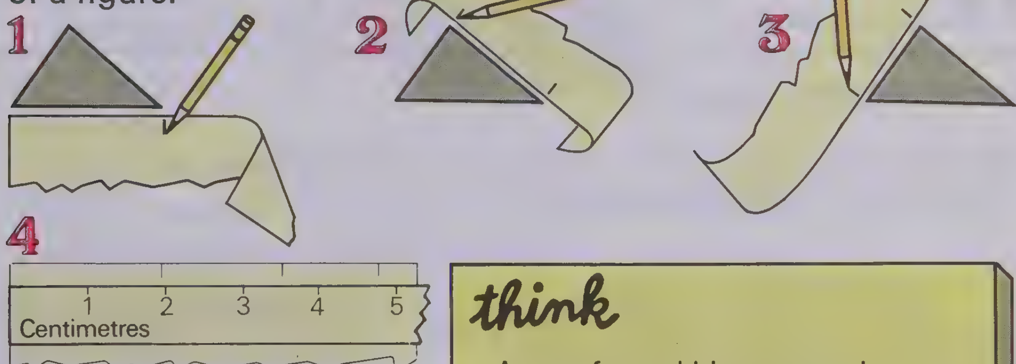


1. Use your centimetre ruler to find the perimeter of each polygon.

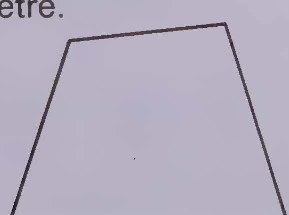


2. Draw a triangle, a quadrilateral, a pentagon, and a hexagon, and find the perimeter of each.

3. Here is a way to use a strip of paper to help you find the perimeter of a figure.



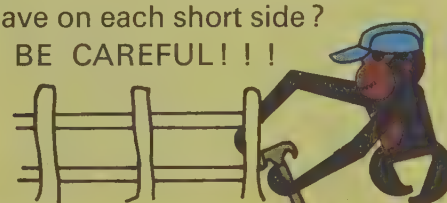
Use this method to find the perimeter of this polygon to the nearest centimetre.



think

A man fenced his rectangular shaped garden and used 40 posts. He had 14 posts on each long side of the garden. How many did he have on each short side?

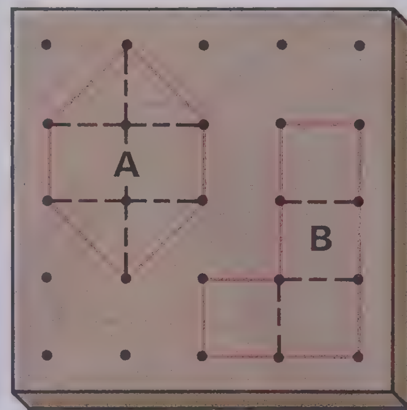
BE CAREFUL! ! !



More practice, page A-6, Set 11

Investigating the Ideas

If each small square on the geoboard is **1 unit of area**, the **area** of each of the polygons **A** and **B** is 4 square units.

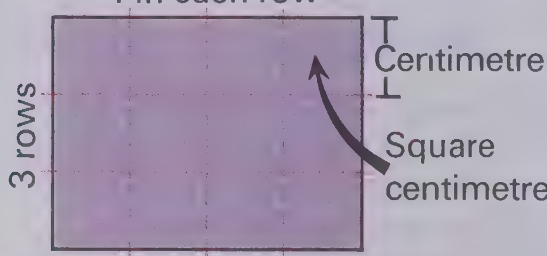


? Can you find at least 8 more differently shaped regions that each have an area of 4 square units?	Record your regions on graph paper.

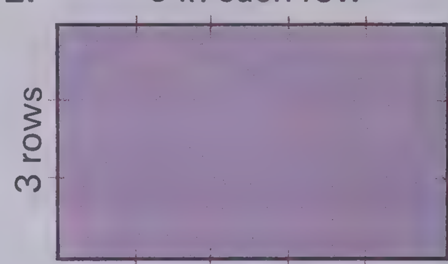
Discussing the Ideas

Explain how you would find the area of each rectangle.
The unit is a square centimetre.

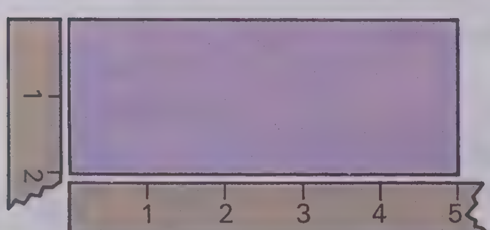
1. 4 in each row



2. 5 in each row

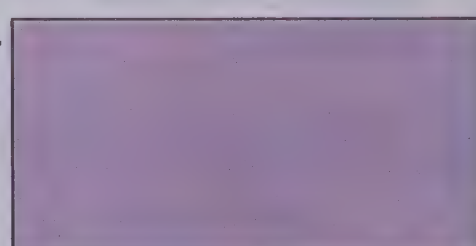


3.



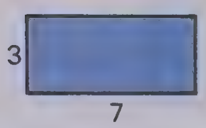
Centimetre ruler

4.

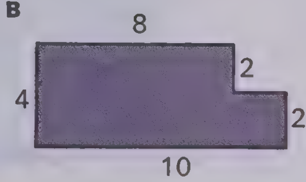


1. Find the areas of each polygon. Pretend the unit of area is the square centimetre and the lengths given are in centimetres.

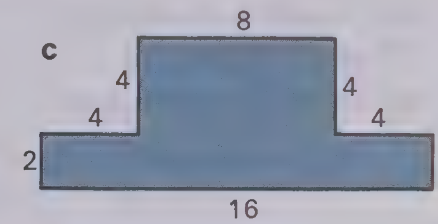
A



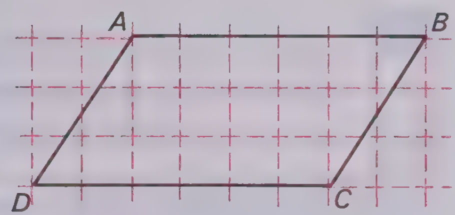
B



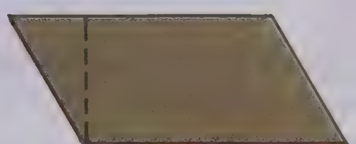
C



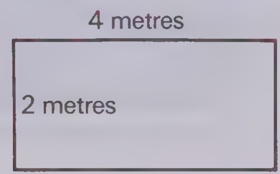
2. Estimate the area of parallelogram *ABCD*.



3. Trace the rectangular "hole" and the parallelogram of "wood."



"wood"



"hole"

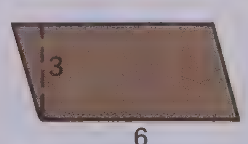
Cut out the piece of "wood", and then cut along the dotted line. Paste the pieces so they completely "fill" the hole.

A What is the area of the rectangle?

B What is the area of the parallelogram?

4. Think about forming a rectangle as in exercise 3. Then give the area.

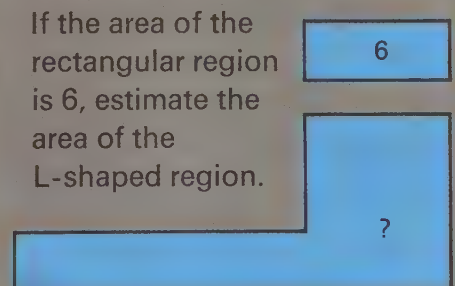
A



B

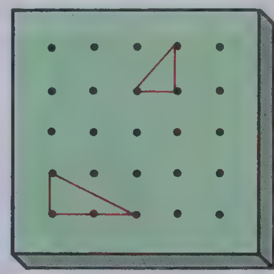
think

If the area of the rectangular region is 6, estimate the area of the L-shaped region.



Investigating the Ideas

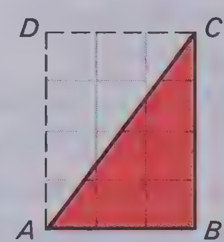
Two **right triangles** of different shapes are shown on the geoboard.



? How many right triangles of different sizes or shapes can you find on a geoboard?	Show your triangles on dot paper.

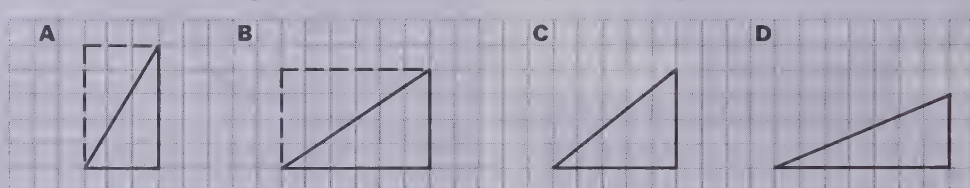
Discussing the Ideas

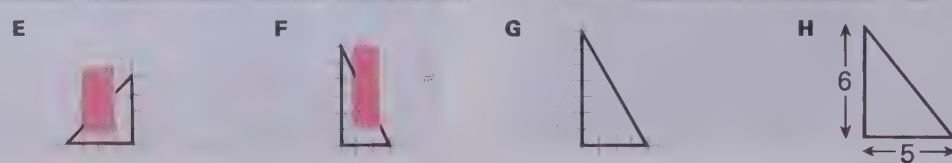
1. Can you give the area of any of the triangular regions you found in the Investigation?



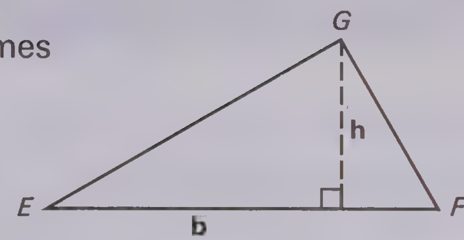
2. Every right triangular region is one half of some rectangular or square region.
   - A What is the area of the region *ABCD*?
   - B The area of region *ABC* is what part of the area of the rectangular region?
   - C What is the area of triangular region *ABC*?

3. Give the area of each triangular region. Can you find a rule for finding the area of any right triangle?



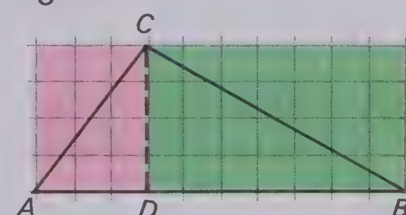


The length of segment EF is sometimes called the base (**b**).
The length indicated by the dotted line is called the height (**h**).

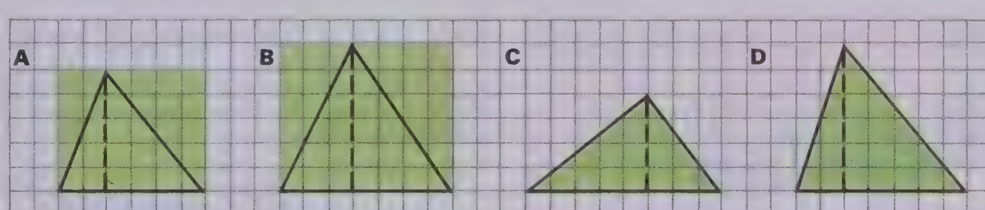


These exercises will help you find the area of a triangular region when you know the base and the height from that base.

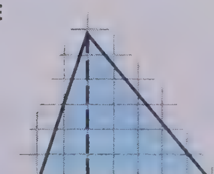
1. A What is the area of the region shaded pink?
B What is the area of the region shaded green?
C What is the area of the two regions together?
D What is the area of triangle ADC?
E What is the area of triangle BDC?
F What is the area of triangle ABC?
G The area of triangle ABC is what part of the entire shaded region?



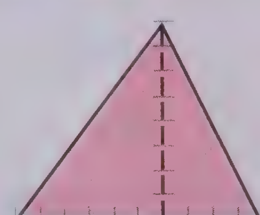
2. Find the area of each large triangular region.



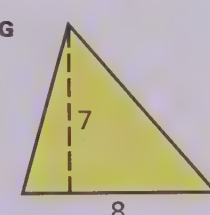
E



F



G



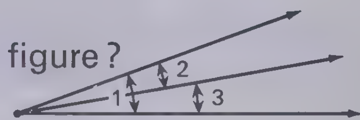
3. Find the areas of the following triangular regions. Only the base and height are given.

A $b = 10$, $h = 4$ B $b = 18$, $h = 4$ C $b = 12$, $h = 10$ D $b = 24$, $h = 36$ E $b = 48$, $h = 35$

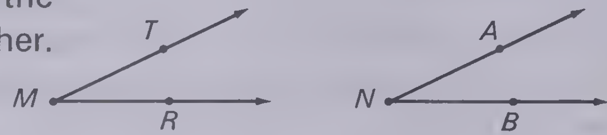
More practice, page A-6, Set 12

1. Only one of the following could be true for this figure. Which one?
   A $\overline{AB} \cong \overline{AC}$ B $\overline{AC} \cong \overline{BC}$ C $\overline{AB} \cong \overline{BC}$

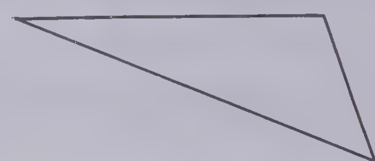
2. Which one of the following could be true for this figure?
   A $\angle 2 \cong \angle 1$ B $\angle 2 \cong \angle 3$ C $\angle 1 \cong \angle 3$



3. The four segments shown on the rays are congruent to each other. Tell whether or not $\angle TMR$ is congruent to $\angle ANB$.

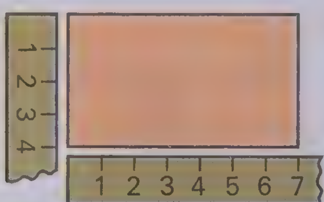


Use your centimetre ruler to find the perimeter of this triangle.

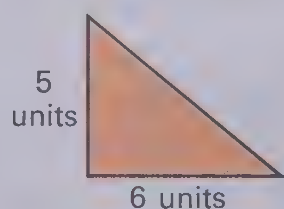


5. Find the area of each region by using the unit indicated.

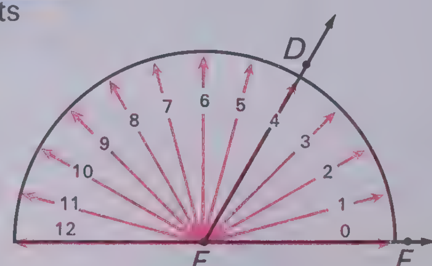
A



B

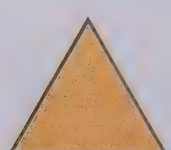


6. A Use the unit shown on the protractor to find the measure of $\angle DEF$.
   B Find the degree measure of $\angle DEF$.

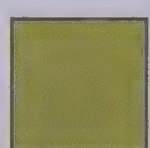


7. How many lines of symmetry does each figure have?

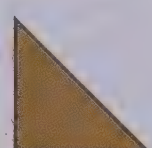
A



B



C



D

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Place value, Inequalities

1. Find the sums.

A $4000 + 300 + 60 + 9$ C $70\,000 + 3000 + 10 + 8$

B $50\,000 + 8000 + 700 + 20 + 1$ D $600\,000 + 90\,000 + 4000$

2. Write each number as in the example.

Example: $3254 = 3000 + 200 + 50 + 4$

A 4695 B 8329 C 17 264 D 29 435 E 843 672

3. Give the sign $<$ or $>$ for each ●.

A 4280 ● 4279 B 32 496 ● 32 500 C 416 837 ● 417 213

4. Solve the equations.

A $5 \times 6 = n$ E $4 \times 9 = c$ I $5 + 9 = d$ M $5 \times 8 = a$

B $8 + 7 = r$ F $8 + 9 = t$ J $7 \times 6 = n$ N $56 \div 7 = t$

C $15 - 9 = s$ G $13 - 8 = b$ K $32 \div 8 = b$ O $35 \div 5 = r$

D $48 \div 6 = a$ H $9 \times 8 = m$ L $49 \div 7 = s$ P $8 \times 8 = f$

5. Find the products.

A 26×10 C 60×40

B 49×100 D 30×200

6. Find the quotients.

A $240 \div 6$ D $1800 \div 3$

B $320 \div 4$ E $1800 \div 300$

C $240 \div 30$ F $1800 \div 30$

7. Estimate the products.

A 99×46 C 48×52

B 19×31 D 199×31

think

The dots below should help you see why 3, 6, and 10 are sometimes called triangular numbers.

3 6 10

Give the next 5 triangular numbers.

You are invited to explore

ACTIVITY CARD 4
Page 335

5 Estimation

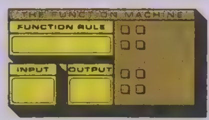
What is the nearest multiple of 10 or 100?

Investigating the Ideas

What are the last four numbers in each function table?

A

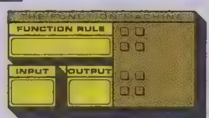
??? RULE ???



n	32	33	34	35	36	37	38	39	40	41	76	92	134	275
output	30	30	30	40	40	40	40	40	40	40				

B

??? RULE ???



n	689	695	721	749	750	758	1243	1250	1276	3284	4516
output	700	700	700	700	800	800	1200				

? Can you write or explain a function rule for each table?

Discussing the Ideas

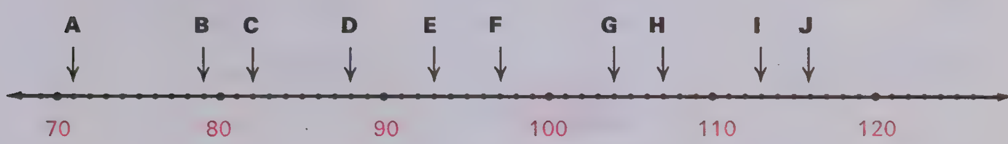
1. Use the function rule for Function Machine A to give the output for each of these inputs.
   A 52 C 84 E 64 G 138 I 654 K 1273
   B 57 D 76 F 65 H 132 J 655 L 1276

2. Use the function rule for Function Machine B to give the output for each of these inputs.
   A 278 C 316 E 650 G 651 I 2346 K 7239
   B 244 D 361 F 649 H 987 J 2356 L 7293

3. State each rule in your own words.

The number-line pictures and exercises will help you choose multiples of 10 and 100 that you will use in estimation.

1. Give the numbers (A through J) that go with the points on the number line.

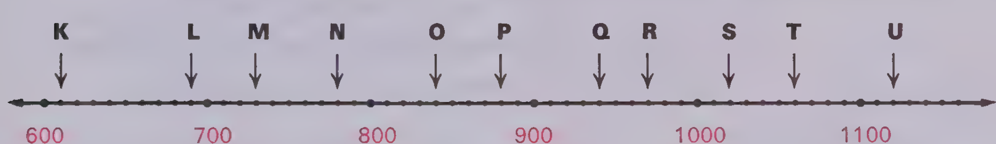


2. Give the multiples of 10 that are closest to the numbers for A through J. If the number is halfway between, give the larger multiple of 10.

3. Give the multiple of 10 that is closest to each number.

A 27	C 45	E 89	G 127	I 136	K 298
B 44	D 99	F 36	H 133	J 289	L 1256

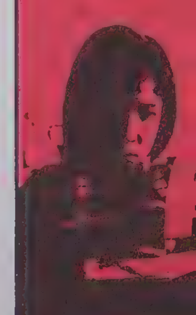
4. Give the numbers (K through U) that go with the points on the number line.



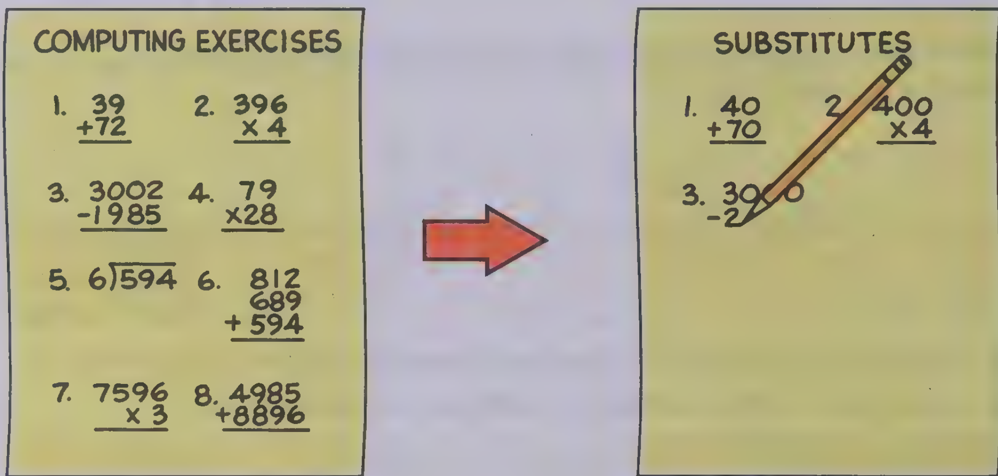
5. Give the multiples of 100 that are closest to the numbers for K through U. If a number is halfway between, give the larger multiple of 100.

6. Give the multiple of 100 that is closest to each number.

A 207	C 84	E 573	G 649	I 2651	K 4059
B 489	D 326	F 456	H 1286	J 3438	L 37



Investigating the Ideas



? Can you copy and complete the page of "substitute" problems? Use multiples of 10 and 100 so that the answers to the two sets will be "very close" to each other.

Discussing the Ideas

1. Your substitute problems will help you find estimates—often without pencil and paper. How many of your substitute problems above can you solve without pencil and paper?

2. The closer you can get to the correct answer, the better your estimate is. Which problem—1, 2, or 3—will give the best estimate for the one in red?

		1	2	3
A	$38 + 84$	$40 + 90$	$40 + 80$	$30 + 80$
B	79×8	70×8	80×8	80×10
C	$604 - 289$	$600 - 300$	$700 - 300$	$600 - 200$
D	68×71	60×70	60×60	70×70

1. Give the multiple of 10 that you think should go in each ▥.
   - A To estimate 42 + 59, we can find the sum 40 + ▥.
   - B To estimate 25 + 72, we can find the sum ▥ + 70.
   - C To estimate 81 − 48, we can find the difference 80 − ▥.
   - D To estimate 148 − 29, we can find the difference 150 − ▥.
   - E To estimate 173 − 36, we can find the difference ▥ − 40.
   - F To estimate 245 + 328, we can find the sum ▥ + 330.
   - G To estimate 29 × 82, we can find the product ▥ × 80.
   - H To estimate 54 × 26, we can find the product ▥ × 30.
   - I To estimate 323 ÷ 83, we can find the quotient ▥ ÷ 80.
   - J To estimate 207 ÷ 65, we can find the quotient 210 ÷ ▥.

2. Give the multiple of 100 that you think should go in each ▥.
   - A To estimate 213 + 487, we can find the sum ▥ + 500.
   - B To estimate 528 + 1176, we can find the sum 500 + ▥.
   - C To estimate 969 + 850, we can find the sum 1000 + ▥.
   - D To estimate 806 − 479, we can find the difference 800 − ▥.
   - E To estimate 1146 − 357, we can find the difference ▥ − 400.
   - F To estimate 950 − 373, we can find the difference ▥ − 400.
   - G To estimate 93 × 123, we can find the product 100 × ▥.
   - H To estimate 549 × 451, we can find the product ▥ × 500.
   - I To estimate 3214 ÷ 789, we can find the quotient ▥ ÷ 800.
   - J To estimate 4461 ÷ 850, we can find the quotient ▥ ÷ 900.

think

Kevin is three times as old as his sister Karen. In four years, he will only be twice as old as Karen. How old are both children now?

Let's explore some special products.

Investigating the Ideas

Study the **flow chart** to help you review finding products like 4×60.

Input box | Instruction boxes | Output box

Problem 4×60 → First find 4×6. → Then multiply by 10. → The product is 240.

$$4 \times 60 = 24 \times 10 = 240$$

? Can you make your own flow chart for one of these products?

1. 5×30 2. 3×500 3. 30×50 4. 5×3000

Discussing the Ideas

1. Find these products.

A 7×10	D 46×10	G 37×100	J 32×1000
B 10×35	E 8×100	H 48×100	K 1000×14
C 276×10	F 100×24	I 7×1000	L 87×1000

2. Give a simple rule for multiplying by 10. By 100. By 1000.

3. Finding the product for ***a*** will help you understand a simple rule for finding ***b***.

A $3 \times 8 \times 100 = a$
$3 \times 800 = b$

B $4 \times 10 \times 7 \times 10 = a$
$40 \times 70 = b$

C $3 \times 10 \times 6 \times 100 = a$
$30 \times 600 = b$

D $4 \times 100 \times 7 \times 10 = a$
$400 \times 70 = b$

4. Give a rule for finding each type of product in exercise 3.

1. Find the products.

A 32×10
B 63×10
C 42×100
D 30×100
E 1000×42
F 20×100
G 342×100
H 10×675

2. Give the number for *a*. Then give the number for *b*.

A $4 \times 2 \times 10 = a \rightarrow 4 \times 20 = b$
B $9 \times 7 \times 10 = a \rightarrow 9 \times 70 = b$
C $7 \times 6 \times 10 = a \rightarrow 7 \times 60 = b$
D $8 \times 6 \times 100 = a \rightarrow 8 \times 600 = b$
E $6 \times 9 \times 100 = a \rightarrow 6 \times 900 = b$
F $8 \times 5 \times 100 = a \rightarrow 8 \times 500 = b$

3. Give the number for *a*. Then give the number for *b*.

A $3 \times 10 \times 2 \times 10 = a \longrightarrow 30 \times 20 = b$
B $4 \times 10 \times 3 \times 10 = a \longrightarrow 40 \times 30 = b$
C $5 \times 10 \times 7 \times 10 = a \longrightarrow 50 \times 70 = b$
D $6 \times 10 \times 2 \times 10 = a \longrightarrow 60 \times 20 = b$
E $4 \times 10 \times 9 \times 10 = a \longrightarrow 40 \times 90 = b$
F $5 \times 10 \times 5 \times 10 = a \longrightarrow 50 \times 50 = b$

4. Find the products.

A 20×30
B 50×30
C 40×50
D 60×50
E 70×20
F 60×60
G 80×40
H 40×70
I 70×60
J 70×70
K 70×80
L 80×80
M 30×90
N 80×90
O 90×90
P 70×90

5. Find the products.

A 40×30
B 40×300
C 30×60
D 30×600
E 300×600
F 50×70
G 50×700
H 500×700
I 50×7000
J 20×600
K 60×200
L 70×300
M 700×30
N 3×7000
O 300×700
P 60×800
Q 600×800
R 60×8000

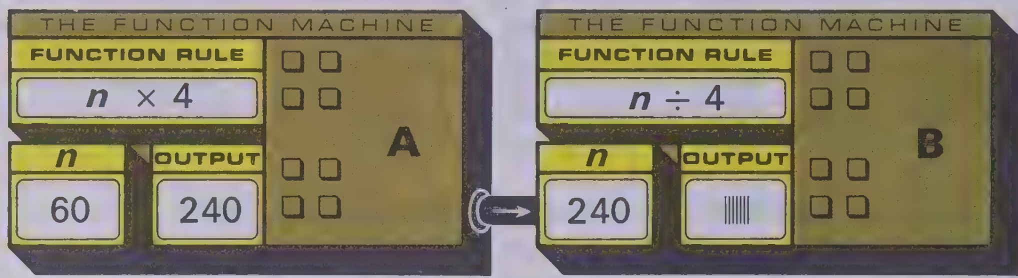
think

A special number pair are we.
100 is our sum, you see.
Our product lets you
know some more.
It's our sum times
twenty-four.

More practice, page A-7, Set 14

Discussing the Ideas

1. The two function machines help you see how special quotients are related to special products. What is the output number for the second machine?



2. You can find special quotients by thinking of missing factors. Solve the two equations.

You have found this quotient $630 \div 10 = n$	$n \times 10 = 630$ when you find this factor.

To find $630 \div 10$, think
"What number times 10 gives 630?"

3. Solve the equations.

A $34 \times 10 = a$ — $340 \div 10 = b$; $340 \div 34 = c$

B $100 \times 69 = a$ — $6900 \div 69 = b$; $6900 \div 100 = c$

C $10 \times 587 = a$ — $5870 \div 587 = b$; $5800 \div 10 = c$

D $7 \times 90 = a$ — $630 \div 90 = b$; $630 \div 7 = c$

E $6 \times 80 = a$ — $480 \div 80 = b$; $480 \div 6 = c$

F $80 \times 40 = a$ — $3200 \div 40 = b$; $3200 \div 80 = c$

1. Solve the equations.

A $a \times 10 = 560 \rightarrow 560 \div 10 = b$
B $a \times 48 = 480 \rightarrow 480 \div 48 = b$
C $a \times 10 = 720 \rightarrow 720 \div 10 = b$
D $a \times 100 = 5600 \rightarrow 5600 \div 100 = b$

think

Find 40 times me.
Divide the product by 4.
It's 700 you'll see.
I will tell no more.

WHO AM I?

2. Give the number for ***a***.
Then give the number for ***b***.

A $a \times 40 = 240 \rightarrow 240 \div 40 = b$
B $a \times 9 = 360 \rightarrow 360 \div 9 = b$
C $a \times 70 = 490 \rightarrow 490 \div 70 = b$
D $a \times 300 = 2700 \rightarrow 2700 \div 300 = b$
E $a \times 6 = 4800 \rightarrow 4800 \div 6 = b$
F $a \times 7 = 5600 \rightarrow 5600 \div 7 = b$

3. Give the quotients.

A $28 \div 7$
B $280 \div 7$
C $2800 \div 7$
D $280 \div 70$
E $280 \div 4$
F $560 \div 70$
G $3600 \div 400$
H $6300 \div 7$
I $5400 \div 900$
J $36\,000 \div 6$
K $1000 \div 10$
L $810 \div 90$
M $42\,000 \div 7000$
N $4800 \div 6$
O $400 \div 8$
P $5600 \div 800$

4. Solve the equations.

A $a \times 30 = 1500 \rightarrow 1500 \div 30 = b$
B $a \times 50 = 3500 \rightarrow 3500 \div 50 = b$
C $a \times 60 = 4200 \rightarrow 4200 \div 60 = b$
D $a \times 40 = 2800 \rightarrow 2800 \div 40 = b$
E $a \times 70 = 2100 \rightarrow 2100 \div 70 = b$
F $a \times 90 = 5400 \rightarrow 5400 \div 90 = b$
G $a \times 80 = 3200 \rightarrow 3200 \div 80 = b$

5. Give the quotients.

A $2400 \div 60$
B $1600 \div 40$
C $2700 \div 30$
D $1400 \div 70$
E $1000 \div 50$
F $2400 \div 80$
G $1200 \div 30$
H $4800 \div 60$
I $3600 \div 90$
J $1800 \div 20$
K $6300 \div 70$
L $7200 \div 80$
M $8100 \div 90$
N $3200 \div 40$

More practice, page A-8, Set 15

Discussing the Ideas

Each problem below requires an **estimate**. In order to estimate the answers to problems such as these, it is helpful to add, subtract, multiply, or divide, using multiples of 10 or 100 that are "close" to the numbers in the problems.

1. Suppose you purchase items costing the amounts shown. Without finding the exact total amount, how would you decide quickly whether or not you could pay the bill with $20?

$1.98
3.49
6.52
4.98
1.06

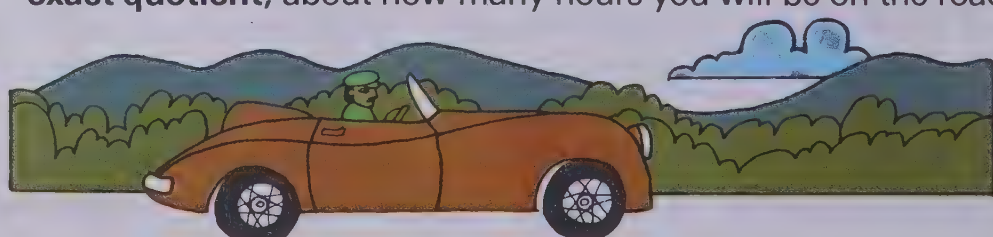
2. Suppose you were going to plant clover on a plot of ground that is 98 metres long and 56 metres wide. To decide how much grass seed to buy, you must find the area of the plot of ground. How could you quickly find **about** how many square metres of ground you have?

98 metres

56 metres

3. Suppose that there are 9026 people in a city and that 2987 of them are men. How could you quickly find, without finding the exact difference, **about** how many women and children live in this city?

4. Suppose you are driving 327 kilometres. If you travel 81 kilometres each hour, how could you tell, **without finding the exact quotient,** about how many hours you will be on the road?



Write an equation that shows how you estimate the answer for each exercise. Use multiples of 10 for exercise 1 and multiples of 100 for exercise 2.

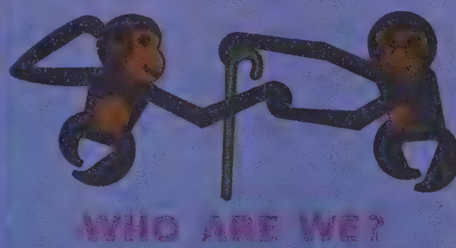
1. A $19 + 18 + 42$
Answer:
$20 + 20 + 40 = 80$
B 45×79
Answer:
$50 \times 80 = 4000$
C $48 + 51 + 47$
D $29 + 68 + 97$
E $323 + 639$
F $163 - 88$
G $361 \div 39$
H 19×48
I 78×57
J $265 \div 91$
K $104 - 48$
L 45×65
M 36×34
N $548 \div 9$
O 403×69
P 98×11
Q $526 - 109$

2. A $387 + 416 + 721$
Answer:
$400 + 400 + 700 = 1500$
B $5617 \div 811$
Answer:
$5600 \div 800 = 7$
C $519 + 787$
D $706 - 289$
E 95×116
F $2431 \div 590$
G $5489 - 2496$
H $409 + 688 + 716$
I $1607 - 569$
J $2050 \div 271$
K 97×683
L 667×123
M $2348 + 653$
N $543 + 553$
O $4029 \div 817$
P $5359 \div 94$
Q 350×738
R $5327 - 1965$
S $3276 + 3341$
T 651×749

3. **Estimate** the missing factor. Then **estimate** the quotient.
A $n \times 28 = 308$
$308 \div 28 = n$
B $n \times 31 = 1519$
$1519 \div 31 = n$
C $n \times 97 = 5917$
$5917 \div 97 = n$

think

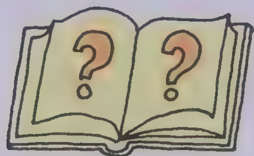
I'm ninety and a little more.
My partner is a little less.
You miss our product by just 4
When 90 nineties is your guess.



Let's use our estimating skills.

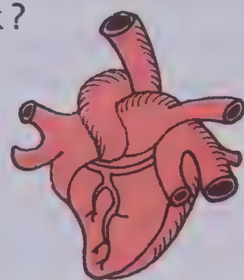
Investigating the Ideas

Choose one of these estimation projects.

1. 

   How many problems are in your mathematics book?

2. What is the total weight of all the children in your classroom?



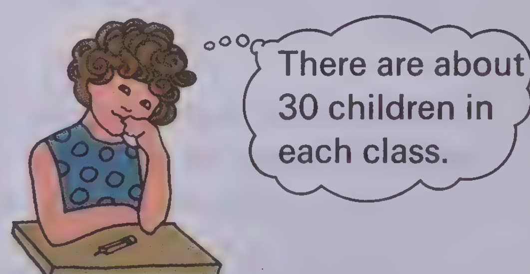
3. How many times does your heart beat in a year?

? Can you give an estimate and write an explanation of how you made your estimate?

Discussing the Ideas

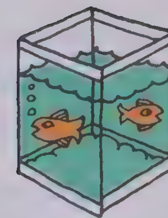
1. A Estimate the number of children in your school.
   B Explain how you made your estimate.

2. In Della's school there are 4 fifth-grade classes. What should be Della's estimate for the number of children in the fifth grade?

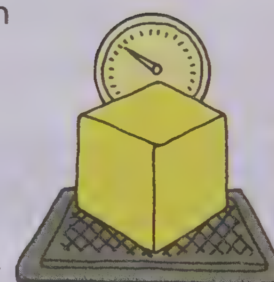


3. If there are 1760 bricks in 1 pile, which is the best estimate of the number of bricks in 5 similar piles?
   A 5000 bricks B 9000 bricks C 12 000 bricks

Choose the best estimate from the three estimates given for each problem.



1. One tank of water can hold about 12 fish. Choose the best estimate for the number of fish 88 tanks would hold:
   A 800 fish B 100 fish C 900 fish



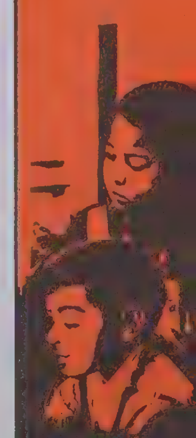
2. A piece of gold weighs about 19 times as much as a piece of ice the same size. If an ice cube weighs 62 g, choose the best estimate for the weight of a similar block of gold.
   A 80 g B 1000 g C 1200 g

3. A piece of aluminum weighs about 169 grams. A lead pipe weighs 850 grams. Estimate the difference in the weights:
   A 600 grams B 700 grams C 800 grams

4. The distance between the posts is about 39 centimetres. Estimate the distance in centimetres between 72 such posts:
   A 2800 cm B 2100 cm C 280 cm

5. One truck holds 36 barrels. Estimate the number of trucks needed to hold 278 barrels:
   A 7 B 9 C 80

6. A car travelling 96 kilometres per hour is travelling about 27 metres per second. Estimate the distance travelled in 55 seconds:
   A 4000 metres B 1 kilometre C 1400 metres



7. Sound travels 322 metres per second in air. Estimate how many seconds it takes sound to travel one kilometre in air:
   A 2 B 3 C 4

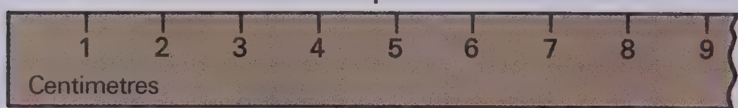
More practice, page A-8, Set 16

Estimation for Fun

1. Count the number of times your heart beats in one minute. Estimate the number of beats in one hour.

2. One of the longest words in the Oxford Dictionary is shown below. Count the numbers of letters in a 1-centimetre space. Then estimate the number of letters in the word.

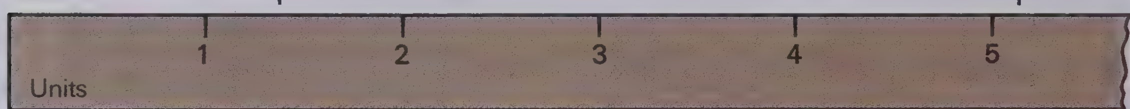
   f l o c c i n a u c i n i h i l i p i l i f i c a t i o n



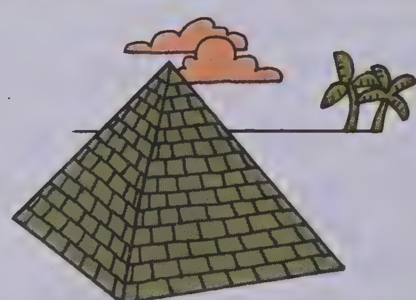
(The Oxford Dictionary defines this word as "the action of estimating as worthless.")

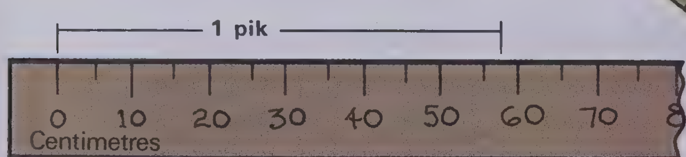
3. One of the longest names known is that of a Hawaiian girl. Estimate the number of letters in her name.

   Kuuleikailialohaopiilaniwailauokekoaulumahiehiekealaomaonaopiikea



4. The picture shows the length of a unit, called a **pik**, which is used in Egypt. The pyramid of Cheops, near Cairo, was originally about 253 piks high. Estimate the height of the pyramid in centimetres.

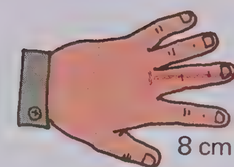




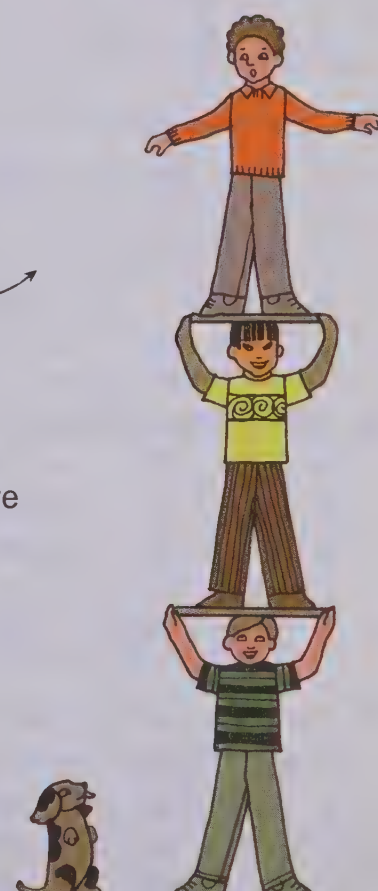
★ 5. Use your mathematics book and your ruler to estimate the number of pages in a book 3 metres thick.

6. It takes about 48 marbles (of approximately 1 cm diameter) to cover the bottom of a litre milk carton. Estimate the number of marbles the carton holds when it is filled to the red arrow.

7. The height of a full-grown human is about 21 times the length of his middle finger.
   A Estimate the height of a person whose middle finger is the length shown.
   ★ B Estimate the heights of a tall adult and a short adult by measuring the lengths of their middle fingers. Check your estimates by finding their correct heights.



8. Measure the height (in centimetres) of three of your classmates.
   A Estimate the height of a tower formed by the three classmates as shown.
   B If all the children in your class formed a tall tower, about how many metres high would it be ?



★ 9. Make the following estimates and then compare your answers with those of your classmates.
   A Estimate the number of litres of milk you drink in a week, a month, and a year.
   B Estimate in kilograms the total weight of all the children in your class.
   C Estimate in metres and in kilometres the distance you walk in one week.
   D Estimate in square metres the area of your classroom floor.

1. Find the products.

A 10×10 B 10×100 C 100×100 D 10×1000

2. Find the products.

A 10×6	C 10×28	E 6×100	G 18×100	I 100×28
B 17×10	D 10×35	F 100×9	H 27×100	J 54×100

3. Give the number for ***a***.
Then give the number for ***b***.

A $40 \times 3 \times 10 = a \longrightarrow 40 \times 30 = b$

B $50 \times 7 \times 10 = a \longrightarrow 50 \times 70 = b$

C $90 \times 6 \times 10 = a \longrightarrow 90 \times 60 = b$

D $70 \times 9 \times 10 = a \longrightarrow 70 \times 90 = b$

4. Find the products.

A 30×30	F 80×50
B 40×20	G 90×6
C 60×10	H 10×90
D 4×70	I 90×90
E 70×40	J 7×30

5. Find the quotients.

A $900 \div 30$	F $4000 \div 50$
B $800 \div 40$	G $540 \div 6$
C $600 \div 10$	H $900 \div 90$
D $280 \div 7$	I $8100 \div 90$
E $2800 \div 40$	J $210 \div 30$

think

Copy the last six equations and give the numbers covered by the red screens.

1, 2, 3 $\quad 1 \times 3 = (2 \times 2) - 1$

2, 3, 4 $\quad 2 \times 4 = (3 \times 3) - 1$

3, 4, 5 $\quad 3 \times 5 = (4 \times 4) - 1$

4, 5, 6 $\quad 4 \times 6 = (5 \times 5) - 1$

$5 \times 7 = (\square \times \square) - 1$

$7 \times 9 = (\square \times \square) - 1$

$9 \times 11 = (\square \times \square) - 1 = \square$

$19 \times 21 = (\square \times \square) - 1 = \square$

$49 \times 51 = (\square \times \square) - 1 = \square$

$99 \times 101 = (\square \times \square) - 1 = \square$

6. Find the products and quotients.

A 50×300	E 800×50	I 60×90	M $5400 \div 90$
B 70×400	F 90×400	J $3000 \div 6$	N $40\,000 \div 80$
C 600×40	G 600×70	K $28\,000 \div 70$	O $42\,000 \div 70$
D 50×600	H 80×900	L $15\,000 \div 300$	P $81\,000 \div 90$

7. Give the multiple of 10 that is closest to each number.
   A 39 C 67 E 866
   B 41 D 134 F 1068

8. Give the multiple of 100 that is closest to each number.
   A 509 C 86 E 6945
   B 197 D 1367 F 5999

think

How many 1-digit numbers are there? How many 2-digit numbers are there? Estimate the number of 3-digit numbers.

9. Write an equation that shows how to estimate the answer for each exercise. Use multiples of 10.

A $39 + 78$	D 27×52	G 14×98	J $801 - 98$
B $397 + 88$	E $423 \div 58$	H 65×35	K 48×54
C $503 - 59$	F 85×46	I $719 \div 83$	L $561 \div 72$

10. Write an equation that shows how to estimate the answer for each exercise. Use multiples of 100.

A $513 + 978$	C 92×112	E $6729 - 3687$
B $1503 - 498$	D $3543 \div 726$	F $4864 \div 697$

11. Ironwood is one of the heaviest types of wood. It weighs about 1450 grams per cubic decimetre. Balsa is one of the lightest types of wood. It weighs about 125 grams per cubic decimetre.
   A Estimate the weight of a cubic metre of ironwood.
   B Estimate the weight of a cubic metre of balsa wood.

12. Some large cars are nearly 6 metres long. One of the largest buses ever built was $17\frac{1}{2}$ metres long. Estimate the number of cars that could fit into a space long enough for the bus.

★ 13. One of the longest bicycles ever built was a 10-seater built in 1898. It was 763 centimetres long. Estimate the difference between the length of this bicycle and the total length of 10 ordinary bicycles, each 175 centimetres long.

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Place value

1. Solve the equations.

A $9 \times 6 = n$ D $63 \div 9 = n$ G $72 = n \times 8$ J $8 = 56 \div n$
B $8 + 7 = n$ E $50 = n \times 10$ H $n = 54 \div 9$ K $49 \div 7 = n$
C $16 - 7 = n$ F $67 = n + 47$ I $n = 9 \times 7$ L $n \times 10 = 90$

2. Find the products.

A 7×4 D 5×8 G 7×5 J 8×8 M 5×9
B 4×9 E 9×8 H 7×9 K 6×7 N 7×7
C 8×6 F 3×7 I 8×4 L 9×6 O 7×8

3. Solve the equations.

A $837 = 800 + n + 7$ D $864 = 800 + 50 + n$
B $973 = n + 70 + 3$ E $358 = 200 + n + 8$
C $796 = n + 90 + 700$ F $592 = n + 190 + 2$

4. Solve the equations.

A $843 = (n \times 100) + 43$ H $803 = (80 \times 10) + n$
B $843 = (n \times 10) + 3$ I $803 = (79 \times 10) + n$
C $369 = (n \times 100) + 69$ J $700 = (70 \times 10) + n$
D $369 = (n \times 10) + 9$ K $700 = (69 \times 10) + n$
E $4582 = (4 \times n) + 582$ L $3005 = (300 \times 10) + n$
F $4582 = (45 \times n) + 82$ M $3005 = (299 \times 10) + n$
G $4582 = (458 \times n) + 2$ N $3023 = (29 \times 100) + n$

5. Solve the equations.

A $3 \times 57 = (3 \times 50) + (3 \times n)$ C $5 \times 84 = (5 \times n) + (5 \times 4)$
B $4 \times 36 = (n \times 30) + (n \times 6)$ D $3 \times 24 = (3 \times n) + (3 \times 4)$
E $3 \times 124 = (3 \times n) + (3 \times 20) + (3 \times 4)$
F $3 \times 5124 = (3 \times n) + (3 \times 100) + (3 \times 20) + (3 \times 4)$

You are invited to explore

ACTIVITY CARD 5
Page 335

Sequoia Trees

The giant sequoia trees have been said to be the "oldest and largest living things on earth." Some are from 3000 to 4000 years old. One of the largest sequoias, called the "General Sherman," is 83 metres tall. Another type of sequoia, called a redwood tree, grows taller but not as large. A tree called the "New Tree" is one of the tallest of the redwoods. It is 112 metres tall.

1. The Toronto Dominion Centre is 219 metres tall. Find the difference between the height of the building and the height of the "New Tree."

2. How much taller is the "New Tree" than the "General Sherman"?

3. Is the Toronto Dominion Centre 2 times as tall as the "New Tree"?

4. It takes 17 men with outstretched arms to encircle the "General Sherman." Suppose a man with outstretched arms can reach 183 cm. Give the circumference of the "General Sherman."

5. The "General Sherman" contains enough wood to build 35 small five-room houses. Find the number of houses that could be built with the lumber from 24 such trees.

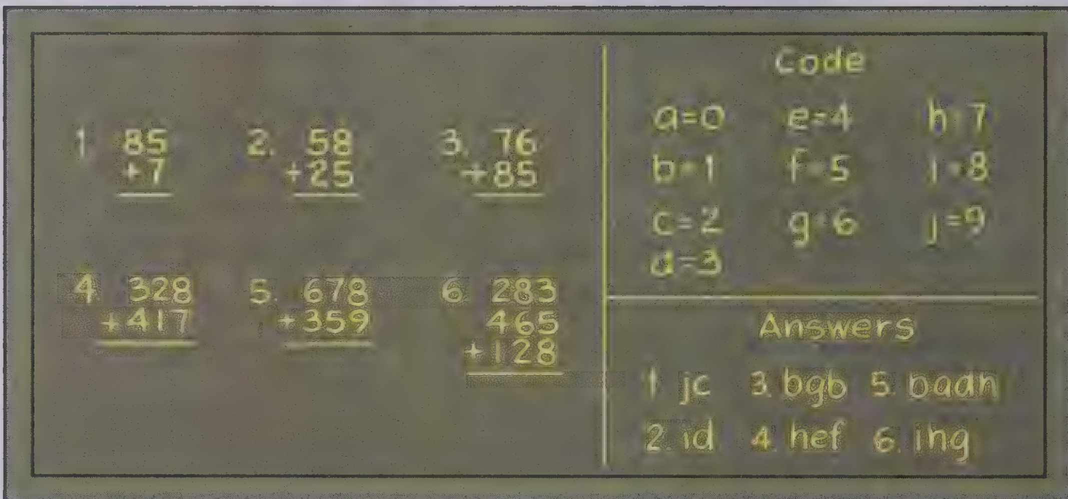
6. The oldest dated giant sequoia was 3212 years old when it was cut down in 1892. Find the date when this tree started to grow.

6 Computing

How are your adding skills?

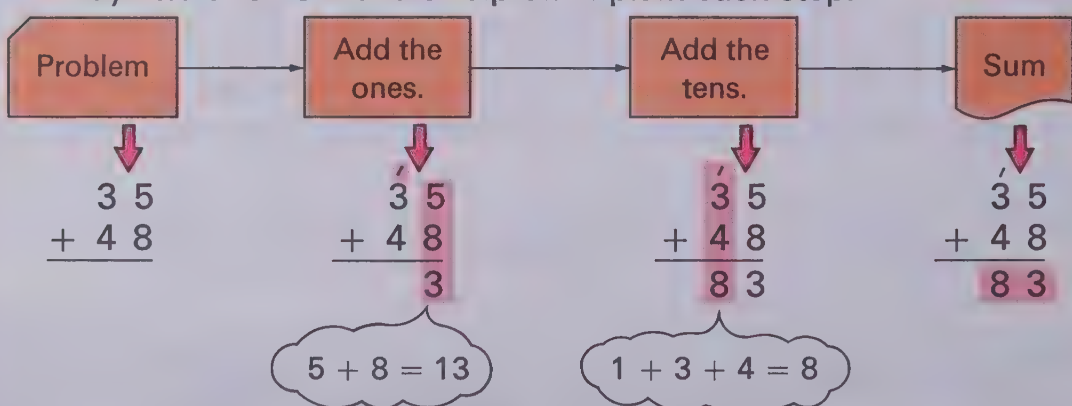
Investigating the Ideas

? Can you find the sums without missing more than one?	Use the code to grade your own paper.



Discussing the Ideas

1. If you missed more than one of the addition problems, you may find this flow chart helpful. Explain each step.



2. Can you correct each mistake you made in the Investigation?

1. Find the sums.

A 87 + 6	B 8 + 54	C 9 + 36	D 18 + 7	E 35 + 8	F 24 + 7

2. Find the sums.

A 64 + 21	B 48 + 36	C 67 + 39
D 58 + 67	E 95 + 87	F 64 + 46

think

Find the missing digits.

2 9 ▒7
5 35 ▒
7 624
+ ▒ 446
23 ▒95

3. Do not copy the exercises below. Just find each sum and write it on your paper.

A 56 + 8	D 5 + 9 + 8	G 6 + 7 + 8 + 7
B 47 + 9	E 6 + 7 + 5	H 8 + 9 + 7 + 8
C 89 + 8	F 6 + 7 + 8	I 6 + 9 + 5 + 8

4. Find the sums.

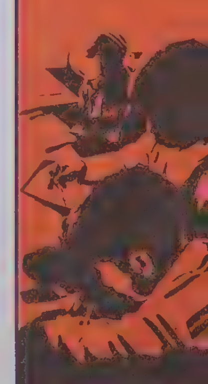
A 348 + 692 + 843	B 784 + 65 + 892	C 743 + 806 + 59	D 984 + 376 + 977	E 856 + 972 + 800
F 9283 + 7651 + 8420 + 9165	G 9037 + 8066 + 579 + 8432	H 8651 + 784 + 97 + 8465	I 982 + 7655 + 8 + 93	J 7836 + 965 + 1749 + 88

5. Find the sums.

A 6784 + 932 + 89 + 7864 + 35
B 6748 + 297 + 3608 + 94
C 76 528 + 9328 + 657 + 9827
D 6427 + 15 348 + 19 + 384

★ **6.** In these exercises, some of the digits are covered. Give all possible digits for the ▒.

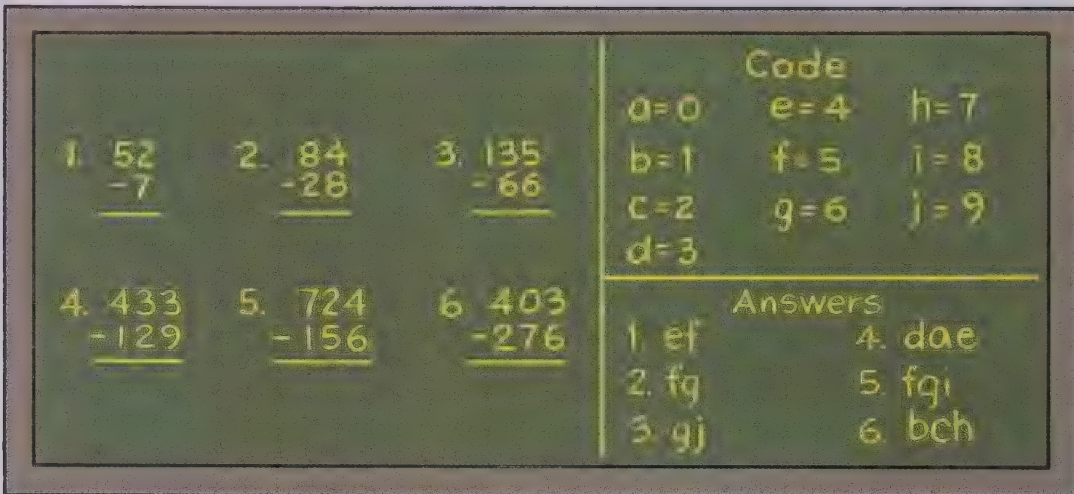
A 5▒ + 2▒ = ▒ ▒
B 4▒ + 8▒ = ▒▒▒
C ▒9▒ + ▒5▒ = ▒▒▒



More practice, page A-9, Set 17

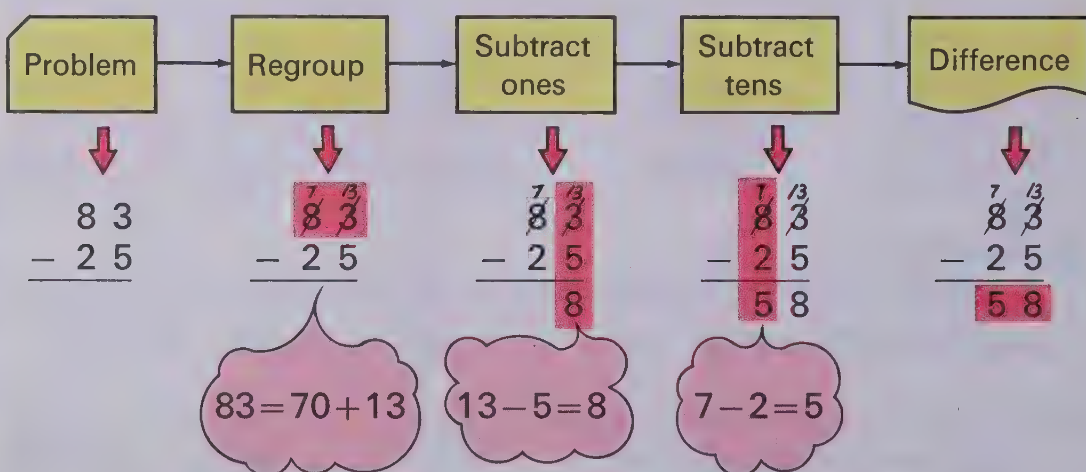
Investigating the Ideas

? Can you find the differences without missing more than one?	Use the code to grade your own paper.



Discussing the Ideas

1. If you missed more than one of the subtraction problems, you may find this flow chart helpful. Explain each step.



2. Can you correct each mistake you made in the Investigation?

1. Find the differences.

A $\begin{array}{r} 64 \\ -28 \\ \hline \end{array}$	B $\begin{array}{r} 72 \\ -34 \\ \hline \end{array}$	C $\begin{array}{r} 83 \\ -29 \\ \hline \end{array}$	D $\begin{array}{r} 27 \\ -18 \\ \hline \end{array}$	E $\begin{array}{r} 95 \\ -75 \\ \hline \end{array}$	F $\begin{array}{r} 61 \\ -54 \\ \hline \end{array}$
G $\begin{array}{r} 158 \\ -69 \\ \hline \end{array}$	H $\begin{array}{r} 137 \\ -58 \\ \hline \end{array}$	I $\begin{array}{r} 62 \\ -21 \\ \hline \end{array}$	J $\begin{array}{r} 157 \\ -78 \\ \hline \end{array}$	K $\begin{array}{r} 93 \\ -65 \\ \hline \end{array}$	L $\begin{array}{r} 80 \\ -56 \\ \hline \end{array}$
M $\begin{array}{r} 182 \\ -91 \\ \hline \end{array}$	N $\begin{array}{r} 156 \\ -87 \\ \hline \end{array}$	O $\begin{array}{r} 327 \\ -164 \\ \hline \end{array}$	P $\begin{array}{r} 643 \\ -156 \\ \hline \end{array}$	Q $\begin{array}{r} 827 \\ -287 \\ \hline \end{array}$	R $\begin{array}{r} 650 \\ -193 \\ \hline \end{array}$
S $\begin{array}{r} 723 \\ -327 \\ \hline \end{array}$	T $\begin{array}{r} 8427 \\ -6543 \\ \hline \end{array}$	U $\begin{array}{r} 6513 \\ -2465 \\ \hline \end{array}$	V $\begin{array}{r} 4327 \\ -1651 \\ \hline \end{array}$	W $\begin{array}{r} 7214 \\ -3888 \\ \hline \end{array}$	

2. The two examples show a convenient way to regroup when there are zeros involved. Complete each subtraction.

Think 39 tens and 12.

A $\begin{array}{r} 402 \\ -136 \\ \hline \end{array}$ → $\begin{array}{rrrr} & {\scriptstyle 3} & {\scriptstyle 9} & {\scriptstyle 12} \\ & \cancel{4} & \cancel{0} & \cancel{2} \\ - & 1 & 3 & 6 \\ \hline \end{array}$

Think 699 tens and 14.

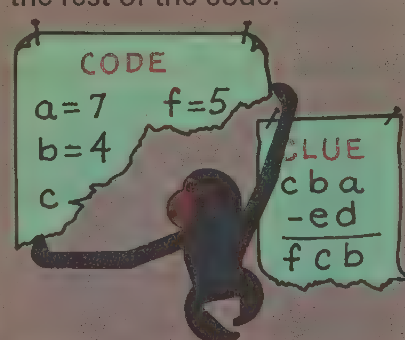
B $\begin{array}{r} 7004 \\ -4265 \\ \hline \end{array}$ → $\begin{array}{rrrrr} & {\scriptstyle 6} & {\scriptstyle 9} & {\scriptstyle 9} & {\scriptstyle 14} \\ & \cancel{7} & \cancel{0} & \cancel{0} & \cancel{4} \\ - & 4 & 2 & 6 & 5 \\ \hline \end{array}$

3. Find the differences.

A $\begin{array}{r} 701 \\ -137 \\ \hline \end{array}$	B $\begin{array}{r} 600 \\ -133 \\ \hline \end{array}$	C $\begin{array}{r} 507 \\ -489 \\ \hline \end{array}$
D $\begin{array}{r} 5726 \\ -4314 \\ \hline \end{array}$	E $\begin{array}{r} 6592 \\ -1399 \\ \hline \end{array}$	F $\begin{array}{r} 8462 \\ -3597 \\ \hline \end{array}$
G $\begin{array}{r} 7302 \\ -1765 \\ \hline \end{array}$	H $\begin{array}{r} 8000 \\ -1672 \\ \hline \end{array}$	I $\begin{array}{r} 5001 \\ -3679 \\ \hline \end{array}$
J $\begin{array}{r} 600 \\ -257 \\ \hline \end{array}$	K $\begin{array}{r} 800 \\ -69 \\ \hline \end{array}$	L $\begin{array}{r} 6034 \\ -1655 \\ \hline \end{array}$
M $\begin{array}{r} 8006 \\ -1739 \\ \hline \end{array}$	N $\begin{array}{r} 6205 \\ -1984 \\ \hline \end{array}$	O $\begin{array}{r} 7026 \\ -1550 \\ \hline \end{array}$

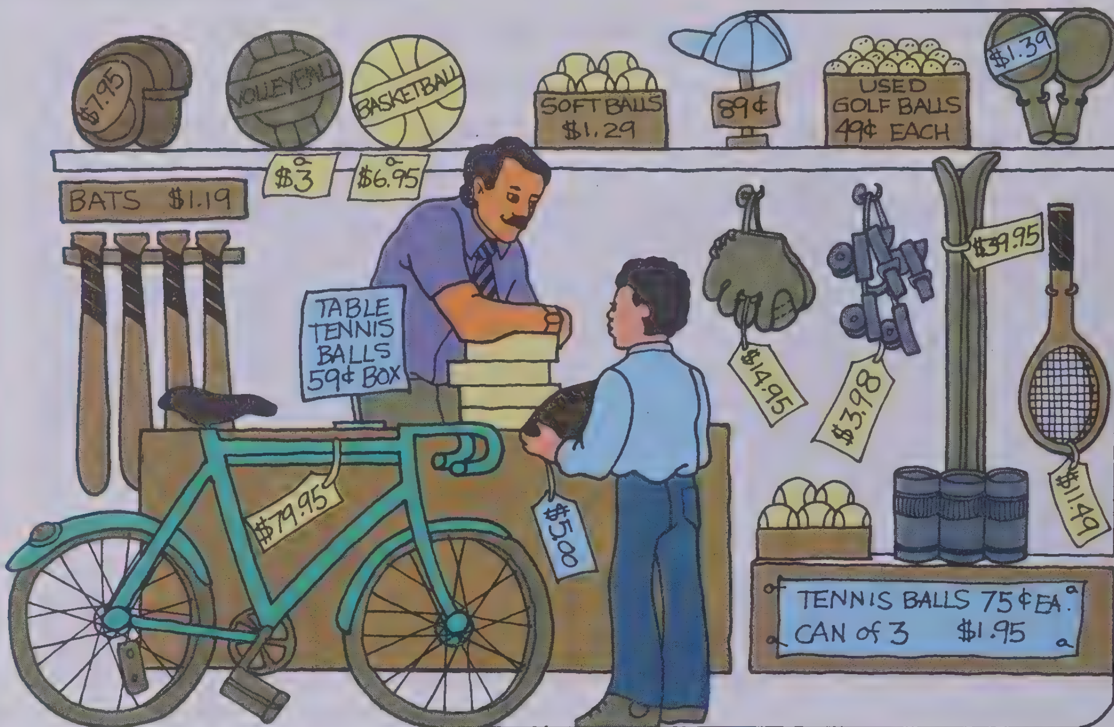
think

Here is part of a code and a clue. See if you can find the rest of the code.



More practice, page A-9, Set 18

Investigating the Ideas



? You have $20.00 to spend. Can you figure out a way to spend nearly all of it, except about a dollar for tax?

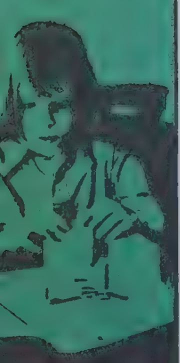
Discussing the Ideas

1. When you add and subtract amounts of money, what must you be sure to do when you write your problem on paper? Why?

2. A What two items are more than $20.00?
B What items are between 10 and 20 dollars?

3. Explain why the total is not correct. How would you write the problem?

ball cap	89¢
volleyball	$3
total	92¢



1. How much more does the football cost than the roller skates?

2. How much would the tennis racket and a can of 3 tennis balls cost?

3. The bicycle costs how much more than the skis?

4. How much would it cost to buy the ball glove, a softball, and a bat?

5. If you bought the basketball for \$6.95 and gave the clerk \$20, how much money should you get back?

6. How much money do you save by getting a can of 3 tennis balls rather than 3 separate balls?

7. How much would it cost to buy 4 table tennis paddles and a box of table tennis balls?

★ 8. What two things could you buy (without tax) so that you would get 5 cents in change if you gave the clerk 20 dollars?

9. Find the total amounts.

A	B	C	D	E
\$3.27	\$9.23	\$2.39	\$7.45	\$5.67
4.68	6.58	6.98	1.68	0.85

F	G
\$4.26	\$7.37
3.78	0.06
4.65	5.04

H \$6.73, \$5.86, \$.95, \$10.75, \$64.30

I \$46.20, \$34.95, \$68.50, \$94.27

10. Find the difference in the amounts.

A	B	C
\$6.78	\$21.95	\$7.98
2.39	16.56	4.06

D \$78.60, \$21.78

E \$2.98, \$54.50

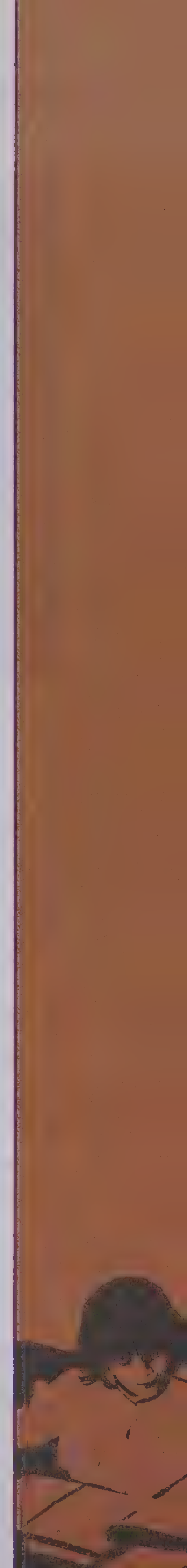
F \$6.75, \$29.95

G \$60.00, \$29.88

think

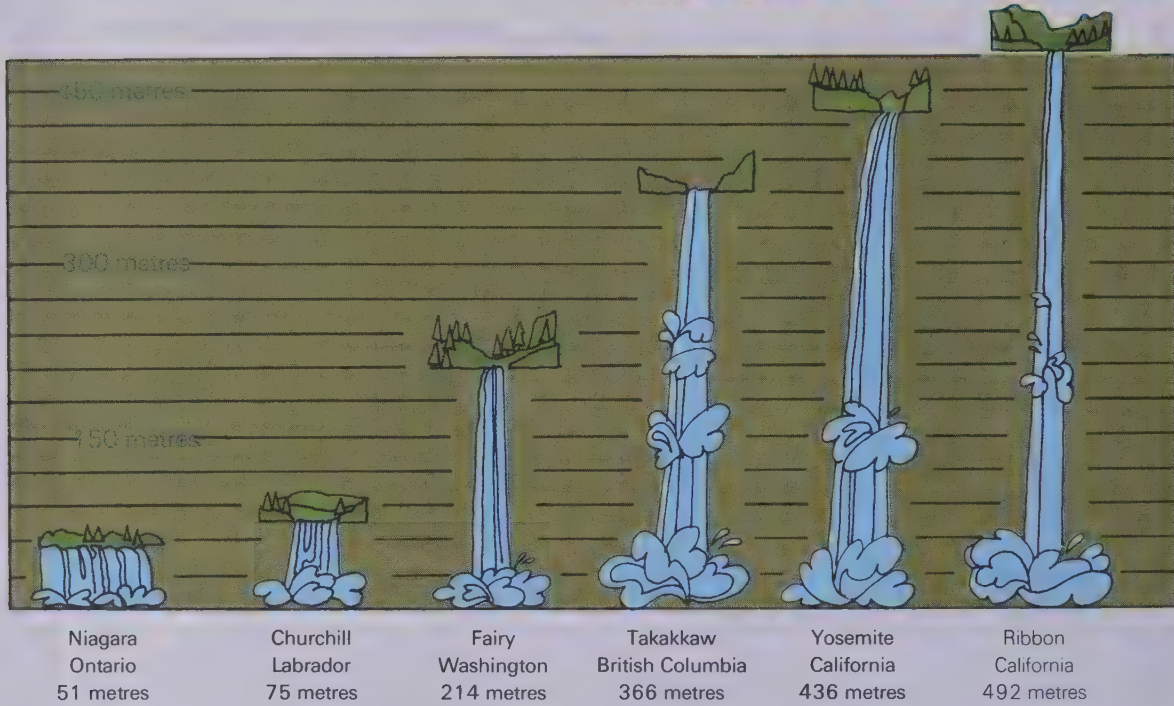
Replace each ▥ with the same digit.

```
   3 ▥52
   1 07▥
   5 4▥2
+  ▥73▥
  1▥92▥
```



Solving Story Problems

Waterfalls of North America



1. Tell how much farther the water falls in the first waterfall than in the second.

- **A** Ribbon, Yosemite
- **B** Ribbon, Takakkaw
- **C** Yosemite, Niagara
- **D** Churchill, Niagara
- **E** Fairy, Churchill
- **F** Takakkaw, Churchill
- **G** Takakkaw, Niagara
- **H** Fairy, Niagara
- **I** Ribbon, Niagara

2. At Angel Falls in Venezuela the water falls 488 metres farther than at Ribbon Falls. Give the number of metres for Angel Falls.

3. Yosemite Falls has 3 sections. The upper falls is shown above. The water drops 206 metres in the middle section and 97 metres in the lower falls. Give the total distance that the water drops

4. At Multnomah Falls in Oregon the water drops a distance 176 metres less than at Takakkaw Falls. What is the height of Multnomah Falls ?

★ **5.** Place Ville-Marie in Montreal is 117 metres higher than Churchill Falls. How much higher is Takakkaw than Place Ville-Marie ?

SUSPENSION BRIDGES

Bridge	Location	Year built	Height above water (Metres)	Length of main span (Metres)
Verrazano-Narrows	New York City	1964	70	1300
Golden Gate	San Francisco Bay	1937	73	1280
Tacoma	Washington	1952	56	854
Pierre Laporte	Quebec	1972	45	668
Lion's Gate	Vancouver	1939	46	473
Narrows Bridge	Halifax	1969	48	421

1. How much longer is the main span of the Verrazano-Narrows Bridge than the Pierre Laporte ?

2. How many years old was the Golden Gate Bridge when the Verrazano-Narrows Bridge was built ?

3. How much longer is the main span of the Golden Gate Bridge than that of Halifax's Narrows Bridge ?

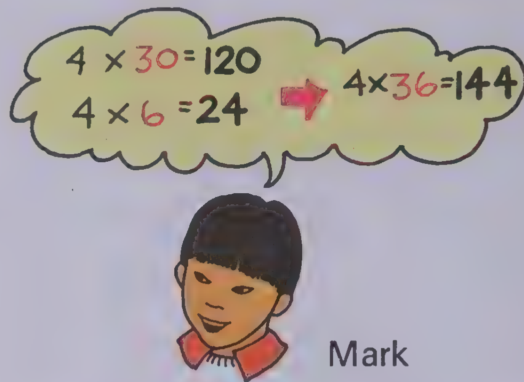
4. The Lion's Gate Bridge was built how many years before the Tacoma Bridge ?

5. The highest bridge listed in the chart is how much higher than the lowest ?

6. What is the total length of the main spans for these three bridges: the Lion's Gate, the Pierre Laporte, and the Narrows ?

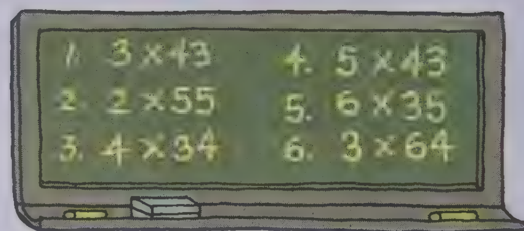
Investigating the Ideas

The distributive principle helps you find products like 4×36 without doing any writing. Study the example. Now test yourself and grade your own paper.



?

Can you write just the answers for these problems in 2 minutes?



Discussing the Ideas

1. Use this example of the distributive principle to explain how Mark found the product 4×36.

$$4 \times 36 = (4 \times 30) + (4 \times 6)$$

2. Give the numbers for a and b. Then give the product for c.

A $3 \times 40 = a$, $3 \times 2 = b$; $3 \times 42 = c$

B $2 \times 50 = a$, $2 \times 4 = b$; $2 \times 54 = c$

C $4 \times 20 = a$, $4 \times 3 = b$; $4 \times 23 = c$

D $6 \times 40 = a$, $6 \times 5 = b$; $6 \times 45 = c$

E $3 \times 50 = a$, $3 \times 6 = b$; $3 \times 56 = c$

F $2 \times 90 = a$, $2 \times 6 = b$; $2 \times 96 = c$

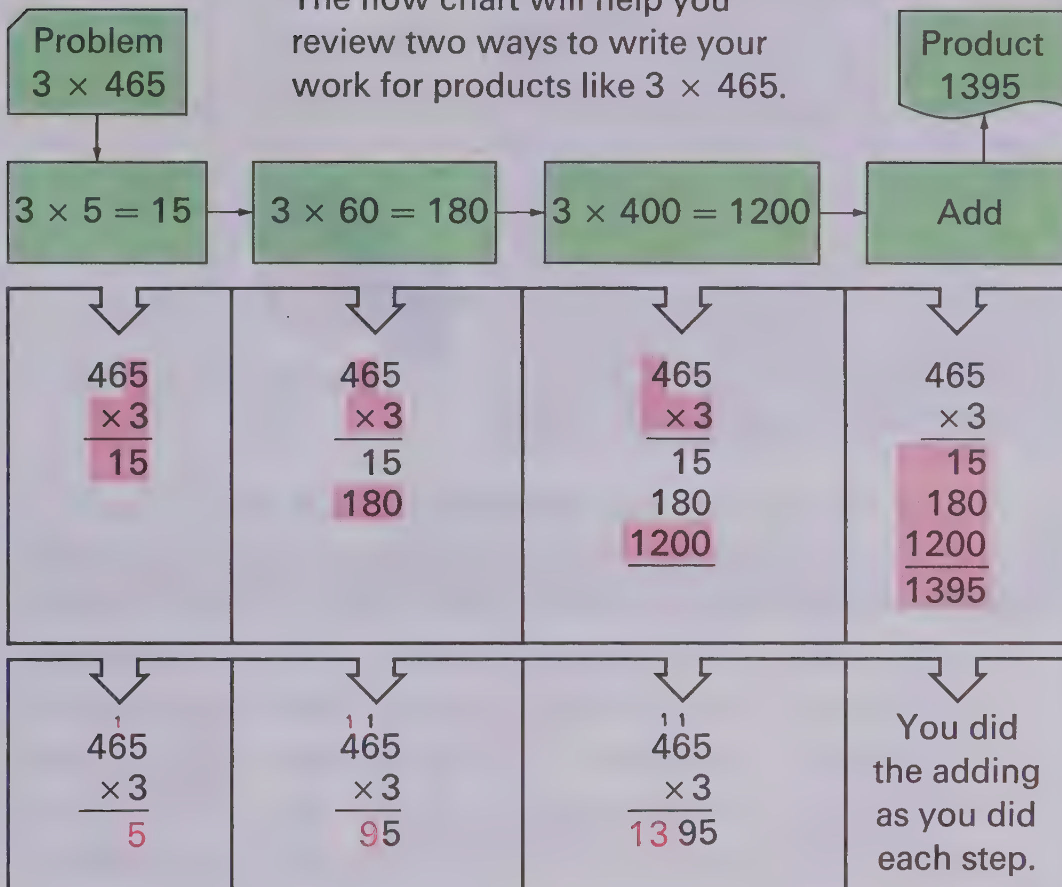
G $5 \times 70 = a$, $5 \times 3 = b$; $5 \times 73 = c$

H $4 \times 70 = a$, $4 \times 4 = b$; $4 \times 74 = c$

I $3 \times 60 = a$, $3 \times 5 = b$; $3 \times 65 = c$

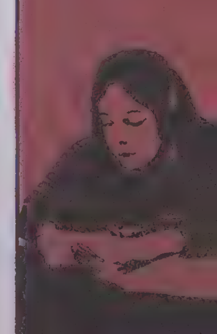
J $2 \times 60 = a$, $2 \times 7 = b$; $2 \times 67 = c$

The flow chart will help you review two ways to write your work for products like 3 × 465.



Find the products.

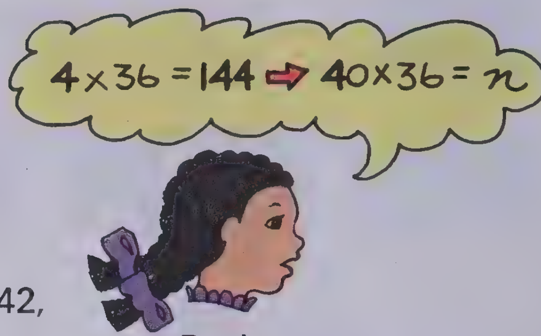
1. 34 × 4	**2.** 65 × 3	**3.** 38 × 6	**4.** 67 × 4	**5.** 75 × 7
6. 94 × 8	**7.** 185 × 4	**8.** 359 × 5	**9.** 287 × 4	**10.** 651 × 9
11. 704 × 8	**12.** 839 × 7	**13.** 534 × 9	**14.** 576 × 8	**15.** 680 × 9
16. 3276 × 2	**17.** 5843 × 3	**18.** 8439 × 4	**19.** 8037 × 4	**20.** 9206 × 8



More practice, page A-10, Set 19

Discussing the Ideas

1. If you can find products like 4×36, you can easily find products like 40×36. Can you solve the equation for Becky?



Becky

2. If you know the product 3×42, what must you multiply this product by to find 30×42?

3. Describe how you can find the product 40×23.

4. Solve the equations.

A $3 \times 54 = 162 \longrightarrow 30 \times 54 = m$	F $5 \times 67 = 335 \longrightarrow 50 \times 67 = v$
B $4 \times 26 = 104 \longrightarrow 40 \times 26 = p$	G $6 \times 83 = 498 \longrightarrow 83 \times 60 = x$
C $7 \times 34 = 238 \longrightarrow 70 \times 34 = r$	H $9 \times 54 = 486 \longrightarrow 90 \times 54 = w$
D $2 \times 93 = 186 \longrightarrow 20 \times 93 = s$	I $7 \times 86 = 602 \longrightarrow 86 \times 70 = y$
E $8 \times 35 = 280 \longrightarrow 80 \times 35 = t$	J $8 \times 49 = 392 \longrightarrow 80 \times 49 = z$

5. Explain each step in the example below.

Step 1	Step 2	Step 3
$\begin{array}{r} 37 \\ \times\ 48 \\ \hline 296 \end{array}$	$\begin{array}{r} 37 \\ \times\ 48 \\ \hline 296 \\ 1480 \\ \hline \end{array}$	$\begin{array}{r} 37 \\ \times\ 48 \\ \hline 296 \\ 1480 \\ \hline 1776 \end{array}$
$8 \times 37 = 296$	$40 \times 37 = 1480$	$296 + 1480 = 1776$

1. Find the products.

A 24×36	B 36×25	C 52×41
D 14×56	E 35×24	F 46×37
G 62×26	H 75×33	I 83×57
J 49×68	K 91×37	L 38×49

think

Each letter stands for a digit. Can you find each digit?

$$\begin{array}{r} \text{a a a a} \\ \times\ \text{a a} \\ \hline \text{a a a a} \\ \text{a a a a}\ \ \\ \hline \text{a b b b a} \end{array} \qquad \begin{array}{r} \text{c d d} \\ \times\ \text{c d d} \\ \hline \text{c d d d d} \end{array}$$

2. Study the example. Then find the products.

$34 \times 526 = (30 \times 526) + (4 \times 526)$		
Step 1	Step 2	Step 3
$\begin{array}{r} 526 \\ \times\ 34 \\ \hline 2104 \end{array}$	$\begin{array}{r} 526 \\ \times\ 34 \\ \hline 2104 \\ 15780 \\ \hline \end{array}$	$\begin{array}{r} 526 \\ \times\ 34 \\ \hline 2104 \\ 15780 \\ \hline 17884 \end{array}$
$4 \times 526 = 2104$	$30 \times 526 = 15\,780$	$2104 + 15\,780 = 17\,884$

A 256×25	B 324×36	C 514×23	D 137×45	E 284×62	F 891×34

3. Find the products.

A 341×3 | 341×90 | 341×200 → 341×293

B 956×8 | 956×40 | 956×300 → 956×348

More practice, page A-10, Set 20

Distances

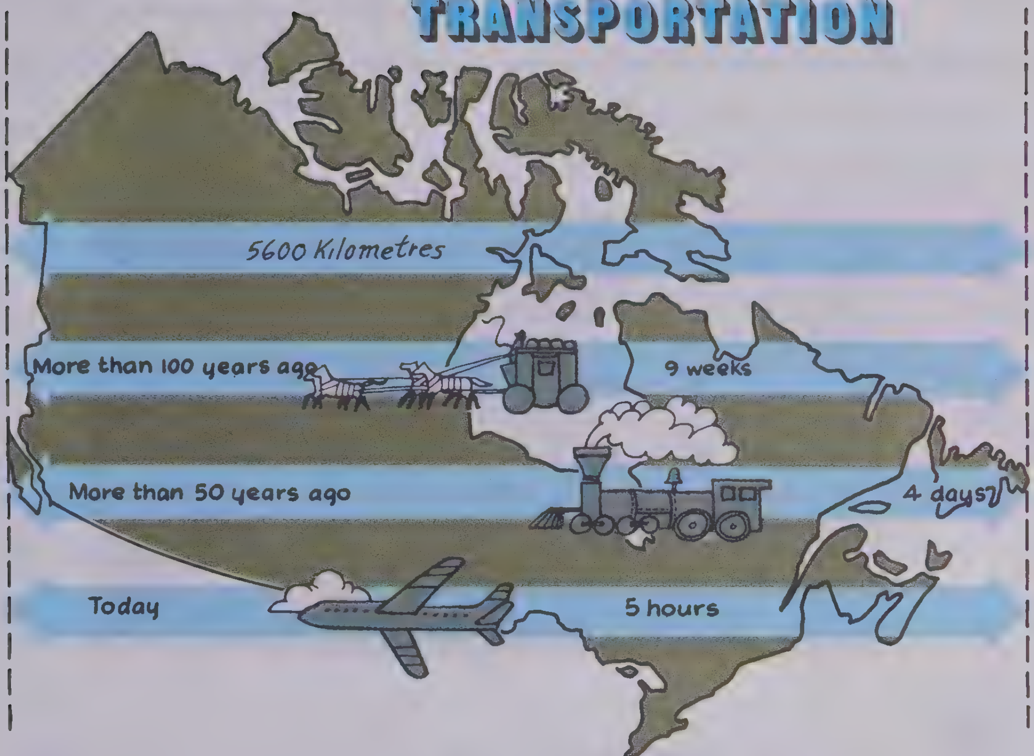
1. Give the total distance for each trip.
   A Montreal to Quebec to Halifax
   B Vancouver to Regina to Winnipeg to Sault Sainte Marie
   C Edmonton to Whitehorse to Vancouver to Edmonton

2. Use the map to plan these trips for the smallest number of kilometres.
   A Vancouver to Winnipeg
   B Edmonton to Sault Sainte Marie
   C Halifax to Sault Sainte Marie
   D Quebec to Whitehorse

3. Which distance is about 4 times as far as the distance from Montreal to Quebec ?

★ 4. Mr. Brown had driven his new car 1247 kilometres before leaving for a trip. When he got to one of the cities on the map, he had driven his car a total of 1817 kilometres. He went from this city to another city on the map, and by then he had driven 2617 kilometres. From what city on the map did Mr. Brown start?

TRANSPORTATION



1. About how many hours did it take the train to cross Canada ?

2. About how many minutes does it take the jet plane to cross the country ?

3. How many days did it take the stagecoach to cross Canada ? How many hours would this be? How many minutes ?

4. More than 100 years ago, the first train between Montreal and Vancouver took 5 days. How many hours was this ?

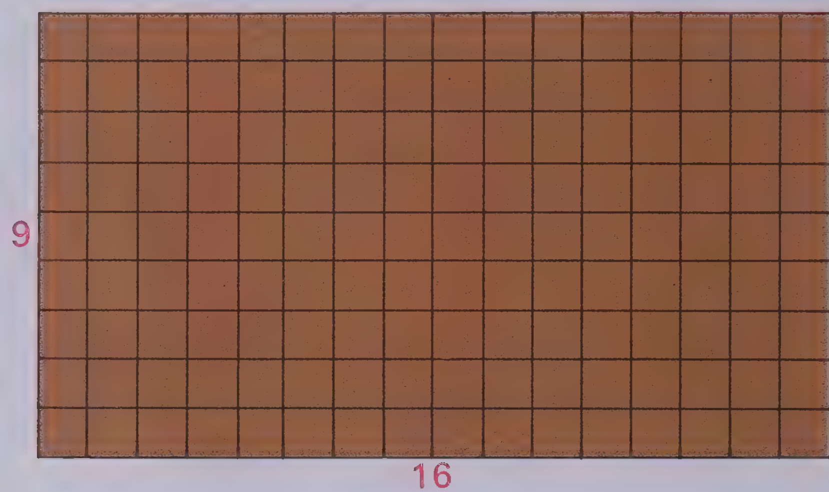
5. The trip from Montreal to Vancouver is about 869 kilometres less than a trip all the way across the country. About how far is the trip ?

6. Today a train can cross Canada in about 58 hours. Could a modern train make it across and back while the old train is going across ?

Investigating the Ideas

Here is a rectangular region that has an area of 144 square units.

$9 \times 16 = 144$

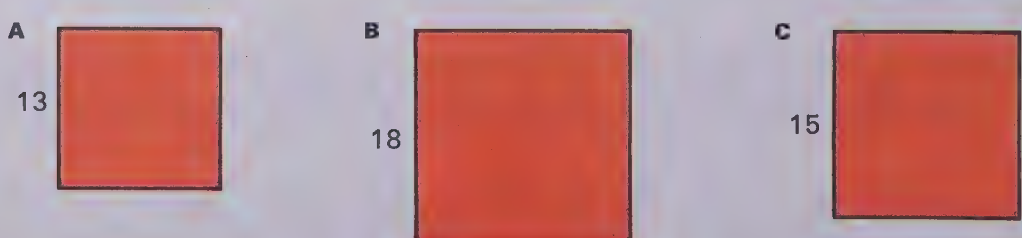


?

How many other **rectangular** regions that have an area of 144 square units can you find?	Draw and color at least one of the regions on graph paper.

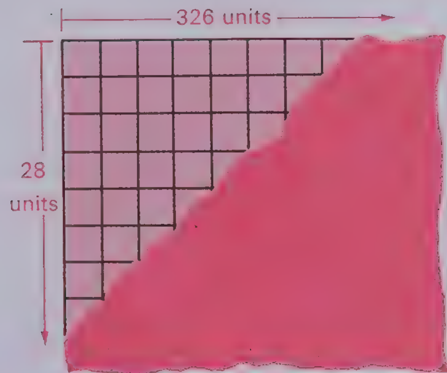
Discussing the Ideas

1. What is the perimeter of the region shown in the Investigation?
2. Can you give the perimeter of each of the regions you found in the Investigation?
3. What are the area and perimeter of each square?

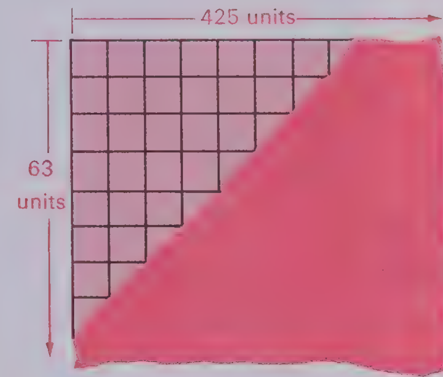


1. Find the area of the rectangle indicated in each exercise.

A

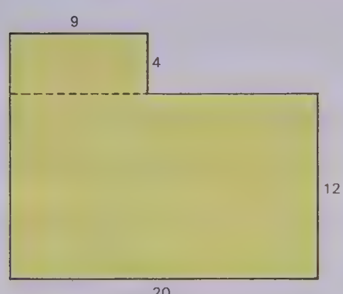


B



2. Find the area of each figure.

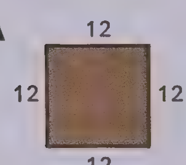
A



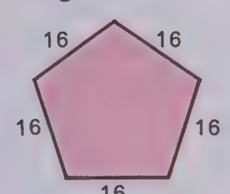
B

3. Multiply to find the perimeter of each figure.
The lengths of the sides are given.

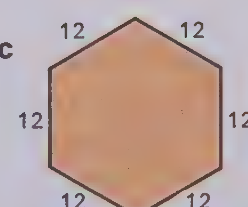
A



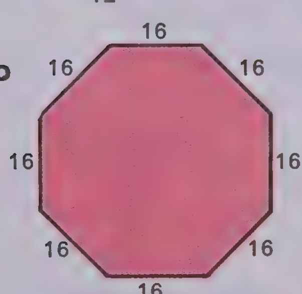
B



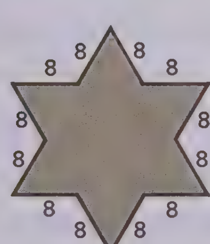
C



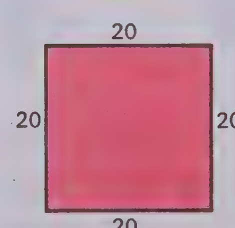
D



E



F

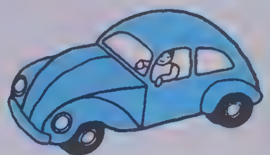


Short Stories Weights

1 Baby elephant: 89 kilograms. Full-grown elephant: 57 times that much. How many kilograms does a full-grown elephant weigh?

2 Baby whale: weighs twice as much as a full-grown elephant. How much does a baby whale weigh?

3 Full-grown whale: weighs 954 times as much as a baby elephant. How much does a full-grown whale weigh?



4 Automobile: weighs 21 times as much as the baby elephant. How much does the automobile weigh?

5 Professional football player: 31 kg heavier than the baby elephant. What is the football player's weight?

6 Beauty queen: 35 kg lighter than the baby elephant. How much does the beauty queen weigh?

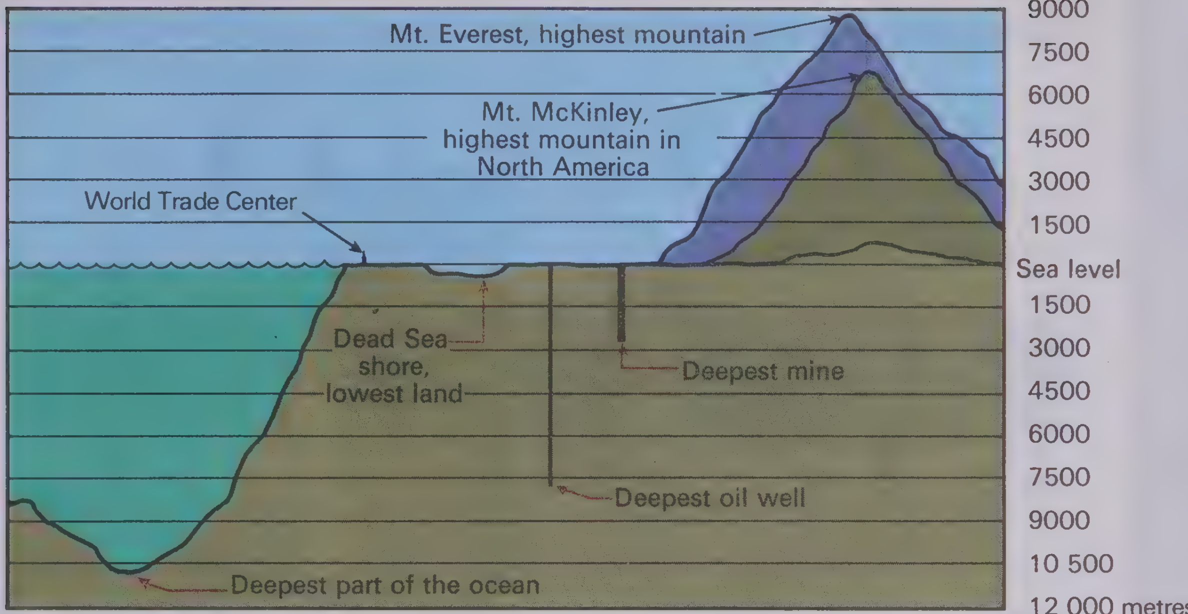
7 One tonne: 68 kilograms less than 12 times as much as the baby elephant. How many kilograms?

8 Sigma 7 (a Mercury space capsule): weighed 763 kilograms less than 30 times the weight of the baby elephant. How much did Sigma 7 weigh?

9 Circus fat man: weighs 30 kilograms more than three times as much as the baby elephant. How much does the fat man weigh?

10 Strong man: lifts twice as much as 8 kilograms more than the baby elephant weighs. How much does he lift?

HEIGHT AND DEPTH



1. Use the graph above to estimate the height or depth of each item listed. Write the name of each item and your estimate beside it.

2. The World Trade Center is 412 metres tall. Mount McKinley is 5749 metres higher than the World Trade Center. How high is Mount McKinley ?

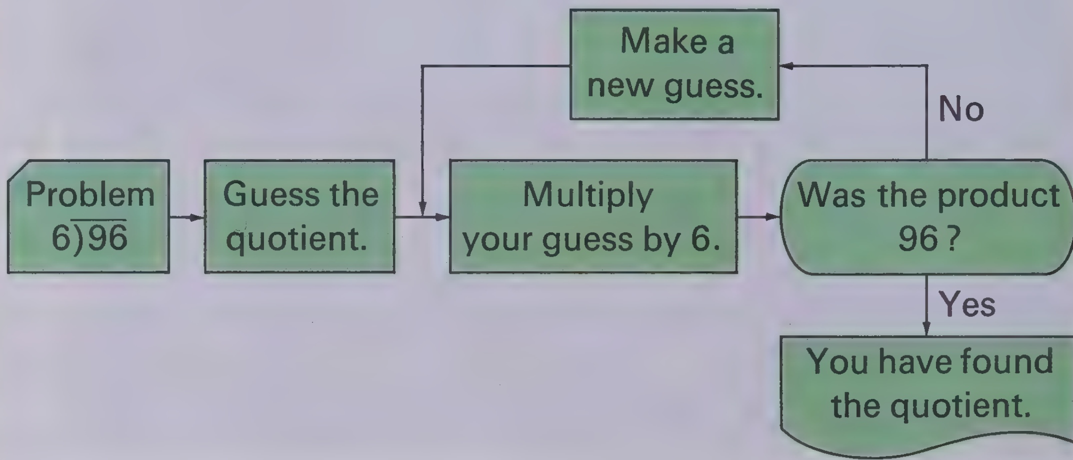
3. Mount Everest is almost 22 times higher than the World Trade Center. Use the height of the building, given in exercise 2, to estimate the height of Mount Everest.

4. The lowest land is 395 metres below sea level. The deepest mine is 2406 metres deeper. How deep is the deepest mine ?

5. How deep is the deepest oil well if it is 4929 metres deeper than the mine ?

6. The deepest part of the ocean is about 305 metres deeper than 27 times the depth of the lowest land. Use this information and the depth of the lowest land, given in exercise 4, to estimate the depth of the deepest part of the ocean.

Investigating the Ideas



? Can you follow the flow chart and find the quotient ?

Discussing the Ideas

1. How many estimates did you make before you found the quotient ?

2. How could subtraction help you make your new guess ?

3. The example on the right will help you review the meaning of divisor, dividend, quotient, and remainder.

   $$\begin{array}{rrl} & 8 & \text{quotient} \\ \text{divisor}\ \ 4 & \overline{)35} & \text{dividend} \\ & \underline{32} & \\ & 3 & \text{remainder} \end{array}$$

   A What number was the divisor in the Investigation ?
   B What number was the dividend ?
   C What was the remainder ?

Give the number pair for each gray space in exercises 1 to 6.
Then find the quotient and remainder.

Pairs:

1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10

1. A $\square \times 4 < 15$
$\square \times 4 > 15$
B $4\overline{)15}$

2. A $\square \times 6 < 40$
$\square \times 6 > 40$
B $6\overline{)40}$

3. A $\square \times 5 < 47$
$\square \times 5 > 47$
B $5\overline{)47}$

4. A $\square \times 3 < 16$
$\square \times 3 > 16$
B $3\overline{)16}$

5. A $\square \times 7 < 48$
$\square \times 7 > 48$
B $7\overline{)48}$

6. A $\square \times 6 < 34$
$\square \times 6 > 34$
B $6\overline{)34}$

7. For each exercise, first tell how many digits you think the product has; then estimate the product; finally, find the product to see how you did on your estimates.

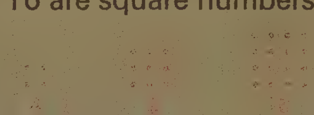
Example: 4×49 (Answer: Number of digits, 3; estimate, 200; correct product, 196)

A 40×49 D 8×97 G 90×75 J 4×197
B 6×51 E 80×97 H 90×750 K 40×197
C 60×51 F 9×75 I 92×756 L 42×197

8. From the set {32, 19, 51, 97, 79, 11}, choose a number for the missing factor. Find the product to check your work.

A $n \times 5 = 95$
B $a \times 11 = 121$
C $6 \times b = 582$
D $t \times 8 = 408$
E $s \times 8 = 776$
F $608 \times d = 6688$
G $r \times 21 = 399$
H $22 \times m = 704$
I $49 \times b = 2499$

think The dots below show why 4, 9, and 16 are square numbers.



Give the next 10 square numbers.
Find a 2-digit number that is both square and triangular.

Discussing the Ideas

1. To find 262 ÷ 6, you find the greatest number of sixes that can be subtracted from 262.
   A Can you subtract 10 sixes? B Can you subtract 100 sixes?

2. $40 \times 6 < 262$
   $50 \times 6 > 262$
   The quotient must be between __?__ and __?__.

3. Study each step below. Then try this one on your own: 5)212

	Think:	Write:
Step 1 6)262 240 ← 40 × 6	I can subtract 40 sixes from 262.	6)262 240 (40) 22
Step 2 6)262 240 22 18 ← 3 × 6 4	Then I can subtract 3 more sixes.	6)262 240 (40) 22 18 (3) 4
Step 3 43 6)262 240 ← 40 22 18 ← 3 4	The **quotient** is 43. The **remainder** is **4**.	43 6)262 240 (40) 22 18 (3) 4

4. Explain this check for 262 ÷ 6. Check:

$$\begin{array}{r} 43 \\ \times 6 \\ \hline 258 \end{array} \qquad \begin{array}{r} 258 \\ +4 \\ \hline 262 \end{array}$$

1. Copy the two inequalities in each part. Put in the correct number pair. Then find the quotient and remainder.

Pairs:

10	20	30	40	50	60	70	80	90
20	30	40	50	60	70	80	90	100

Example

$\square \times 5 < 173$
$\square \times 5 > 173$ $\rightarrow 5\overline{)173}$

Answer

$30 \times 5 < 173$
$40 \times 5 > 173$

$$\begin{array}{rrl} & 34 & \\ 5)\overline{173} & & \\ & 150 & 30 \\ \hline & 23 & \\ & 20 & 4 \\ \hline & 3 & \end{array}$$

A $\square \times 4 < 143$
$\square \times 4 > 143$ $\rightarrow 4\overline{)143}$

B $\square \times 3 < 258$
$\square \times 3 > 258$ $\rightarrow 3\overline{)258}$

C $\square \times 6 < 273$
$\square \times 6 > 273$ $\rightarrow 6\overline{)273}$

D $\square \times 4 < 299$
$\square \times 4 > 299$ $\rightarrow 4\overline{)299}$

E $\square \times 5 < 280$
$\square \times 5 > 280$ $\rightarrow 5\overline{)280}$

F $\square \times 7 < 456$
$\square \times 7 > 456$ $\rightarrow 7\overline{)456}$

G $\square \times 9 < 293$
$\square \times 9 > 293$ $\rightarrow 9\overline{)293}$

2. From the set {10, 20, 30, . . .}, find the largest number that will make the inequality true. Then find the quotient and remainder.

A $r \times 4 < 143 \rightarrow 4\overline{)143}$

B $m \times 3 < 258 \rightarrow 3\overline{)258}$

C $t \times 6 < 273 \rightarrow 6\overline{)273}$

D $a \times 4 < 299 \rightarrow 4\overline{)299}$

E $q \times 5 < 324 \rightarrow 5\overline{)324}$

F $s \times 8 < 324 \rightarrow 8\overline{)324}$

G $n \times 7 < 203 \rightarrow 7\overline{)203}$

H $d \times 9 < 306 \rightarrow 9\overline{)306}$

Discussing the Ideas

1. To find $1428 \div 4$, you find the greatest number of fours that can be subtracted from 1428.
   A Can you subtract 100 fours?
   B Can you subtract 1000 fours?

2. $300 \times 4 < 1428$
   $400 \times 4 > 1428$
   The quotient must be between __?__ and __?__.

3. Study the example below.
   Then try this one on your own: $5\overline{)1733}$

Think:		Write:	
357		357	
$4\overline{)1428}$	I can subtract	$4\overline{)1428}$	
1200 ← 300 × 4	300 fours from 1428.	1200	300
228	Then I can subtract	228	
200 ← 50 × 4	50 more fours.	200	50
28	Finally, I can subtract	28	
28 ← 7 × 4	7 more fours.	28	7
0		0	

4. Find and check the quotient for this example.

$6\overline{)1847}$
1800 ← ___ × 6
47
42 ← ___ × 6
5

1. For each part, copy the two inequalities. Put in the correct number pair. Then find the quotient and remainder.

Pairs:

100 200	200 300	300 400	400 500	500 600	600 700	700 800	800 900	900 1000

A $\square \times 3 < 738$, $\square \times 3 > 738$ $\longrightarrow 3\overline{)738}$

B $\square \times 6 < 2074$, $\square \times 6 > 2074$ $\longrightarrow 6\overline{)2074}$

C $\square \times 5 < 3963$, $\square \times 5 > 3963$ $\longrightarrow 5\overline{)3963}$

D $\square \times 4 < 3008$, $\square \times 4 > 3008$ $\longrightarrow 4\overline{)3008}$

E $\square \times 7 < 1372$, $\square \times 7 > 1372$ $\longrightarrow 7\overline{)1372}$

F $\square \times 3 < 2820$, $\square \times 3 > 2820$ $\longrightarrow 3\overline{)2820}$

G $\square \times 8 < 5030$, $\square \times 8 > 5030$ $\longrightarrow 8\overline{)5030}$

H $\square \times 9 < 7134$, $\square \times 9 > 7134$ $\longrightarrow 9\overline{)7134}$

2. From the set (100, 200, 300, . . .), find the largest number that will make the inequality true. Then find the quotient and remainder.

A $n \times 4 < 1427 \longrightarrow 4\overline{)1427}$

B $y \times 6 < 3125 \longrightarrow 6\overline{)3125}$

C $r \times 5 < 1826 \longrightarrow 5\overline{)1826}$

D $m \times 3 < 2256 \longrightarrow 3\overline{)2256}$

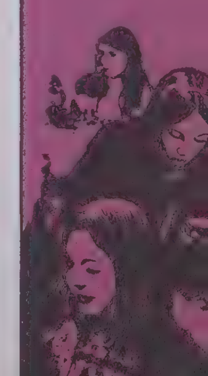
E $t \times 7 < 4488 \longrightarrow 7\overline{)4488}$

F $p \times 8 < 5000 \longrightarrow 8\overline{)5000}$

think

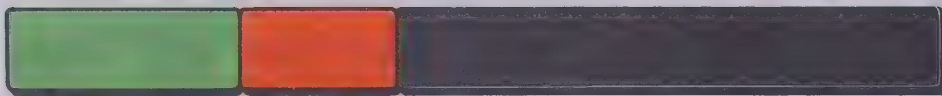
The missing digits are all different. Find them.

$$\begin{array}{r} \times 9 \\ \hline 111\ 111\ 111 \end{array}$$

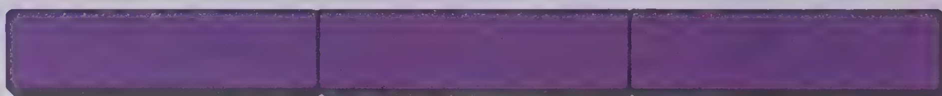


Investigating the Ideas

Here is a "train" of three strips, **not all the same.**



Here is a "matching train" of three strips, **all the same.**



? Can you make some pairs of "matching trains" so that
1. matching trains have the same number of strips,
2. one train has strips all the same, and
3. one train has strips not all the same?

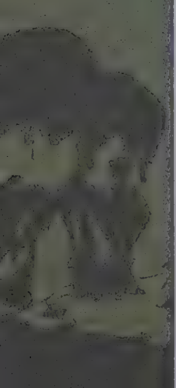
Discussing the Ideas

1. How are these two equations like the trains above?
   A $3 + 2 + 7 = 12$
   B $4 + 4 + 4 = 12$

We say that **4** is the **average** of the numbers 3, 2, and 7.

2. Give the averages you found for your matching trains.

3. Can you show with your strips that 3 is the average of 5, 4, 2, and 1?

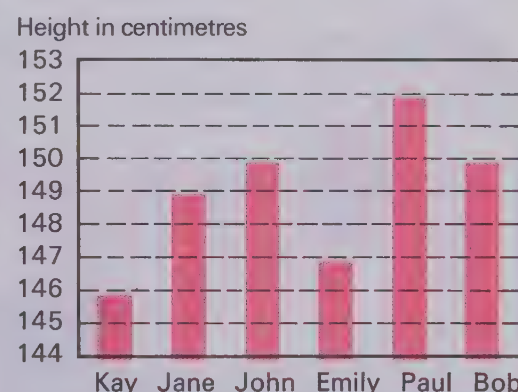


1. Find the sums. Then find the **average** of the addends.
   - A $12 + 7 = r$
   - B $3 + 8 + 2 = n$
   - C $5 + 9 + 10 + 4 = t$
   - D $6 + 9 + 12 + 5 + 8 = m$
   - E $20 + 32 + 36 = y$
   - F $63 + 75 + 47 + 27 = p$

2. Complete the sentences. Then find the average.
   - A To find the average of 4, 5, and 9, divide __?__ by 3.
   - B To find the average of 6, 10, 13, and 3, divide __?__ by 4.
   - C To find the average of 55 and 29, divide __?__ by __?__.

3. Give the missing numbers.
   - A The average of 5, 7, and 12 is ▯.
   - B The average of 4, 12, 7, 9, 13, and 3 is ▯.

4. The bar graph shows the heights in centimetres of 6 children. Find the average height of these children.



5. Jan is 3834 days old. Fran is 4015 days old. Nan is 3923 days old. Find the average age (in days) of the 3 girls.

6. This graph shows how far some boys threw a softball. Find the average distance.

★ 7. A Find the average weight (to the nearest whole number) of the children in your class.
   B Find their average height in centimetres.

More practice, page A-11, Set 21

Discussing the Ideas

Study the steps in the example below. Use the same steps to find this quotient: $4\overline{)1815}$

Long Way

Step 1

```
8 ) 2 9 1 5
    2 4 0 0   (300)
        5 1 5
```

Step 2

```
8 ) 2 9 1 5
    2 4 0 0   (300)
      5 1 5
      4 8 0   (60)
        3 5
```

Step 3

```
      3 6 4
8 ) 2 9 1 5
    2 4 0 0   (300)
      5 1 5
      4 8 0   (60)
        3 5
        3 2   (4)
          3
```

1st Shortcut

```
      3
8 ) 2 9 1 5
    2 4 0 0
      5 1 5
```

This quotient has **3 hundreds.**

```
      3 6
8 ) 2 9 1 5
    2 4 0 0
      5 1 5
      4 8 0
        3 5
```

This quotient has **6 tens.**

```
      3 6 4
8 ) 2 9 1 5
    2 4 0 0
      5 1 5
      4 8 0
        3 5
        3 2
          3
```

This quotient has **4 ones.**

2nd Shortcut

$$\begin{array}{r} 3 \\ 8\overline{)2\,9^{5}1\,5} \end{array}$$

29 ÷ 8
The quotient is **3.**
The remainder is **5.**

$$\begin{array}{r} 3\,6 \\ 8\overline{)2\,9^{5}1^{3}5} \end{array}$$

51 ÷ 8
The quotient is **6.**
The remainder is **3.**

$$\begin{array}{r} 3\,6\,4\text{ R3} \\ 8\overline{)2\,9^{5}1^{3}5} \end{array}$$

35 ÷ 8
The quotient is **4.**
The remainder is **3.**

1. Find each quotient and remainder.

A $7\overline{)58} \longrightarrow 7\overline{)581}$

B $8\overline{)34} \longrightarrow 8\overline{)340}$

C $7\overline{)44} \longrightarrow 7\overline{)4494}$

D $6\overline{)33} \longrightarrow 6\overline{)3366}$

E $8\overline{)42} \longrightarrow 8\overline{)4296}$

F $4\overline{)25} \longrightarrow 4\overline{)257}$

G $6\overline{)38} \longrightarrow 6\overline{)384}$

H $7\overline{)67} \longrightarrow 7\overline{)6776}$

I $9\overline{)29} \longrightarrow 9\overline{)2943}$

J $9\overline{)57} \longrightarrow 9\overline{)5769}$

2. Find the quotients and remainders.

A $4\overline{)329}$ B $8\overline{)969}$ C $8\overline{)47\ 562}$ D $9\overline{)6498}$

E $6\overline{)4356}$ F $6\overline{)2738}$ G $3\overline{)1824}$ H $8\overline{)31\ 254}$

I $9\overline{)4635}$ J $9\overline{)6666}$ K $5\overline{)2437}$ L $7\overline{)5000}$

M $7\overline{)3246}$ N $5\overline{)23\ 647}$ O $4\overline{)2603}$ P $6\overline{)56\ 213}$

3. The table gives weights of 8 children. Find the average weight of the children.

Jay	35	Ann	37
Nancy	32	Ted	39
Susan	33	Bill	43
Steve	41	Mary	36

4. Here are the distances for 6 boys in the standing broad jump:

165 centimetres
140 centimetres
185 centimetres
161 centimetres
145 centimetres
180 centimetres
173 centimetres
170 centimetres

Find the average length of the jumps.

think

The numbers in the sets are consecutive numbers.

{5, 6, 7} {58, 59, 60, 61}

Now find the three consecutive numbers in this riddle.

A fine trio are we.
Consecutive numbers, too.
Our sum is sixty-three.
The rest is up to you.

More practice, page A-11, Set 22

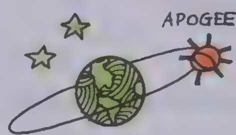
Short Stories

1 688 tractors.
8 on each flatcar.
How many flatcars?

2 52 weeks → 1 year.
19 years.
How many weeks?

3 4 pitchers fill 1 jug. 2344 pitchers. How many jugs?

4 9 tiles cover 1 square metre.
6615 tiles. How many square metres?

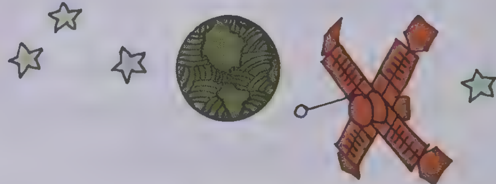


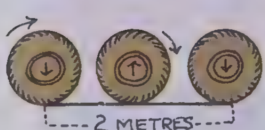
5 Satellite.
Apogee (farthest distance from earth), 3304 kilometres.
Perigee (closest distance from earth), 554 kilometres.

A What is the difference in these distances?

B What is the average of these distances?

6 Satellite makes 867 orbits. 9 orbits each day. How many full days? How many extra orbits?





7 Car wheel.
One revolution → 2 metres
How many revolutions for 1 kilometre?

8 6 wheels on a truck. 35 958 wheels.
How many trucks?

9 Relay team → 4 runners. 112 runners. How many teams?

10 Drove 3 hours. Average speed, 104 km/h. Then drove 3 more hours. Average speed, 88 km/h. Travelled how far?

11 Neon sign blinks 7 times each minute.

A How many blinks in an hour?

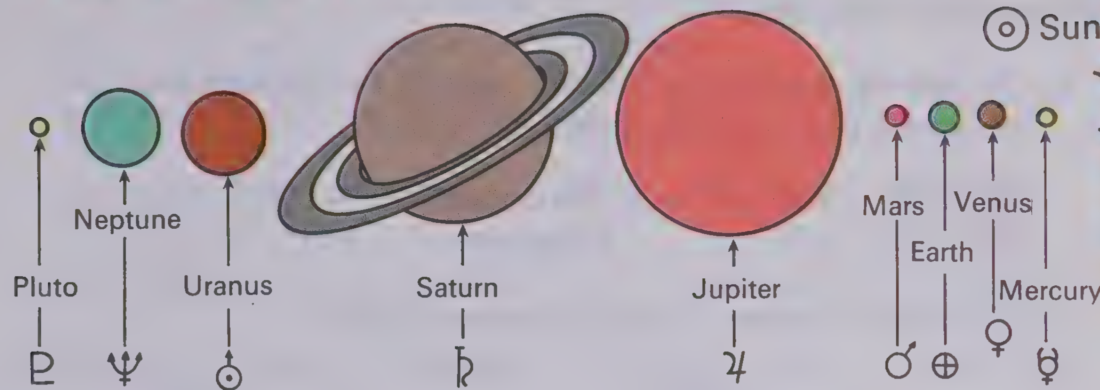
B How many blinks in a 24-hour day?

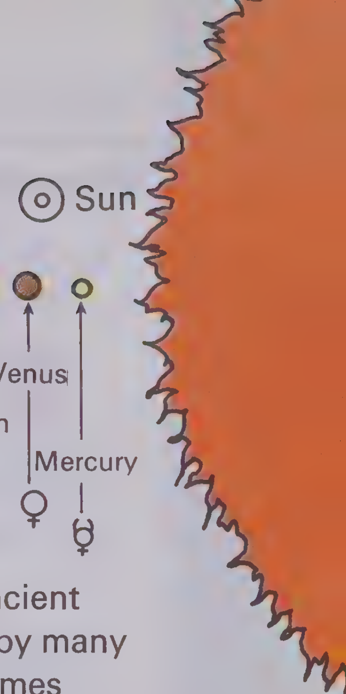
12 Satellite into orbit.
Speed, 8 km per second.

A How many km per hour?

B How many km per day?

The Planets





The relative sizes of the nine planets are shown above. The ancient symbols shown below the names of the planets are still used by many astronomers to represent these planets. The word "planet" comes from a Greek word meaning "wanderer." Can you explain why a word with this meaning was used?

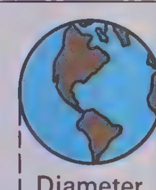
Planets	Millions of kilometres from the sun
Mercury	58
Venus	108
Earth	150
Mars	228
Jupiter	778
Saturn	1427
Uranus	2870
Neptune	4500
Pluto	5909

1. The average distances of the planets from the sun are given in the table.

A How much farther from the sun is Mars than Earth?

B Which planet is about 100 times as far from the sun as Mercury?

C Is the distance of Neptune from the sun more or less than four times the distance of Jupiter from the sun? How much more or less?



2. The diameters of the four planets closest to the sun are given beside the symbols for the planets. Find the average diameter of these planets.

☿ 4866 km ♀ 12 106 km ♂ 6760 km ⊕ 12 742 km

★ **3.** Find the average diameter in kilometres of the four largest planets.

♆ 45 430 ⛢ 46 940 ♄ 116 440 ♃ 139 580

Discussing the Ideas

1. To find the quotient for $30\overline{)287}$, we find the greatest number of thirties that can be subtracted from 287.

$9 \times 30 < 287$
$10 \times 30 > 287$

The quotient must be between __?__ and __?__.

2. Study the example. Then try this one: $40\overline{)256}$

Think:		Write:
$\begin{array}{r} 9 \\ 30\overline{)287} \\ \underline{270} \leftarrow 9 \times 30 \\ 17 \end{array}$	We can subtract 9 thirties from 287. The quotient is 9. The remainder is 17.	$\begin{array}{rl} 9 & \\ 30\overline{)287} & \\ \underline{270} & 9 \\ 17 & \end{array}$

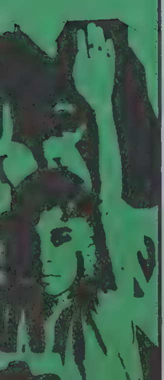
3. To find the quotient for $40\overline{)3168}$, we find the greatest number of forties that can be subtracted from 3168.
   A Can you subtract 10 forties?
   B Can you subtract 100 forties?

4. $70 \times 40 < 3168$
$80 \times 40 > 3168$

The quotient must be between __?__ and __?__.

5. Study the example. Then try this one on your own: $60\overline{)2145}$

Think:		Write:
$\begin{array}{r} 79 \\ 40\overline{)3168} \\ \underline{2800} \leftarrow 70 \times 40 \\ 368 \\ \underline{360} \leftarrow 9 \times 40 \\ 8 \end{array}$	We can subtract 70 forties from 3168. Then we can subtract 9 more forties. The quotient is 79. The remainder is 8.	$\begin{array}{rl} 79 & \\ 40\overline{)3168} & \\ \underline{2800} & 70 \\ 368 & \\ \underline{360} & 9 \\ 8 & \end{array}$



1. Find the largest whole number that will make the inequality true. Then find the quotient and remainder.

A $n \times 30 < 287 \rightarrow 30\overline{)287}$ E $c \times 60 < 317 \rightarrow 60\overline{)317}$

B $a \times 70 < 371 \rightarrow 70\overline{)371}$ F $k \times 20 < 197 \rightarrow 20\overline{)197}$

C $b \times 40 < 213 \rightarrow 40\overline{)213}$ G $f \times 90 < 563 \rightarrow 90\overline{)563}$

D $r \times 80 < 650 \rightarrow 80\overline{)650}$ H $s \times 30 < 284 \rightarrow 30\overline{)284}$

2. From the set $\{10, 20, 30, \ldots\}$, find the largest number that will make the inequality true. Then find the quotient and remainder.

A $s \times 40 < 3168 \rightarrow 40\overline{)3168}$ E $p \times 50 < 4111 \rightarrow 50\overline{)4111}$

B $t \times 60 < 1378 \rightarrow 60\overline{)1378}$ F $q \times 70 < 2971 \rightarrow 70\overline{)2971}$

C $n \times 80 < 3396 \rightarrow 80\overline{)3396}$ G $f \times 90 < 8406 \rightarrow 90\overline{)8406}$

D $a \times 30 < 1008 \rightarrow 30\overline{)1008}$ H $r \times 20 < 297 \rightarrow 20\overline{)297}$

3. Find the quotients and remainders.

A $20\overline{)130}$ B $40\overline{)276}$ C $30\overline{)187}$

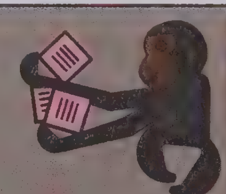
D $70\overline{)513}$ E $60\overline{)269}$ F $50\overline{)420}$

G $20\overline{)646}$ H $40\overline{)2500}$ I $40\overline{)999}$

J $80\overline{)7580}$ K $40\overline{)3700}$ L $70\overline{)4970}$

M $50\overline{)3863}$ N $90\overline{)8834}$ O $50\overline{)2507}$

think



$\square \times \square \times \square = 720$

The red screens cover three consecutive whole numbers whose product is 720.
Find these numbers.
Find three other numbers whose product is 720.

More practice, page A-12, Set 23

Discussing the Ideas

1. To find the quotient for $52\overline{)378}$, think → $? \times 50 < 378$
   - A What is the largest number that will make the sentence true?
   - B Explain how the dividing is completed when you use the number found in exercise 1A.

$$\begin{array}{rl} 7 & \\ 52\overline{)378} & \\ \underline{364} & 7 \\ 14 & \end{array}$$

2. To find the quotient for $63\overline{)368}$, think → $? \times 60 < 368$
   - A What is the largest number that will make the sentence true?
   - B Why is the number you found in exercise 2A not the correct quotient?
   - C Explain how to choose the correct quotient and complete the dividing.

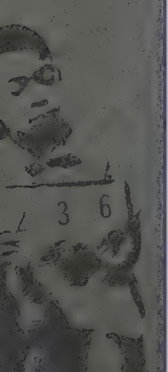
$$\begin{array}{rl} 5 & \\ 63\overline{)368} & \\ \underline{315} & 5 \\ 53 & \end{array}$$

3. To find the quotient for $35\overline{)218}$, think → $? \times 40 < 218$
   - A What is the largest number that will make the sentence true?
   - B Multiply the divisor by the number you found in exercise 3A; then subtract. Is the remainder less than the divisor?
   - C Explain how the dividing is completed.

$$\begin{array}{rl} 6 & \\ 35\overline{)218} & \\ \underline{175} & 5 \\ 43 & \\ \underline{35} & 1 \\ 8 & \end{array}$$

4. Find the quotients and remainders. Be ready to explain your work.

A $81\overline{)486}$	B $59\overline{)420}$	C $42\overline{)265}$
D $65\overline{)500}$	E $74\overline{)289}$	F $26\overline{)209}$



1. Copy the two inequalities with the correct number pair. Then find the quotient and remainder.

Pairs:

1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10

Think: $? \times 40 < 218$

Think: $? \times 50 < 378$

A $\square \times 35 < 218$, $\square \times 35 > 218$ → $35\overline{)218}$

B $\square \times 61 < 327$, $\square \times 61 > 327$ → $61\overline{)327}$

C $\square \times 43 < 371$, $\square \times 43 > 371$ → $43\overline{)371}$

D $\square \times 37 < 223$, $\square \times 37 > 223$ → $37\overline{)223}$

E $\square \times 52 < 378$, $\square \times 52 > 378$ → $52\overline{)378}$

F $\square \times 49 < 368$, $\square \times 49 > 368$ → $49\overline{)368}$

G $\square \times 72 < 585$, $\square \times 72 > 585$ → $72\overline{)585}$

H $\square \times 89 < 555$, $\square \times 89 > 555$ → $89\overline{)555}$

2. Find the largest whole number that will make the inequality true. Then find the quotient and remainder.

A $n \times 29 < 178$ → $29\overline{)178}$

B $p \times 61 < 253$ → $61\overline{)253}$

C $q \times 42 < 137$ → $42\overline{)137}$

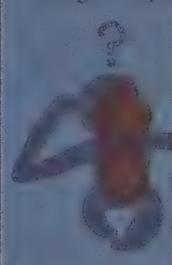
D $a \times 39 < 226$ → $39\overline{)226}$

E $n \times 51 < 423$ → $51\overline{)423}$

F $r \times 19 < 173$ → $19\overline{)173}$

think

Give the missing numbers so that this will be a magic square.



93		80	90
82	88		
86	84	83	
81			

Reviewing the Ideas

1. Find the sums.

A	B	C	D	E
$\begin{array}{r}594\\637\\+868\\\hline\end{array}$	$\begin{array}{r}976\\86\\+368\\\hline\end{array}$	$\begin{array}{r}898\\407\\+600\\\hline\end{array}$	$\begin{array}{r}9864\\387\\+98\\\hline\end{array}$	$\begin{array}{r}8765\\3479\\+2698\\\hline\end{array}$

2. Find the differences.

A	B	C	D	E
$\begin{array}{r}6007\\-4387\\\hline\end{array}$	$\begin{array}{r}5009\\-4923\\\hline\end{array}$	$\begin{array}{r}3070\\-1487\\\hline\end{array}$	$\begin{array}{r}6508\\-2346\\\hline\end{array}$	$\begin{array}{r}6000\\-1374\\\hline\end{array}$

3. Find the total amounts.

A	B	C	D	E
$\begin{array}{r}\$2.79\\1.39\\\hline\end{array}$	$\begin{array}{r}\$12.50\\23.75\\\hline\end{array}$	$\begin{array}{r}\$0.69\\1.58\\\hline\end{array}$	$\begin{array}{r}\$2.84\\9.76\\8.41\\\hline\end{array}$	$\begin{array}{r}\$18.95\\23.50\\11.66\\\hline\end{array}$

4. Find the products.

A	B	C	D	E
$\begin{array}{r}76\\\times 9\\\hline\end{array}$	$\begin{array}{r}372\\\times 6\\\hline\end{array}$	$\begin{array}{r}58\\\times 47\\\hline\end{array}$	$\begin{array}{r}15\,365\\\times 29\\\hline\end{array}$	$\begin{array}{r}8734\\\times 263\\\hline\end{array}$

5. Find the quotients and remainders.

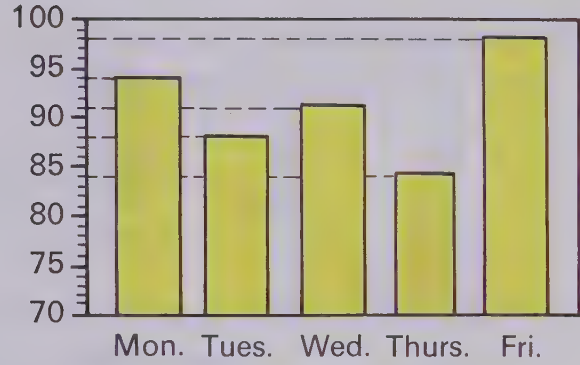
A $9\overline{)414}$ C $30\overline{)195}$ E $50\overline{)4350}$ G $69\overline{)569}$

B $4\overline{)345}$ D $21\overline{)189}$ F $70\overline{)4700}$ H $7\overline{)6769}$

think

Here is an interesting calendar puzzle. Have a friend choose a 3-by-3 "square" of dates on any calendar. Tell him you can find the sum of these dates faster than he can if he will just give you the smallest date in the square. Suppose he chooses the dates colored in the calendar shown. He would say 9. You would add 8 and then multiply by 9. This will always give you the sum of the dates. Can you explain why?

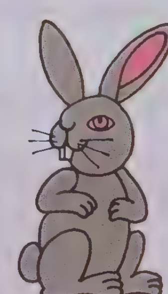
6. The bar graph shows Tom's scores on 5 spelling tests. Give his average score.



7. The population of the city of Winnipeg was 246 246 in 1971. The population of Winnipeg's Metropolitan area was 540 262.
   A Was the metropolitan population more than twice the population of the city?
   B How many more than twice the population of the city?

EAGLES	STARS
68	55
82	84
53	48
94	67
60	69
75	85

8. The Eagles played the Stars in 6 basketball games during the season. Here is a record of the scores. Give the average number of points scored per game by the Eagles and by the Stars. Give the average number of points scored per game.



9. There are 52 weeks in one year. A rabbit is considered fairly old when it has lived 312 weeks. How many years is this?

10. A package of 24 sheets of paper is called 1 **quire** of paper. If you buy 212 sheets of paper, how many quires have you bought? How many extra sheets?

11. About 168 hours after a specially fed grub worm spins a cocoon about itself, a queen bee is formed inside the cocoon. How many days is this?

1. Find the sums.

A $\begin{array}{r} 29 \\ 87 \\ 96 \\ 54 \\ +32 \\ \hline \end{array}$ B $\begin{array}{r} 865 \\ 976 \\ 493 \\ 976 \\ +487 \\ \hline \end{array}$ C $\begin{array}{r} 5764 \\ 987 \\ 8368 \\ 476 \\ +98 \\ \hline \end{array}$ D $\begin{array}{r} 2463 \\ 172 \\ 5432 \\ 1621 \\ +8464 \\ \hline \end{array}$

2. Find the sums, products, or differences.

A $\begin{array}{r} 987 \\ +643 \\ \hline \end{array}$ D $\begin{array}{r} 635 \\ \times 31 \\ \hline \end{array}$ G $\begin{array}{r} 807 \\ -499 \\ \hline \end{array}$ J $\begin{array}{r} 5869 \\ \times 54 \\ \hline \end{array}$

B $\begin{array}{r} 8003 \\ -69 \\ \hline \end{array}$ E $\begin{array}{r} 6597 \\ +9886 \\ \hline \end{array}$ H $\begin{array}{r} 385 \\ \times 267 \\ \hline \end{array}$ K $\begin{array}{r} 8309 \\ -2847 \\ \hline \end{array}$

C $\begin{array}{r} 8596 \\ \times 233 \\ \hline \end{array}$ F $\begin{array}{r} 4020 \\ -1976 \\ \hline \end{array}$ I $\begin{array}{r} 6000 \\ -3986 \\ \hline \end{array}$ L $\begin{array}{r} 6517 \\ \times 436 \\ \hline \end{array}$

3. Solve the equations.

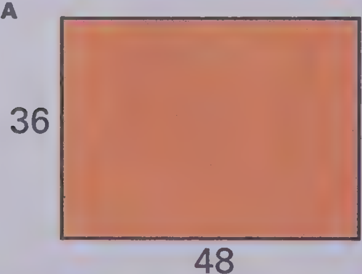
A $(54 - 9) + 9 = y$ D $(90 - d) + 13 = 90$ G $(7 \times 4) \div r = 7$

B $(91 - 7) + m = 91$ E $(320 + 24) - n = 320$ H $(6 \times c) \div 8 = 6$

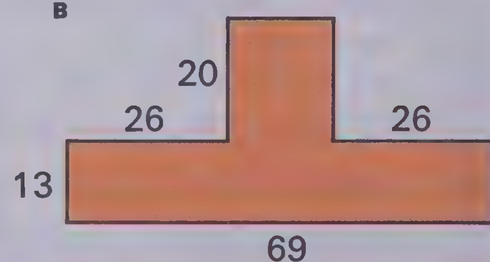
C $(107 + 6) - 6 = b$ F $(9 \times 6) \div 6 = p$ I $(56 \div 8) \times 8 = t$

4. Find the area and perimeter of each region.

A



B



You are invited to explore

ACTIVITY CARD 6
Page 336

Short Stories ECOLOGY

1. School Ecology Club. Collected 1840 aluminum cans. One cent for every 2 cans. How much money?

2. 30 students. Collected 840 kilograms of newspapers for recycling. How many kilograms per person?

3. Aluminum cans: 96 kilograms. 10¢ a kilogram. How many dollars' worth?

4. City: 25 000 people. 2 kilograms garbage per person each day. How many tonnes of garbage each day?

5. Used newspapers: 40¢ for 50 kilograms. How much for one tonne?

6. Estimated world population. Today: 4 billion people. Year 2000: 2 billion more than today. How many people in 2000?

7. World population in 1925: 2 billion. Today: 4 billion. How many years for population to double?

8. Provincial Parks in Canada. 1964: 227 320 square kilometres. 1971: 246 530 square kilometres. How much more space in 1971?

9. City population: 750 000. Cost of new sewage equipment: $300 per person. Total cost?

10. Pollution in the atmosphere each year. Cars: 86 million tonnes. Factories: 43 million tonnes. Heating and burning: 13 million tonnes. How many tonnes in all?

7 Dividing

How fast do you travel?

Investigating the Ideas

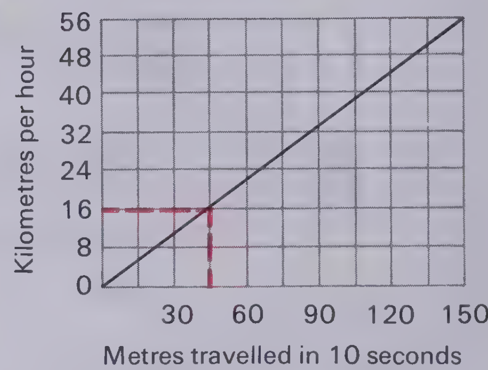
How far do you go in 10 seconds ? Measure one of these in metres.

WALKING

RUNNING

BICYCLING

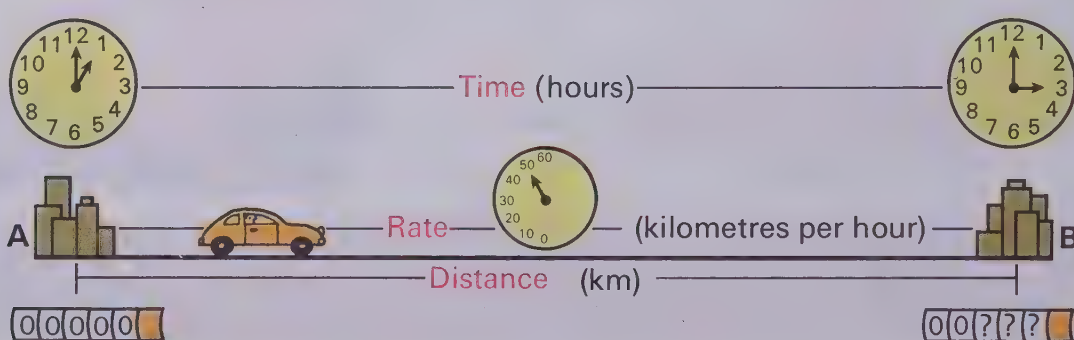
? Can you use the graph to find your **speed** in kilometres per hour ?

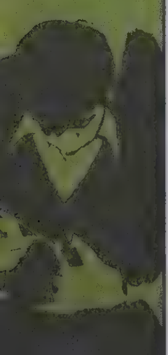


Note: The red dashed line shows that 45 metres in 10 seconds is about 16 kilometres per hour.

Discussing the Ideas

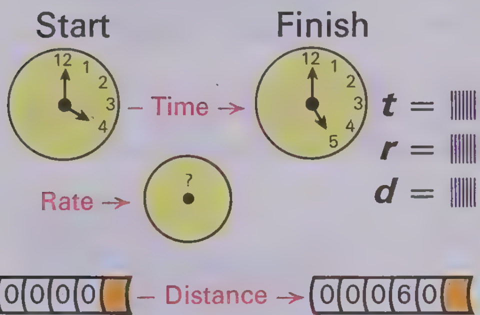
1. If you know how many metres you travel in 10 seconds, how can you use the graph above to find your rate in km / h ?
2. Explain how to use the information in the picture below to find the distance from city **A** to city **B**.

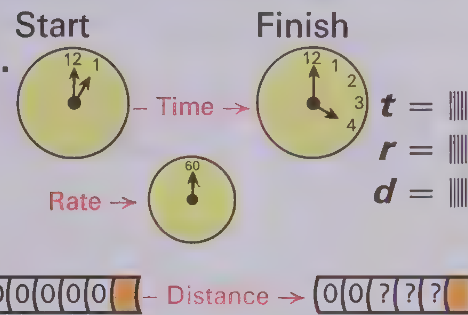


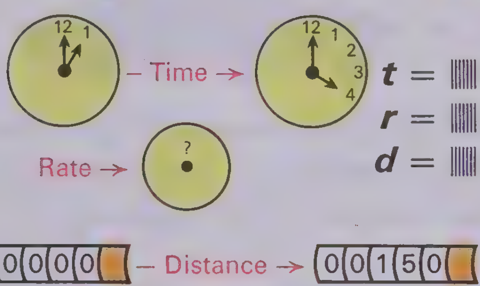


Using the Ideas

In the exercises the **time** is the number of hours that have passed from the time the trip started until the end of the trip. The **rate** is the number of kilometres travelled per hour. The **distance** is the number of kilometres from where the trip started to where it ended. Give the **time** (*t*), **rate** (*r*), and **distance** (*d*), for each trip.

1.


2.


3.


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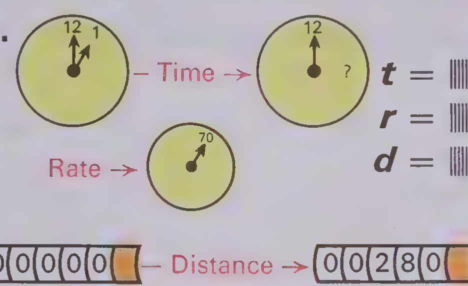

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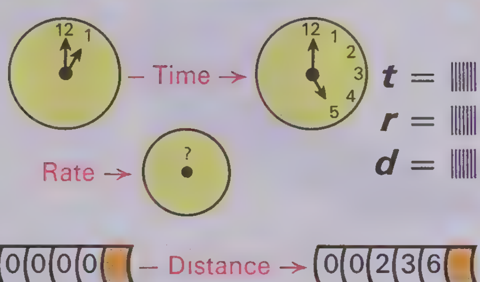

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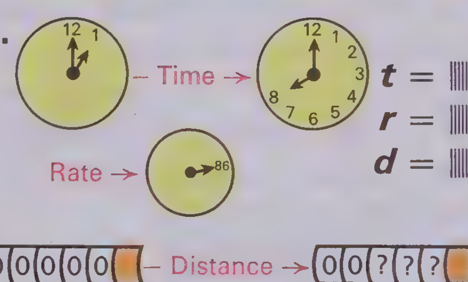

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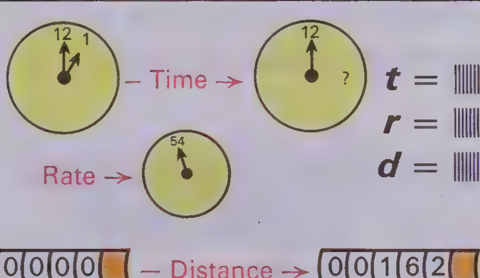
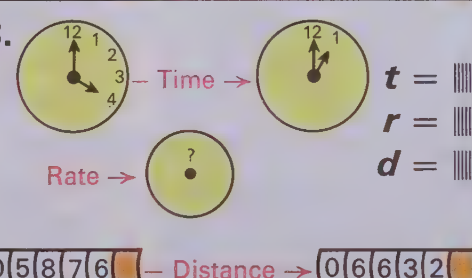
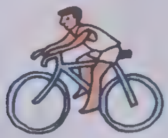


★8.


Finding Time, Rate or Distance

Write equations for exercises 1 through 6. Solve each equation to find the **time**, **rate**, or **distance**.

1. The first successful gas-powered car was a three-wheeler built in 1886 by Carl Benz. It went 84 kilometres in six hours. How fast did it go?

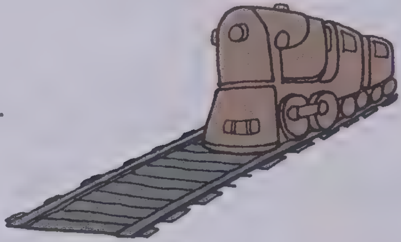


2. The winner of a bicycle race travelled 272 km in 8 hours. What was his average speed?

3. The Queen Elizabeth was one of the largest passenger ocean liners ever built. She travelled about 51 km/h. How far could the Queen Elizabeth travel in a day?

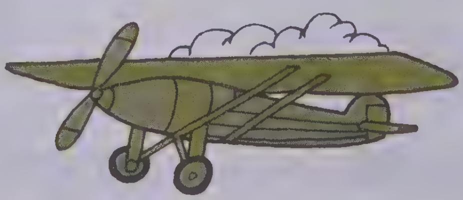
4. How many hours would it take the Queen Elizabeth to travel 408 km at the rate of 51 km per hour?

5. The longest straight railroad in the world is in Australia. This railroad is 524 kilometres long. If a train could average 131 km/h, how long would it take to cover this distance?



6. If a train travelled the 524 km in 7 hours, about what was the average speed?

★ 7. During his historic flight across the Atlantic Ocean, Charles A. Lindbergh flew his plane 5812 km in 33 hours. About what was his average rate?



Let's practice estimation.

These exercises will help you find quotients in division problems such as $62\overline{)2689}$.

1. Give the number pair for each gray space. Some estimation hints are given to help you find the number pair for exercises A through C. Multiply to be sure you have selected the correct number pair.

Pairs:

10	20	30	40	50	60	70	80	90
20	30	40	50	60	70	80	90	100

Think: $? \times 60 < 2689$

A $\square \times 62 < 2689$
$\square \times 62 > 2689$
(Answer: 40, 50)

Think: $? \times 50 < 2734$

B $\square \times 47 < 2734$
$\square \times 47 > 2734$

Think: $? \times 70 < 5160$

C $\square \times 65 < 5160$
$\square \times 65 > 5160$

D $\square \times 31 < 2000$
$\square \times 31 > 2000$

E $\square \times 59 < 3874$
$\square \times 59 > 3874$

F $\square \times 28 < 2689$
$\square \times 28 > 2689$

G $\square \times 45 < 3760$
$\square \times 45 > 3760$

H $\square \times 67 < 3642$
$\square \times 67 > 3642$

I $\square \times 81 < 7438$
$\square \times 81 > 7438$

J $\square \times 36 < 3476$
$\square \times 36 > 3476$

K $\square \times 92 < 6986$
$\square \times 92 > 6986$

L $\square \times 74 < 4835$
$\square \times 74 > 4835$

2. From the set {10, 20, 30, 40, 50, 60, 70, 80, 90}, find the largest number that will make each sentence true.

A $n \times 62 < 2689$
B $n \times 59 < 5874$
C $n \times 31 < 2000$
D $n \times 78 < 5160$
E $n \times 48 < 3760$
F $y \times 63 < 5538$
G $y \times 29 < 2600$
H $y \times 18 < 1307$
I $y \times 66 < 6004$
J $y \times 42 < 1837$

think

To tell you who I am,
I'll give you just one clue.
I'm as much more than 8
As I'm less than 52

WHO AM I?

Discussing the Ideas

1. To find the quotient for $\mathbf{62}\overline{)2689}$, think, → $? \times \mathbf{60} < 2689$

A What is the largest multiple of 10 that will make the inequality true?

B Explain how you can complete the dividing when you use the number found in part A.

C Try this one on your own: $61\overline{)2878}$

$$\begin{array}{r l} 62\overline{)\ 2689} & \\ -2480 & 40 \\ \hline 209 & \\ -186 & 3 \\ \hline 23 & \end{array}$$

2. Sometimes your estimate may be too large. $? \times \mathbf{80} < 5796$

A What is the largest multiple of 10 that will make the inequality true?

B Why is this multiple of 10 not correct for the quotient?

C Explain how to choose the correct multiple of 10 and complete the dividing.

D Try this one on your own: $64\overline{)3130}$

$$\begin{array}{r l} 69 & \\ 84\overline{)\ 5796} & \\ -5040 & 60 \\ \hline 756 & \\ -756 & 9 \\ \hline 0 & \end{array}$$

3. Sometimes your estimate may be too small. $? \times \mathbf{80} < 4668$

A What is the largest multiple of 10 that will make the inequality true?

B Why is that multiple of ten not correct for the quotient?

C Explain how to complete the dividing.

D Try this one on your own: $86\overline{)5240}$

$$\begin{array}{r l} 62 & \\ 75\overline{)\ 4668} & \\ -4500 & 60 \\ \hline 168 & \\ -150 & 2 \\ \hline 18 & \end{array}$$

1. In each exercise, find the missing tens' digit in the quotient.

A $\begin{array}{r}\square 2\\ 9\overline{)288}\end{array}$ B $\begin{array}{r}\square 8\\ 7\overline{)266}\end{array}$ C $\begin{array}{r}\square 1\\ 28\overline{)868}\end{array}$ D $\begin{array}{r}\square 3\\ 41\overline{)2173}\end{array}$

E $\begin{array}{r}\square 0\\ 32\overline{)2240}\end{array}$ F $\begin{array}{r}\square 6\\ 17\overline{)442}\end{array}$ G $\begin{array}{r}\square 2\\ 53\overline{)2226}\end{array}$ H $\begin{array}{r}\square 2\\ 71\overline{)3692}\end{array}$

I $\begin{array}{r}\square 3\\ 48\overline{)1584}\end{array}$ J $\begin{array}{r}\square 9\\ 82\overline{)5658}\end{array}$ K $\begin{array}{r}\square 5\\ 25\overline{)625}\end{array}$ L $\begin{array}{r}\square 5\\ 69\overline{)1035}\end{array}$

M $\begin{array}{r}\square 6\\ 94\overline{)2444}\end{array}$ N $\begin{array}{r}\square 0\\ 88\overline{)4400}\end{array}$ O $\begin{array}{r}\square 2\\ 76\overline{)4712}\end{array}$ P $\begin{array}{r}\square 9\\ 55\overline{)5445}\end{array}$

2. Find the quotients and remainders. Check your work.

A $8\overline{)236}$ B $39\overline{)337}$ C $30\overline{)1642}$ D $62\overline{)5831}$

E $19\overline{)158}$ F $60\overline{)500}$ G $80\overline{)605}$ H $7\overline{)2304}$

I $38\overline{)1420}$ J $50\overline{)2970}$ K $6\overline{)47\,653}$ L $44\overline{)387}$

M $75\overline{)2986}$ N $70\overline{)6734}$ O $86\overline{)7316}$ P $40\overline{)327}$

Q $35\overline{)3163}$ R $54\overline{)4731}$ S $98\overline{)7685}$ T $9\overline{)43\,297}$

3. Solve the equations.

A $272 \div 8 = n$ C $2881 \div 43 = n$ E $3276 \div 7 = n$

B $2940 \div 60 = n$ D $182 \div 26 = n$ F $53\,487 \div 9 = n$

★ **4.** Find the **divisor** for each exercise. The quotient is given.

A $\begin{array}{r}9\\ \square\overline{)225}\end{array}$ B $\begin{array}{r}30\\ \square\overline{)720}\end{array}$ C $\begin{array}{r}75\\ \square\overline{)2175}\end{array}$ D $\begin{array}{r}39\\ \square\overline{)1287}\end{array}$

★ **5.** For each exercise, find one number that can serve as **both** quotient and divisor.

A $\begin{array}{r}\square\\ \square\overline{)25}\end{array}$ B $\begin{array}{r}\square\\ \square\overline{)81}\end{array}$

C $\begin{array}{r}\square\\ \square\overline{)100}\end{array}$ D $\begin{array}{r}\square\\ \square\overline{)900}\end{array}$

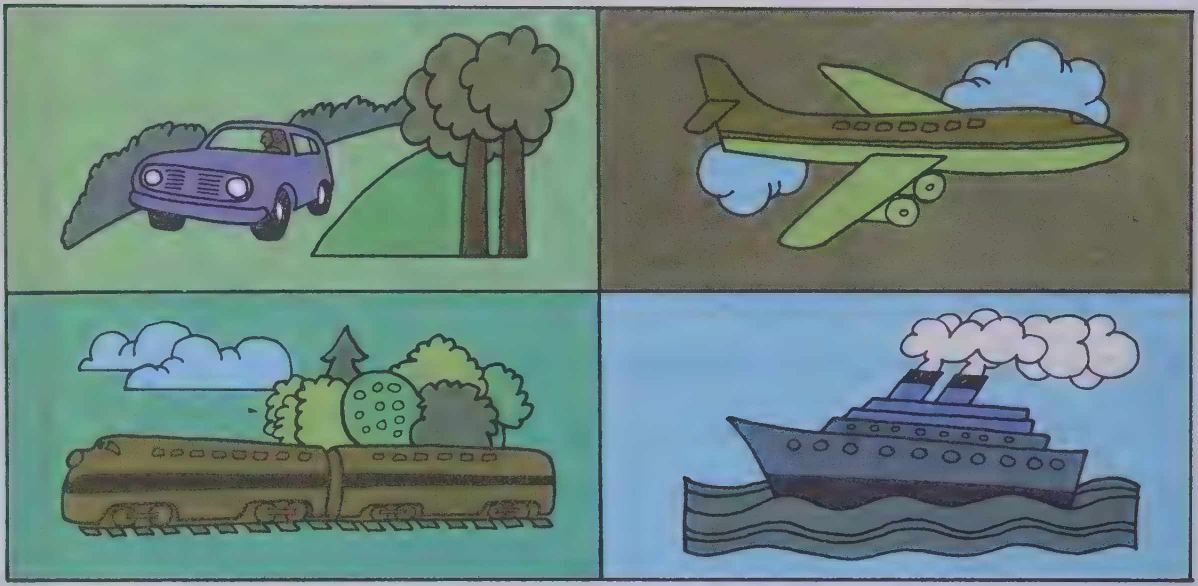
think

$$\begin{array}{r} \star\star \\ \square\overline{)\star\star\star} \\ \underline{\star\star\star} \\ 8\star \\ \underline{\star\star} \\ 0 \end{array}$$

Give the digit that should go in the $\square$. The ★'s are for other digits that you need not find.

More practice, page A-12, Set 24

Travel



1. A large ocean liner averages about 55 km / h. About how many hours would it take this ship to travel 2970 km ?

2. An express train averages 120 kilometres per hour. About how long would a 1380-kilometre trip take ?

3. If a car travels at an average speed of 72 km / h, how long would it take to go 705 kilometres ?

4. A large jet airliner travels 2792 km in 3 hours. About how far does it fly in 1 hour ?

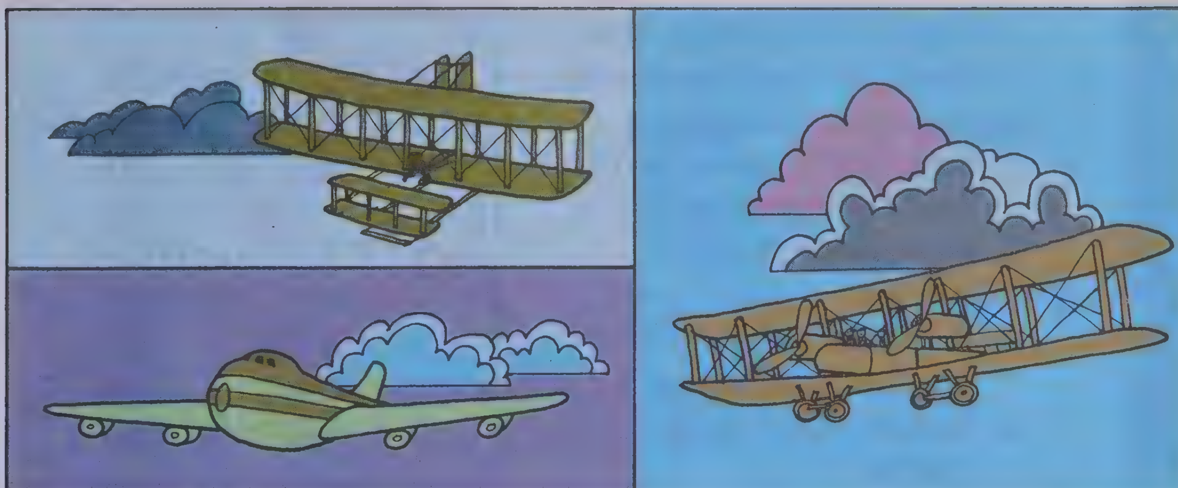
5. How far would an ocean liner travel in 78 hours if it averages 49 kilometres per hour ?

6. Suppose a freight train averages about 42 kilometres per hour. About how long would it take the train to go 1794 kilometres ?

★7. A man drove 616 km at a speed of 88 km / h and 480 km at a speed of 96 km / h. How far did the man drive ? How long did it take ?

★8. A captain wants his ship to go 3612 km in 3 days and 14 hours. How fast (how many km / h) should he plan to travel ?

AIRPLANES



1. If a jet plane flies 4044 kilometres in 4 hours, how fast is it travelling ?

2. If a jet plane carried 98 passengers on each flight, how many flights would it take to transport 3430 passengers from Toronto to Chicago ?

3. We would say that a jet plane travelling 2576 kilometres per hour is going very fast. In 1904 the Wright brothers flew a plane at a top speed of about 56 kilometres per hour. How many times faster is the jet than the plane flown by the Wright brothers ?

4. In 1910 an airplane flying at an altitude of 1800 metres would have been close to the record altitude. In 1960, 34 200 metres was close to record altitude. How many times higher could airplanes fly in 1960 than in 1910 ?

5. A ''747'' jet airliner measures 59 m 50 cm from one wing tip to the other.
   - A Give in centimetres the wing span of a ''747''.
   - B The smallest airplane has a wing span of 2m 15 cm. Give the wing span in centimetres.
   - C Use centimetres to calculate the number of the small airplanes that could fit within the wing span (wing tip to wing tip) of the 747 jet airliner.

Discussing the Ideas

1. The example in the box shows a shortcut for dividing. The method you have been using is shown for comparison.
   - A Explain the estimation in part 1 of the shortcut.
   - B Explain part 1 of the shortcut.
   - C Explain the estimation in part 2 of the shortcut.
   - D Explain part 2 of the shortcut.

Shortcut	Usual method
1 3	
42)1575	42)1575
126	1260 (30)
31	315
2 37	37
42)1575	42)1575
126	1260 (30)
315	315
294	294 (7)
21	21

2. In this example, if you use 22 ÷ 3 to get your estimate, your first quotient figure will be too small.
   - A Explain what you must do in each method when your first estimate is too small.
   - B Explain the estimation in part 3 of the shortcut.
   - C Explain part 3 of the shortcut.

Shortcut	Usual method
1 30 7	
26)2208	26)2208
182	1820 (70)
38	388
2 8	260 (10)
26)2208	128
208	84
12	26)2208
3 84	1820 (70)
26)2208	388
208	260 (10)
128	128
104	104 (4)
24	24

3. Sometimes your first estimate may be too large. You might estimate 6 for this problem: **64)3776**
   - A What must you do when your first estimate is too large?
   - B Use the shortcut to complete the dividing.

1. Use the heavy black numerals to help you estimate the first quotient figure for each exercise below.

40 50

A $52\overline{)209}$ B $39\overline{)245}$ C $23\overline{)115}$ D $45\overline{)1760}$

90 70

E $88\overline{)4148}$ F $42\overline{)1575}$ G $65\overline{)28\,293}$ H $73\overline{)38\,574}$

2. For each of the examples above, tell whether the quotient is between 0 and 10, 10 and 100, or 100 and 1000.

3. Find the quotients and remainders. Use the shortcut.

A $39\overline{)1722}$ B $71\overline{)6825}$ C $42\overline{)966}$ D $58\overline{)2842}$ E $18\overline{)967}$

F $27\overline{)918}$ G $63\overline{)4665}$ H $44\overline{)2812}$ I $65\overline{)5146}$ J $89\overline{)8265}$

4. Find the quotients and remainders. Use any method you choose. Check your work.

A $61\overline{)2878}$ B $39\overline{)1092}$ C $64\overline{)3030}$ D $19\overline{)1577}$

E $58\overline{)3700}$ F $47\overline{)987}$ G $85\overline{)4940}$ H $32\overline{)1920}$

think

$1 + \square = (\square \times 3) \div 2$

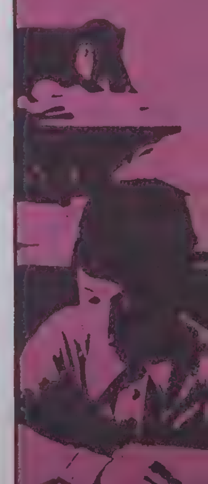
$1 + 2 + \square = (\square \times 4) \div 2$

$1 + 2 + 3 + \square = (\square \times 5) \div 2$

$1 + 2 + 3 + 4 + \square = (\square \times 6) \div 2$

$1 + 2 + 3 + 4 + 5 + \square = (\square \times 7) \div 2$

Check the equations to be sure they are true. Then estimate the sum of the whole numbers 1 through 30.



More practice, page A-13, Set 25

Discussing the Ideas

1. A Can you show that the quotient for $69\overline{)22\,364}$ is less than 1000 and greater than 100?

   B Explain how to estimate the number of hundreds in the quotient.

   C Explain step 1.

   D Explain how to estimate the number of tens in the quotient.

   E Explain step 2.

   F Explain how to estimate the number of ones in the quotient.

   G Explain step 3.

   H What are the quotient and the remainder?

70

$\boxed{1}$
```
        3
69 ) 2 2 3 6 4
     2 0 7
       1 6
```

$\boxed{2}$
```
        3 2
69 ) 2 2 3 6 4
     2 0 7
       1 6 6
       1 3 8
         2 8
```

$\boxed{3}$
```
        3 2 4
69 ) 2 2 3 6 4
     2 0 7
       1 6 6
       1 3 8
         2 8 4
         2 7 6
             8
```

2. A Can you show that the quotient for $42\overline{)12\,768}$ is less than 1000 and greater than 100?

   B Explain how to estimate the number of hundreds in the quotient.

   C Explain step 1.

   D Explain how you can tell that there are 0 tens in the quotient.

   E Explain how to estimate the number of ones in the quotient.

   F Explain step 3.

   G What are the quotient and the remainder?

$\boxed{1}$
```
        3
42 ) 1 2 7 6 8
     1 2 6
         1
```

$\boxed{2}$
```
        3 0
42 ) 1 2 7 6 8
     1 2 6
         1 6
```

$\boxed{3}$
```
        3 0 4
42 ) 1 2 7 6 8
     1 2 6
         1 6 8
         1 6 8
```

1. Study the first two steps in this example. Then copy the problem and see if you can complete it. Notice that the quotient is more than 100 and less than 1000.

$\boxed{1}$

```
        3
79 ) 26897
     237
      31
```

$\boxed{2}$

```
        34
79 ) 26897
     237
      319
      316
        3
```

2. Find the quotients and remainders. Check your work.

A $51\overline{)11\,934}$	B $32\overline{)3648}$	C $64\overline{)14\,784}$	D $39\overline{)21\,879}$
E $62\overline{)22\,630}$	F $65\overline{)40\,764}$	G $75\overline{)41\,625}$	H $23\overline{)9476}$
I $78\overline{)30\,342}$	J $89\overline{)61\,143}$	K $56\overline{)47\,376}$	L $27\overline{)5237}$

3. Find the quotients and remainders. Check your work.

A $31\overline{)3265}$	B $52\overline{)20\,904}$	C $69\overline{)37\,260}$	D $83\overline{)61\,918}$
E $35\overline{)7256}$	F $46\overline{)36\,835}$	★ G $231\overline{)1848}$	★ H $342\overline{)7005}$

4. A There are 28 children in a class. The sum of all their spelling scores is 2436. Find the average score.

B There are 3084 washers in a box. If 2 dozen washers are put into each package, how many packages will there be? How many extra washers?

think

Give the missing digits.

```
      ▯▯
 ▯) ▯12
    64▯
     ▯▯
     ▯▯
      0
```

More practice, page A-13, Set 26

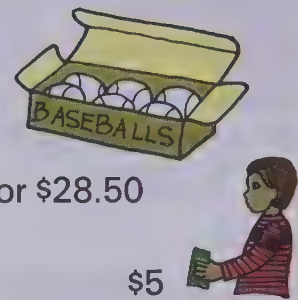
How can multiplication and division be used to solve problems about money?

Investigating the Ideas

$3.98 each

$20

Terry



6 for $28.50

$5

Jose

?	
Can Terry buy five records? Can Jose buy one baseball?	**NO** pencil — **NO** paper

Discussing the Ideas

1. How did you decide your answers to the Investigation questions?

2. Explain the flow chart below for 6 × $3.49.

$3.49	→ Think of the number of cents in $3.49.	→ Multiply by 6.	→ Write in dollar notation.
↓	↓	↓	↓
$3.49	349 cents	$\begin{array}{r} 349 \\ \times \quad 6 \\ \hline 2094 \end{array}$	$20.94

3. Explain the flow chart below for $4.65 ÷ 3.

$4.65	→ Think of the number of cents in $4.65.	→ Divide by 3.	→ Write in dollar notation.
↓	↓	↓	↓
$4.65	465 cents	$3\overline{)465}$ = 155	$1.55

1. Find the answers to these money problems.

A $2.53 × 7

B $5.13 × 27

C $0.56 × 4

D $0.87 × 54

E $3.62 × 435

F $5.64 × 65

G $8.00 × 98

H $36.50 × 7

I $36.98 × 32

J $0.95 × 98

K 5)$1.15

L 3)$0.72

M 7)$6.23

N 4)$13.88

O 6)$272.22

P 9)$77.67

Q 80)$4.00

R 60)$22.80

S 39)$234

T 61)$15.25

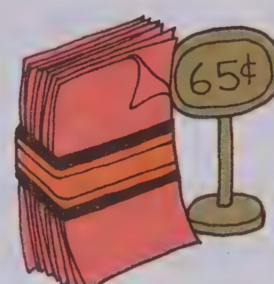
U 45)$41.85

V 36)$20.52

2. Solve the problems.

A Tom received $2.75 for each lawn he mowed. How much did he earn by mowing 8 lawns?

B Jill bought a package of 6 handkerchiefs for $2.16. Find the cost of one of these handkerchiefs.



C The price of a package of colored paper is shown in the picture. Jan bought 8 packages. What was the total cost?

D In exercise 2c, what change did Jan receive when she gave the clerk a $10 bill?

E If 8 kilograms of nuts cost $3.92, find the cost of one kilogram of nuts.

F Mr. Jones bought a TV for $288.90. He paid the same amount each month for 9 months. How much was each payment?

G One of the most expensive fabrics is a gold lace that costs $151.20 per metre (one metre long and 50 cm wide). Find the cost of 8 metres of this fabric.

H If 51 adult tickets were sold and $38.25 was collected, how much did each ticket cost?

I Which costs less per kilogram—an 8-kg package costing $3.92 or a 6-kg package costing $2.88?

More practice, page A-14, Set 27

1. Choose the word that completes each sentence correctly.
   A To estimate the quotient when the divisor ends in 1, 2, 3, or 4 (such as 21, 42, 53, or 34), we usually use the closest multiple of ten __?__ (greater than, less than) the divisor.
   B To estimate the quotient when the divisor ends in 5, 6, 7, 8, or 9 (such as 45, 26, 87, or 19) we usually use the closest multiple of ten __?__ (greater than, less than) the divisor.

2. Find the quotients and remainders. Check your work.

A $29\overline{)174}$	B $35\overline{)243}$	C $57\overline{)250}$	D $33\overline{)296}$	E $85\overline{)267}$
F $47\overline{)260}$	G $23\overline{)158}$	H $51\overline{)423}$	I $61\overline{)253}$	J $29\overline{)266}$
K $38\overline{)368}$	L $86\overline{)518}$	M $24\overline{)223}$	N $75\overline{)600}$	O $65\overline{)565}$

3. Solve the equations.

A $301 \div 7 = n$	C $3658 \div 59 = n$	E $7040 \div 80 = n$
B $1674 \div 62 = n$	D $1143 \div 9 = n$	F $841 \div 29 = n$

4. Find the quotients and remainders. Check your work.

A $79\overline{)10\,047}$	D $39\overline{)16\,602}$
B $71\overline{)29\,983}$	E $43\overline{)41\,252}$
C $95\overline{)20\,746}$	F $84\overline{)58\,225}$

5. Find the answers to these money problems.

A $\begin{array}{r} \$1.79 \\ \times 3 \\ \hline \end{array}$ B $\begin{array}{r} \$0.84 \\ \times 25 \\ \hline \end{array}$ C $\begin{array}{r} \$2.68 \\ \times 19 \\ \hline \end{array}$

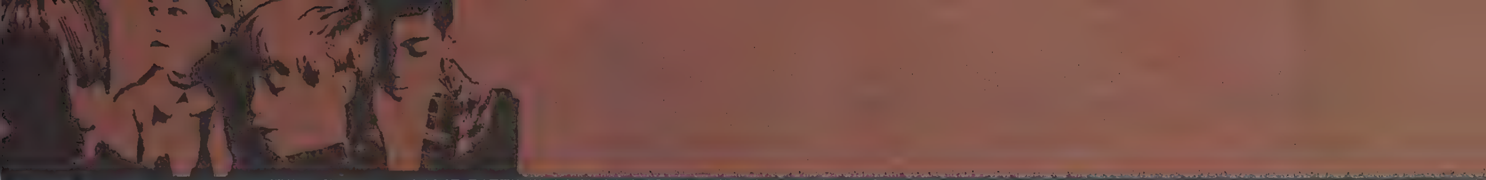
D $6\overline{)\$1.14}$ E $21\overline{)\$7.35}$

think

Have a friend select any row on a calendar that has 7 days. Tell him you can find the sum of the dates faster than he can. Suppose he chooses a week like this:

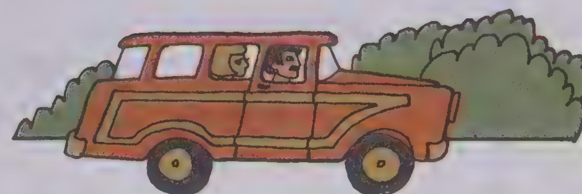
9	10	11	12	13	14	15

Take the smallest number, add 3 and multiply by 7. Can you explain why this gives the sum?



TRAVEL PROBLEMS

1. Solve the problems. Write an equation for each problem.



A A man used 24 l of gasoline while travelling 336 km. What is the average number of km he can travel while using one litre of gasoline ?

B If a man wants to go 195 kilometres in 3 hours, about how fast should he drive ?

C If a motor scooter travels about 36 kilometres per hour, about how long will it take to go 180 kilometres ?

D If Mr. Jones drives about 62 kilometres per hour, about how long will it take him to go 806 kilometres ?

E If a train averaged 83 kilometres per hour and travelled 498 kilometres, how many hours did it travel ?

2. Solve these money problems.

A Mr. Ray bought 5 tires. Each tire cost $24.95. Find the total cost of the tires.

B Mr. Ito paid $93.96 for 4 tires. How much did each tire cost ?

C What is the difference in the cost of Mr. Ray's and Mr. Ito's tires ?

3. On a vacation trip the Williams family travelled these distances in a car.

Monday	505
Tuesday	416
Wednesday	274
Thursday	366
Friday	119

A What was the total distance for the five days ?

B What was the average distance the family travelled in a day ?

C Mr. Williams bought a total of 420 litres of gasoline on the trip. How many kilometres for each litre ?

1. Find the products.

A 10×10 C 40×30 E 40×40 G 80×60

B 30×10 D 60×80 F 90×70 H 50×0

2. Give the number for n.

A $6386 = (63 \times n) + (8 \times 10) + 6$ C $6547 = (654 \times n) + 7$

B $58\,346 = (58 \times n) + (3 \times 100) + (4 \times 10) + 6$

3. Give the numbers for a and b. Then give the number for c.

A $20 \times 4 = a$, $6 \times 4 = b$ → $26 \times 4 = c$

B $50 \times 6 = a$, $8 \times 6 = b$ → $58 \times 6 = c$

C $280 \div 7 = a$, $14 \div 7 = b$ → $294 \div 7 = c$

D $240 \div 8 = a$, $32 \div 8 = b$ → $272 \div 8 = c$

4. Estimate the answers to these exercises.

A $594 + 316 + 891$ D 48×31 G $903 - 396$

B $5876 + 2946$ E 59×22 H 198×204

C $39 + 49 + 59 + 69$ F 81×99 I $363 \div 92$

5. Give the missing numbers in the table.

Function Rule: $30 \times n$

n	Input
9	270
6	A
10	B
80	C
D	1800

★ **6.** Find the missing digits.

A □□6 − 23□ = 368

B □705 − 4□□9 = 4466

C 5□□6 − □53□ = 1768

D 5□□4 − □73□ = 3004

E □□□□ − 3658 = 3342

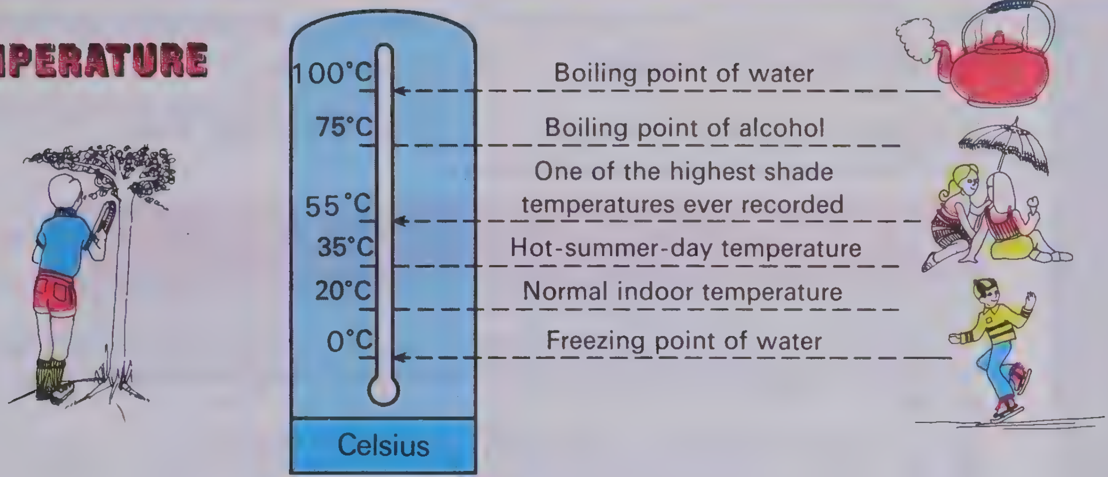
F □8□7 − 3□7□ = 4828

G 7□2□ − □664 = 3058

H □00□ − 6767 = 1□□1

I 7125 − □□□□ = 3478

TEMPERATURE

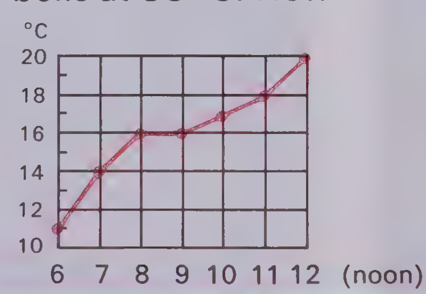


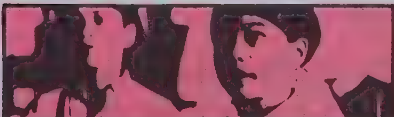
1. A How many Celsius degrees greater is the boiling point of water than the freezing point ?
   B How many Celsius degrees greater is the boiling point of water than the highest temperature recorded in shade ?
   C How many Celsius degrees are there between the boiling points of water and alcohol ?

2. How would you write the temperature when it is seven degrees below zero ?

3. Water at the top of a certain high mountain boils at 88 °C. How much less is the boiling point of water at this height than it is at sea level ?

4. Here is a **line graph** that shows the temperature at each hour between 6 A.M. and noon. Study the graph and find the average of the hourly temperatures.





You are invited to explore

ACTIVITY CARD 7
Page 336

8 Number Theory

What are the factors of a number?

Investigating the Ideas

The figure shows an "equals" (all strips the same) train that matches a 12-centimetre strip.

? How many other "equals" trains that match the 12-centimetre strip can you find ?

Discussing the Ideas

1. The figure above shows that 2 is a factor of 12. What other numbers did you find that are factors of 12 ?

2. Can you use your strips to show that 5 is not a factor of 12 ?

3. Study the figures below.

$\overset{F}{8} \times \overset{F}{6} = \overset{P}{48}$ $\begin{cases} \text{8 is a **factor** of 48.} \\ \text{6 is a **factor** of 48.} \end{cases}$	$\overset{P}{48} \div \overset{F}{3} = \overset{F}{16}$ $\begin{cases} \text{3 is a **factor** of 48.} \\ \text{16 is a **factor** of 48.} \end{cases}$
Multiplication can help you find factors of a number.	Division can help you find factors of a number.

   Can you find any other factors of 48 ?

4. Show that 4 is a factor of 48 by finding the quotient for $4\overline{)48}$. What is another factor of 48 ?

5. Show that 5 is **not** a whole number factor of 48 by finding the quotient for $5\overline{)48}$ and by observing that the **remainder** is not zero.

1. Each equation shows 24 written as the product of two factors. Can you write 24 as a product of two factors that are not shown?

$$1 \times 24 = 24$$
$$2 \times 12 = 24$$
$$3 \times 8 = 24$$
$$4 \times 6 = 24$$

2. List all the whole numbers that are factors of 24. The equations from exercise 1 should help you.

3. Tell whether or not the first number is a factor of the second.
   A 3, 27 B 4, 58 C 6, 72 D 7, 91 E 8, 108 F 8, 96

4. A Show all the ways of writing 18 as the product of two factors. Use each factor only once. (There are 3 ways.)
   B List all the factors of 18.

5. Show all the ways of writing each number as the product of two factors. Use each factor only once.
   A 15 (There are 2 ways.)
   B 45 (There are 3 ways.)
   C 42 (There are 4 ways.)
   D 30 (There are 4 ways.)
   E 35 (There are 2 ways.)
   F 60 (There are 6 ways.)

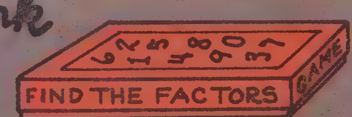
6. List all the factors for each number in exercise 5.

7. Solve the equations. Then list all the factors of 72.
   A $72 \div 1 = n$
   B $72 \div 2 = n$
   C $72 \div 3 = n$
   D $72 \div 4 = n$
   E $72 \div 6 = n$
   F $72 \div 8 = n$

8. List all the factors of each number.
   A 20
   B 13
   C 32
   D 27

9. Give a number that is a factor of every number.

think



Some numbers have exactly 3 factors. For example, 4 has the factors {1, 2, 4} and 9 has the factors {1, 3, 9}. Find some other numbers that have exactly 3 factors.

Investigating the Ideas

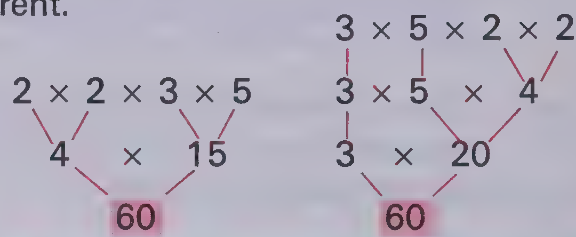
Here are some rules for building **factor trees**.

RULE 1 No 1's allowed.

RULE 2 Each tree grows as tall as possible.

RULE 3 Two trees are different if the factors at any level are different.

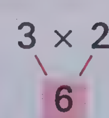
? How many other different trees can you build for 60 ?



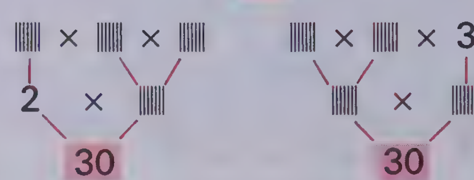
TWO DIFFERENT TREES FOR 60

Discussing the Ideas

1. What factors are in the top row of each of your factor trees for 60 ?
2. Does 6 have a factor tree that is different from this one ? Explain.

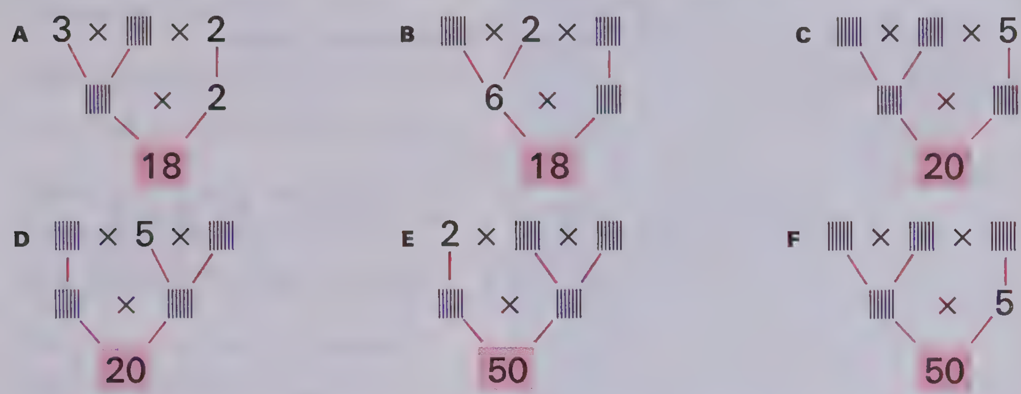


3. Copy each factor tree on your paper and give the missing factors. Explain how you completed the trees.



4. Find another factor tree for 30.
5. Show that each row of each factor tree for 30 contains numbers that are factors of 30.
6. Which row of the three trees contains the same factors of 30 ?

1. Copy each factor tree and give the missing factors.

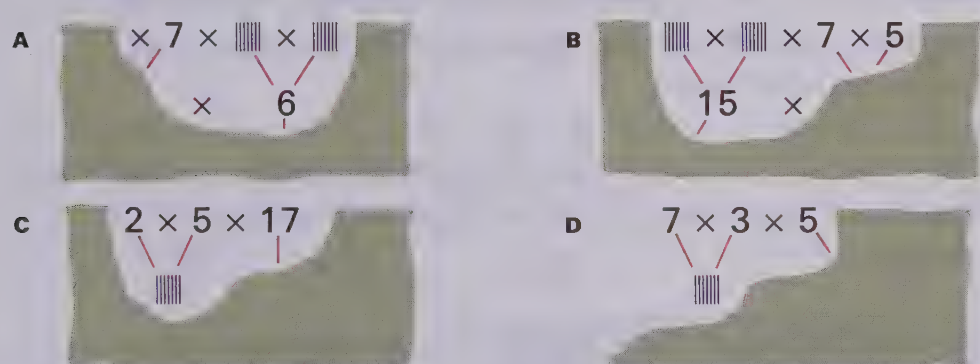


2. How many different trees can you draw for 24?

3. Draw a factor tree for each of these numbers.

A 45 B 28 C 16 D 70 E 36 F 75

4. In each exercise, part of a factor tree is shown.
Give the number for each ▯.



5. Give the missing numbers.

A If 6 is a factor of a number, then ▯ and ▯ are factors of that number.

B If 15 is a factor of a number, then ▯ and ▯ are factors of that number.

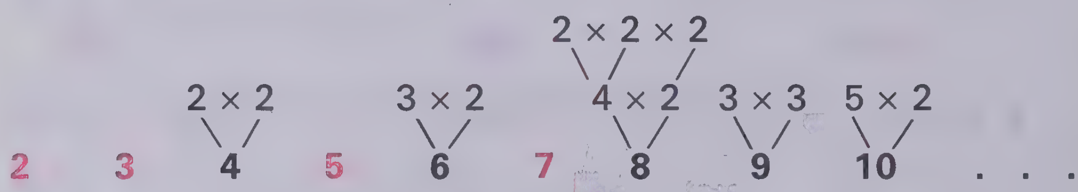
C If 2 and 5 are factors of a number, then ▯ is a factor of the number.

D If 7 and 3 are factors of a number, then ▯ is a factor of the number.

More practice, page A-15, Set 28

Investigating the Ideas

The numbers 2, 3, 5, and 7 do not have factor trees.
Do you see why?



Can you find the other numbers less than 50 that do not have factor trees?

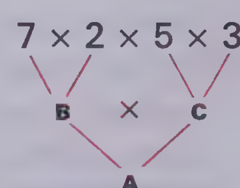
Discussing the Ideas

1. The prime numbers are the whole numbers greater than 1 that have no factor trees. How many factors does each prime number have?

2. The composite numbers are the whole numbers greater than 1 that have factor trees. What are the first 5 composite numbers?

3. The top row of each factor tree should have only prime numbers. Explain why this is so.

4. How can you use the numbers in the top row of this factor tree to find the number for A? Find the missing numbers.



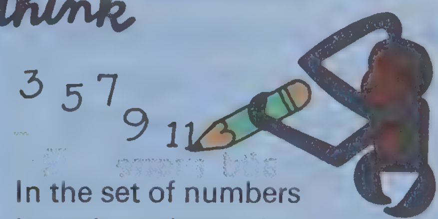
5. Can you use exercise 4 to help you write 210 as the product of prime factors?

1. List all the prime numbers between 1 and 50.

2. What number is both an even number and a prime number?

3. Find two prime numbers whose difference is 1. Are there other such pairs?

4. Find two prime numbers whose difference is 3. Are there other such pairs?

> *think*
>
> 
>
>
> In the set of numbers
> Less than ninety-three,
> I'm the largest one
> That doesn't have a tree.
>
> **WHO AM I?**

5. Copy and complete each factor tree. Then give the number at the bottom as a product of prime factors.

A
$\square \times \square \times \square$
$7 \times \square$
70

B
$\square \times \square \times \square \times \square$
$\square \times 4$
16

C
$\square \times \square \times \square$
$3 \times \square$
105

D
$\square \times \square \times \square \times \square$
$6 \times \square$
150

E
$\square \times \square \times \square \times \square$
$15 \times \square$
150

F
$\square \times \square \times \square \times \square$
$10 \times \square$
150

6. Express each number as a product of prime factors.

A 21	B 42	C 30
D 27	E 72	F 60

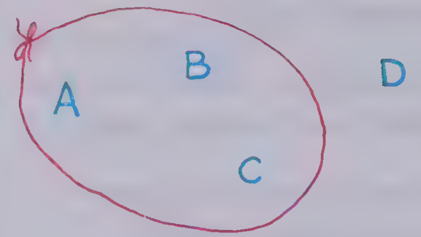
★ 7. Will two different factor trees for the same number give the same factors in the top row?

> *think*
>
> | | |
> |---|---|
> | Twin primes are | 3,5 |
> | primes that differ by | 5,7 |
> | two. Here are some | 11,13 |
> | pairs of twin primes. | 17,19 |
>
> Find a pair of twin primes that is between 100 and 110.

Let's explore union and intersection of sets.

Investigating the Ideas

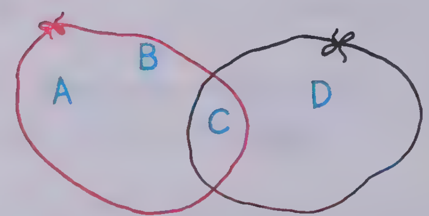
Make a loop out of a 30-cm piece of string. The figure shows another loop with 3 letters inside and 1 letter outside.



?	
How many ways can you place your loop of string on the figure so that **each letter is inside one of the loops** **and** **at least one letter is inside both loops?**	Draw pictures to show the ways you placed your loop.

Discussing the Ideas

1. Each loop in your Investigation contained a set of letters. The **union** of the two sets is all the letters that are in one set or the other or in both sets. What is the union of the two sets in the figure?

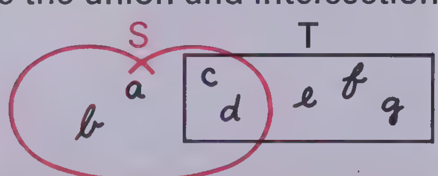


2. A The **intersection** of the two sets is the set of letters that are in **both** sets. What is the intersection of the sets in the figure?

B What letters were in the intersections of the sets you formed in the Investigation?

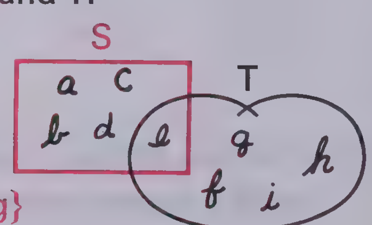
3. Give the union and intersection of sets S and T.

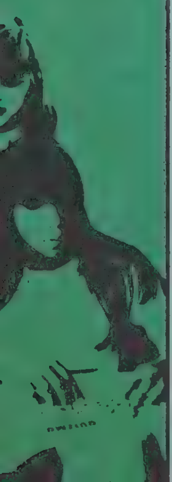
A



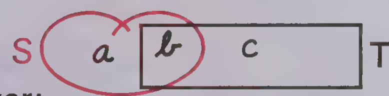
Answer: Union: $S \cup T = \{a, b, c, d, e, f, g\}$
Intersection: $S \cap T = \{c, d\}$

B

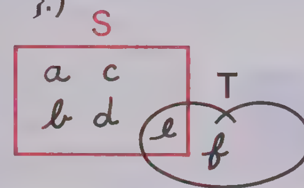


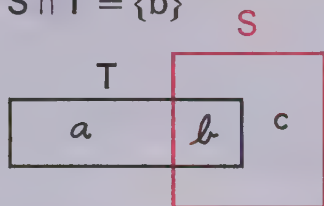


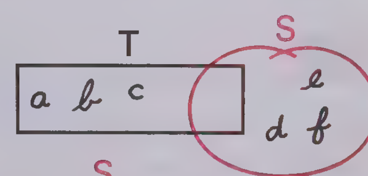
1. For each exercise, give the union and the intersection of the two sets. (Note: If there are no letters in the intersection of the sets, just write S ∩ T = { }.)

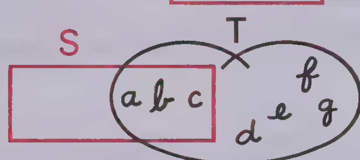
A


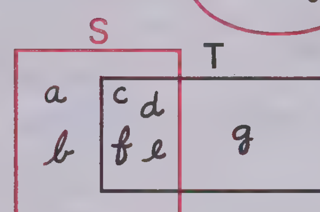
Answer:
S ∪ T = {a, b, c,}
S ∩ T = {b}

B


C


D


E


F


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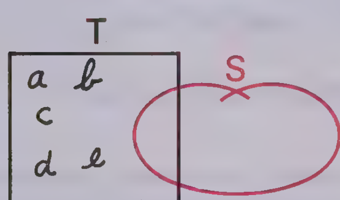
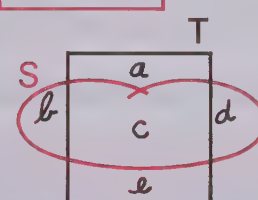


H


2. Study the example. Then give the union and the intersection for each exercise.

Example: S = {1, 2, 3, 4} S ∪ T = {1, 2, 3, 4, 5, 6, 7}
T = {3, 4, 5, 6, 7} S ∩ T = {3, 4}

A S = {1, 2, 3}
T = {3, 4}

B S = {0, 1, 2, 3, 4, 5}
T = {3, 4, 5, 6}

C S = {0, 1, 2}
T = {1, 2, 3, 4, 5, 6}

D S = {0, 1, 2, 3}
T = {4, 5, 6}

E S = {0, 1, 2, 3, 4, 5, 6}
T = {2, 3, 4}

F S = {7, 8, 9}
T = {1, 2, 3}

G S = {5, 6, 7}
T = {2, 3, 4, 5, 6, 7, 8}

H S = {9, 10, 11}
T = {7, 8, 9, 11, 12, 13}

More practice, page A-15, Set 29

What is the greatest common factor of two numbers?

Investigating the Ideas

Don gave the factors of 30. Linda gave the factors of 48.

? Can you find the intersection of Don's and Linda's sets?	Record the largest number in the intersection of the sets.

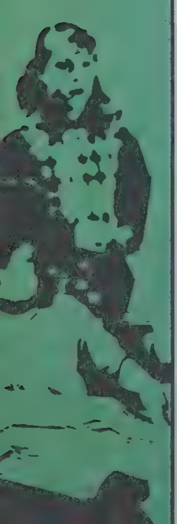
Discussing the Ideas

1. The largest number in the intersection of the children's sets is the **greatest common factor** of 30 and 48. What is it?

2. Study each table and give the greatest common factor of the two numbers.

$S = \{1, 2, 3, 4, 6, 12\}$ ←	The factors of 12
$T = \{1, 2, 4, 5, 10, 20\}$ ←	The factors of 20
$S \cap T = \{1, 2, 4\}$ ←	The common factors of 12 and 20
▮▮▮ is the greatest common factor of 12 and 20.	

$R = \{1, 2, 3, 6, 9, 18\}$ ←	The factors of 18
$Q = \{1, 3, 9, 27\}$ ←	The factors of 27
$R \cap Q = \{1, 3, 9\}$ ←	The common factors of 18 and 27
▮▮▮ is the greatest common factor of 18 and 27.	



1. A List the factors of 12. B List the factors of 18.
   C List the common factors of 12 and 18.
   D What is the greatest common factor of 12 and 18?

2. A List the factors of 18. B List the factors of 20.
   C List the common factors of 18 and 20.
   D What is the greatest common factor of 18 and 20?

3. For each pair of numbers below:
   [A] List the factors of each number.
   [B] List the common factors of the two numbers.
   [C] Give the greatest common factor of the two numbers.

A 12, 4	C 8, 12	E 10, 14	G 5, 15	I 7, 13
B 8, 20	D 9, 27	F 40, 50	H 25, 20	J 16, 24

4. Give the greatest common factor of each pair of numbers.

A 15, 25	D 24, 18	G 9, 10	J 18, 30
B 10, 30	E 3, 21	H 18, 15	K 12, 9
C 18, 8	F 8, 7	I 50, 20	L 15, 8

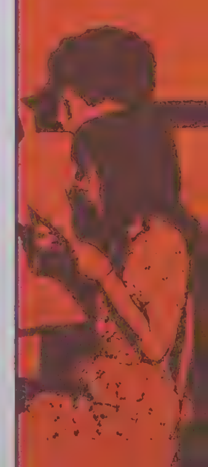
5. What number is a common factor of any pair of numbers?

★ 6. If both of the numbers are prime, what can you say about the greatest common factor of the two numbers?

★ 7. If one of two numbers is prime, what can you say about the greatest common factor of the two numbers?

think

We each would like to be
A composite or a prime.
But when we try to join,
We are turned down
every time.



More practice, page A-16, Set 30

What is the least common multiple of two numbers?

Investigating the Ideas

When your 2-strips and 3-strips "match" at 0, they "match" at 6, 12 . . . .

?

Can you find where your 6-strips and 8-strips first match after 0?

Discussing the Ideas

You were finding multiples and common multiples in the Investigation. The common multiples of 2 and 3 are

{0, 6, 12, 18, . . .}

The first time the strips match after 0 is called the **least common multiple** of the two numbers.

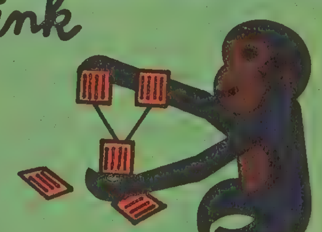
1. What is the least common multiple of 2 and 3?

2. What is the least common multiple of 4 and 6?

1. A List the multiples (to 36) of 3. B List the multiples (to 36) of 4.
   C List the common multiples (to 36) of 3 and 4.
2. A List the multiples (to 24) of 2. B List the multiples (to 24) of 4.
   C List the common multiples (to 24) of 2 and 4.
3. A List the multiples (to 48) of 6. B List the multiples (to 48) of 8.
   C List the common multiples (to 48) of 6 and 8.
4. A List the multiples (to 42) of 3. B List the multiples (to 42) of 6.
   C List the common multiples (to 42) of 3 and 6.
5. A List the multiples (to 60) of 10. B List the multiples (to 60) of 4.
   C List the common multiples (to 60) of 10 and 4.
6. A List the multiples (to 60) of 5. B List the multiples (to 60) of 6.
   C List the common multiples (to 60) of 5 and 6.
7. A List the multiples (to 80) of 8. B List the multiples (to 80) of 10.
   C List the common multiples (to 80) of 8 and 10.
8. A List the multiples (to 70) of 5. B List the multiples (to 70) of 7.
   C List the common multiples (to 70) of 5 and 7.
9. What number is a common multiple for every pair of numbers?
10. Give the least common multiple for:

A 3 and 4	G 8 and 10
B 2 and 4	H 5 and 7
C 6 and 8	I 5 and 2
D 3 and 6	J 9 and 4
E 10 and 4	K 5 and 4
F 5 and 6	L 3 and 5

think



I'm an odd composite.
The smallest one you'll see
That has two different primes
Appearing on its tree.

WHO AM I?

More practice, page A-16, Set 31

Reviewing the Ideas

1. Copy each factor tree and give the missing factors.

   A ▯ × ▯ × ▯
   7 × ▯
   63

   B ▯ × ▯ × ▯ × ▯
   9 × ▯
   54

   C ▯ × ▯ × ▯ × ▯
   10 × ▯
   90

2. Draw a factor tree for each number.
   A 14 B 10 C 28 D 45 E 56

3. Tell whether or not each number is prime. Show a factor tree for each number that is not prime.
   A 21 B 29 C 33 D 67 E 81 F 51 G 47

4. List all the factors of each number.
   A 14 B 12 C 16 D 18 E 20 F 22 G 40

5. Give $S \cup T$ and $S \cap T$ for each pair of sets.
   A $S = \{2, 4, 6, 8, 10\}$
     $T = \{0, 1, 2, 3, 4, 5\}$
   B $S = \{6, 7, 8, 9, 10, 11\}$
     $T = \{0, 1, 2, 3, 4, 5, 6\}$
   C $S = \{0, 2, 4, 6, 8, 10\}$
     $T = \{1, 3, 5, 7, 9, 11\}$
   D $S = \{\ \}$
     $T = \{1, 2, 3\}$
   E $S = \{5, 7, 9\}$
     $T = \{4, 5, 6, 8, 9, 10\}$
   F $S = \{7, 8, 9, 10\}$
     $T = \{7, 8, 9, 10\}$

6. A List all the factors of 30.
   B List all the factors of 18.
   C List the common factors of 18 and 30.
   D What is the greatest common factor of 18 and 30?

7. A List the first 10 multiples of 6.
   B List the first 10 multiples of 8.
   C List some common multiples of 6 and 8.
   D What is the least common multiple of 6 and 8?

8. Give the greatest common factor for each pair of numbers.
   - A 4 and 10
   - B 15 and 25
   - C 18 and 48
   - D 24 and 40

9. Give the least common multiple for each pair of numbers.
   - A 4 and 10
   - B 6 and 15
   - C 8 and 5
   - D 12 and 4

10. If the numbers 1, 2, 3, and 6 are the common factors of two numbers, what is the greatest common factor of the numbers?

think

No one has yet found an even number greater than 2 that is not the sum of two primes. Here are some examples:

$4 = 2 + 2$	$10 = 5 + 5$
$6 = 3 + 3$	$12 = 5 + 7$
$8 = 5 + 3$	$14 = 7 + 7$

You try this for the rest of the even numbers to 50.

11. If the first four common multiples of two numbers are 0, 12, 24, and 36, what is the least common multiple?

12. Give the missing word or number.
   - A No prime number greater than 7 ends with one of the digits 0, 2, 4, 6, 8, or ▥.
   - B If a number is prime, then it has exactly two __?__.
   - C If a number is the product of two smaller numbers, then it is __?__.
   - D The numbers less than 10 that have exactly three factors are 4 and ▥.
   - E ▥ is the smallest 2-digit number that has exactly three factors.
   - F If 7 and 3 are both factors of a number, then ▥ is a factor of the number.
   - G If 10 is a factor of a number, then 2 and ▥ are factors of the number.
   - H If 6 and 5 are factors of a number, then 30, 15, and ▥ are also factors of the number.

1. Solve the equations.

A $9 \times n = 36$	C $n = 23 - 9$	E $46 = (n \times 10) + 6$
B $70 \div 10 = n$	D $35 = n + 28$	F $63 - (n \times 7) = 0$

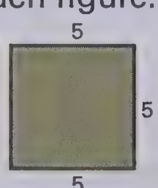
2. Compute the sum, product, difference, or quotient.

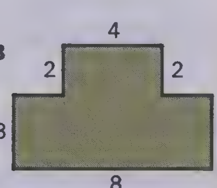
A $\begin{array}{r}576\\+985\\\hline\end{array}$	B $\begin{array}{r}489\\\times 4\\\hline\end{array}$	C $\begin{array}{r}482\\-167\\\hline\end{array}$	D $\begin{array}{r}86\\\times 34\\\hline\end{array}$	E $\begin{array}{r}804\\-457\\\hline\end{array}$
F $\begin{array}{r}583\\\times 26\\\hline\end{array}$	G $\begin{array}{r}999\\+888\\\hline\end{array}$	H $\begin{array}{r}900\\-398\\\hline\end{array}$	I $\begin{array}{r}467\\\times 231\\\hline\end{array}$	J $\begin{array}{r}4002\\-879\\\hline\end{array}$
K $\begin{array}{r}839\\\times 207\\\hline\end{array}$	L $\begin{array}{r}7034\\-2769\\\hline\end{array}$	M $5\overline{)460}$	N $7\overline{)1867}$	

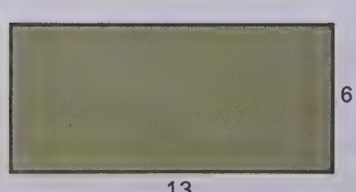
O $8\overline{)46\,592}$ P $32\overline{)256}$ Q $59\overline{)1416}$ R $75\overline{)3075}$

3. A bottle of perfume holds a little more than 29 millilitres of liquid. **Estimate** the number of millilitres of perfume in 8 bottles.

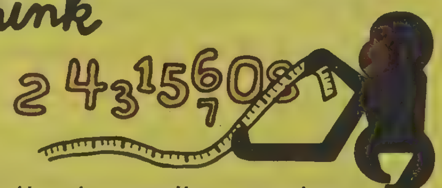
4. Find the **area** and **perimeter** of each figure.

A


B


C


think



I'm the smallest number
Ever to make this claim.
I have 4 prime factors,
With none of them the same.

$a \times b \times c \times d = ?$

WHO AM I?

Solving Story Problems

FIND THE NUMBER

1. Find the number that is 78 more than 296.
2. What number must be added to 346 to get 501 for the sum?
3. Find the number that is 59 less than the product of 26 and 59.
4. What number must you multiply by 36 to get 1512 for the product?
5. Give the number that is 269 less than the sum of 3268, 4297, and 6598.
6. Find the number that is three times the difference of 2003 and 867.
7. Give the number that must be added to the product of 46 and 18 to get 985.
8. What number is six times the sum of 386, 265, 19, and 1268?
9. Find the number that is twice the product of 62 and 7864.
10. Give the number that is 75 less than the sum of 687, 346, 928, 467, 159, 847, and 698.
11. What number is 268 more than the product of 34 and 6925?
12. What number is eight times twice the sum of 6289 and 75 668?
13. Find the number that is 67 more than the quotient of 1728 and 27.
14. Give the number that is 64 times the sum of 326, 547, 832, and 964.

You are invited to explore

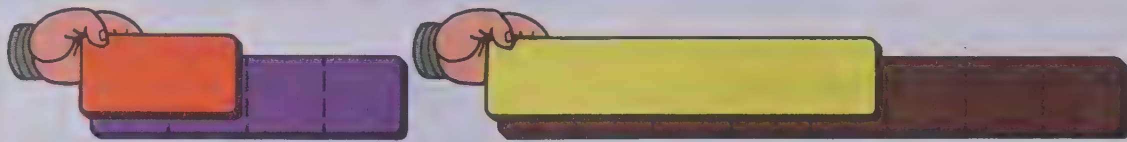
ACTIVITY CARD 8
Page 337

9

Fractions

Let's explore number pairs and fractions.

Investigating the Ideas



2 of the 4 parts are covered.

5 of the 8 parts are covered.

$\frac{2}{4}$ of the purple strip is covered.

$\frac{5}{8}$ of the brown strip is covered.

? Can you use your strips and complete at least 10 more lines of a table like this?

Strips	Number Pair		Fraction of the strip covered
	Parts covered	Parts in all	
red purple	2	4	$\frac{2}{4}$
yellow brown	5	8	$\frac{5}{8}$

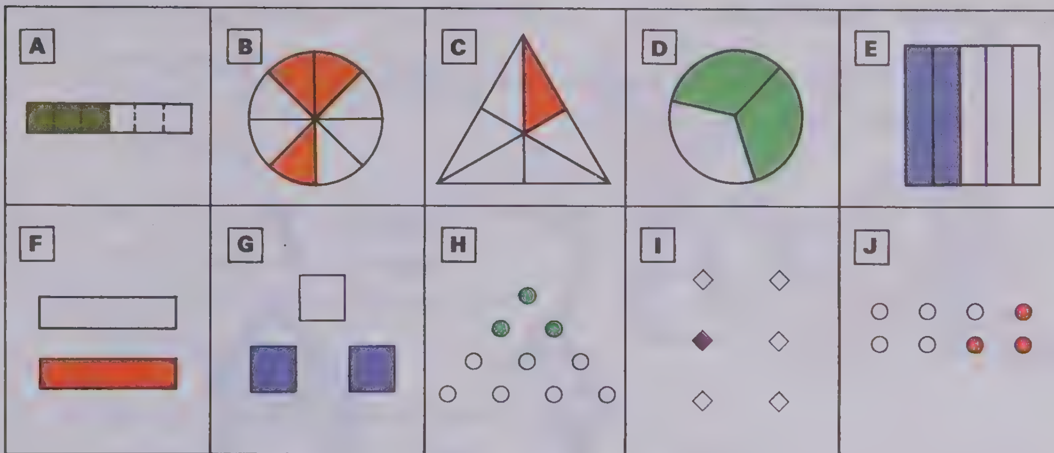
Discussing the Ideas

1. Cover parts of the orange strip with other strips. In this way, how many fractions can you show?

2. Cover parts of the black strip with other strips. Now what fractions can you name?

3. What strips would you use to show each of these?

   A $\frac{7}{8}$ **B** $\frac{5}{7}$ **C** $\frac{3}{5}$ **D** $\frac{6}{9}$ **E** $\frac{5}{6}$ **F** $\frac{2}{3}$ **G** $\frac{8}{8}$

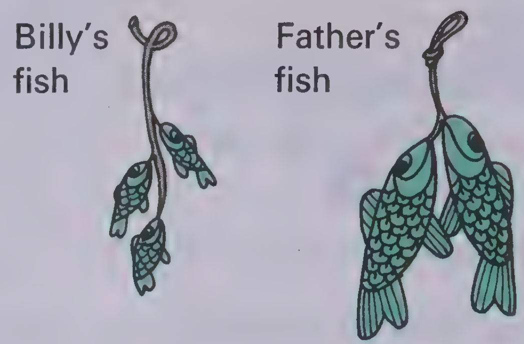


Each row of the table refers to one of the pictures (A through J) above. Give the missing pictures, numbers, or fractions.

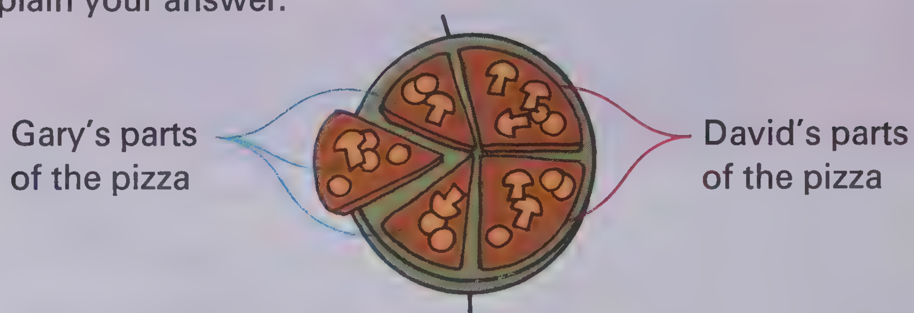
	Picture	Number Pair		Fraction of figure or objects that are colored
		Number colored	Total number	
1.	A	A	6	$\frac{3}{6}$ of the strip is colored.
2.	G	A	B	$\frac{2}{3}$ of the objects are blue.
3.	B	A	B	$\frac{3}{8}$ of the circle is colored.
4.	F	A	B	C of the objects are red.
5.	A	2	3	B of the figure is colored.
6.	H	A	B	C of the objects are green.
7.	C	A	B	C of the triangle is red.
8.	A	2	B	$\frac{2}{5}$ of the figure is colored.
9.	I	A	B	C of the objects are purple.
10.	J	A	B	C of the objects are colored.

Discussing the Ideas

1. A Did Billy catch $\frac{3}{5}$ of the number of fish caught? Explain your answer.



B Did Gary eat $\frac{3}{5}$ of the pizza? Explain your answer.



2. Draw five small circles. Color three of them red.

$$\text{NUMERATOR} \rightarrow 3 \qquad \frac{\;}{}\qquad 5 \leftarrow \text{DENOMINATOR}$$

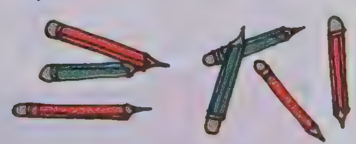
A What fraction of the circles are red?
B What does the numerator of the fraction tell?
C What does the denominator of the fraction tell?

3. What fraction of the children in your room are girls? What denominator did you use? What numerator?

1. A What fraction of the children in this group are girls?
   B What is the numerator of this fraction?

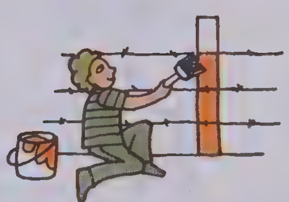
2. A What fraction of the children in the group wear glasses?
   B What is the denominator of this fraction?

3. A What fraction of the pencils in this set are red?

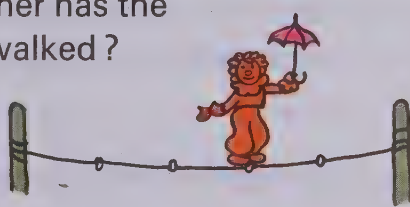


   B What fraction of the pencils in this set are green?

4. A What fraction of the post is painted?
   B What fraction of the post is not painted?
   C Give the numerator and denominator of the fraction in exercise 4B.



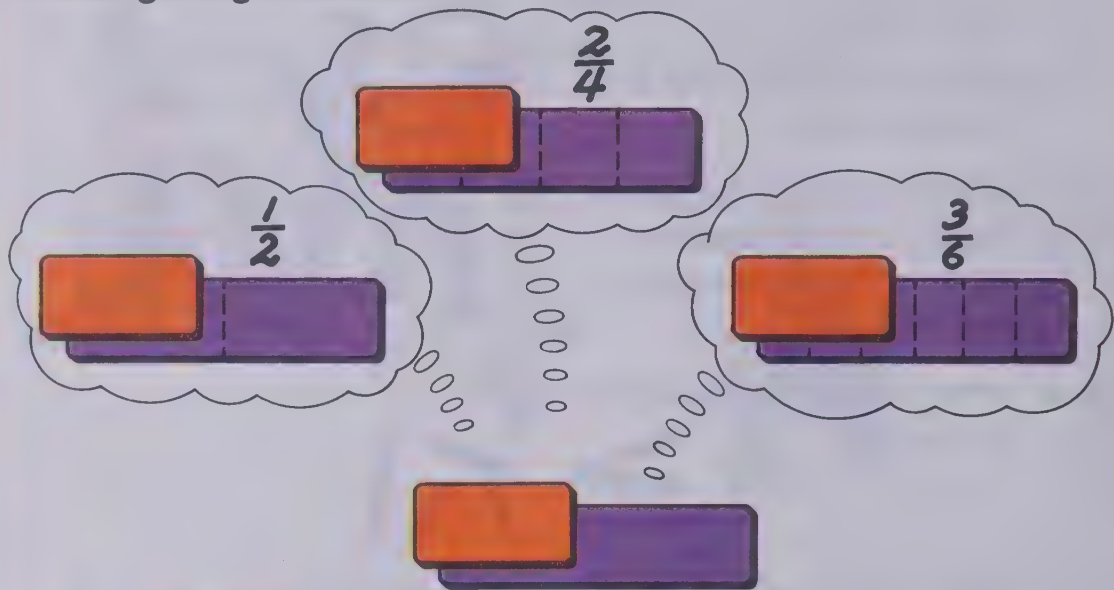
5. What fraction of the way from one post to the other has the tightrope walker walked?



6. What fraction of the window must be replaced?

7. If 2 more window panes had been broken, what would be the numerator of the fraction in exercise 6?

Investigating the Ideas



Imagine the purple strip divided equally in other ways.

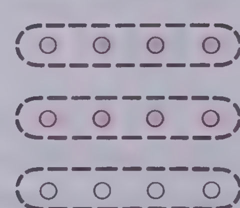
?

How many more fractions that tell what part of the purple strip is covered can you find?

Discussing the Ideas

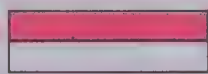
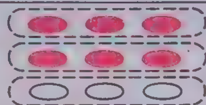
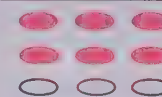
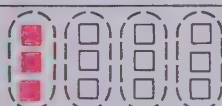
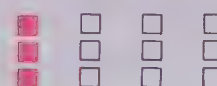
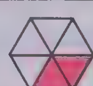
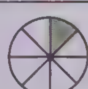
A pair of fractions that suggest the same number of objects in a set or the same part of an object are called equivalent fractions.

1. Use the idea in the Investigation and give some fractions equivalent to $\frac{1}{3}$.



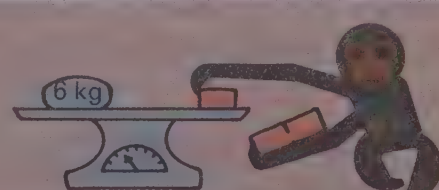
2. A Explain what you might be thinking if you said, "$\frac{8}{12}$ of the dots are pink."
   B Explain what you might be thinking if you said, "$\frac{2}{3}$ of the dots are pink."
   C Explain why $\frac{2}{3}$ is equivalent to $\frac{8}{12}$.

Study examples **A** and **B** in the first two rows of the table. Then copy the statement and give the missing fractions.

A		$\frac{1}{2}$ is equivalent to $\frac{2}{4}$.	
B		$\frac{2}{3}$ is equivalent to $\frac{6}{9}$.	
1.		____ is equivalent to $\frac{6}{8}$.	
2.		$\frac{1}{4}$ is equivalent to ____.	
3.		____ is equivalent to ____.	
4.		____ is equivalent to ____.	
5.		____ is equivalent to ____.	
6.		____ is equivalent to ____.	
7.		____ is equivalent to ____.	
8.		____ is equivalent to ____.	

think

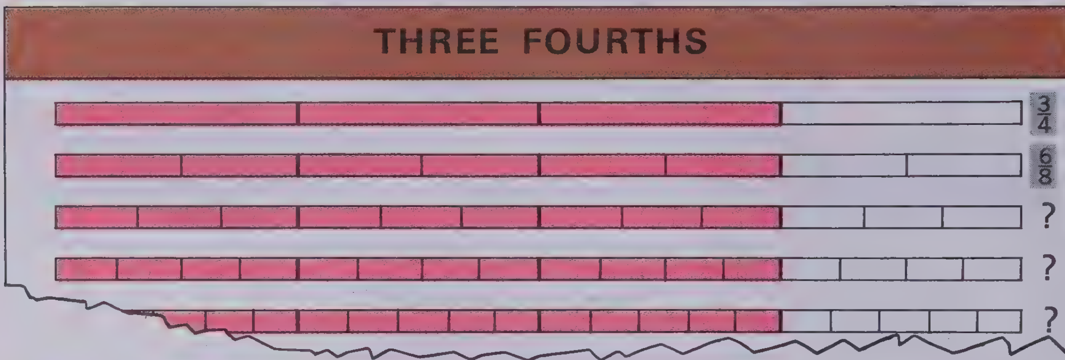
$\frac{1}{2}$ of a brick plus 6 kilograms weighs the same as the brick. How much does the brick weigh?



More practice, page A-17, Set 32

Investigating the Ideas

The same amount of each strip is shaded, but different fractions can be used to describe it.

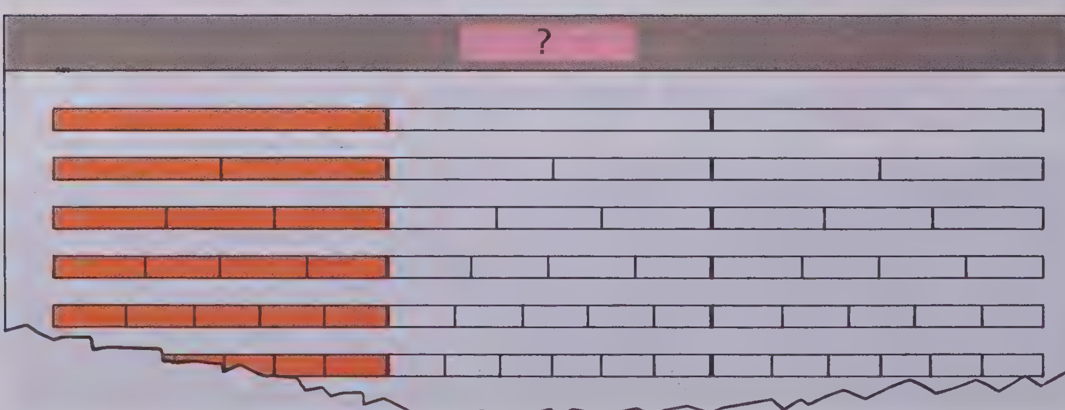


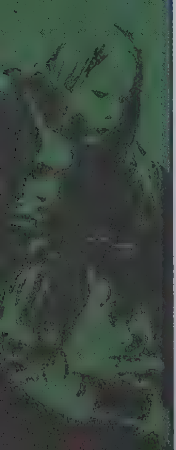
Imagine that the chart goes on and on without end.

? If you think about the chart, can you give eight more fractions in this set of fractions equivalent to $\frac{3}{4}$?
$\{\frac{3}{4}, \frac{6}{8}, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \ldots\}$

Discussing the Ideas

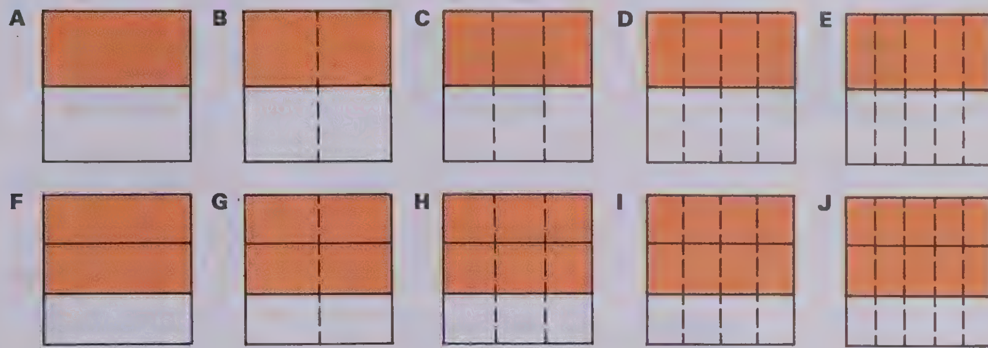
This chart suggests a set of equivalent fractions.
Tell as much as you can about the chart and the set of fractions.





Using the Ideas

1. Give the fraction suggested by each figure.

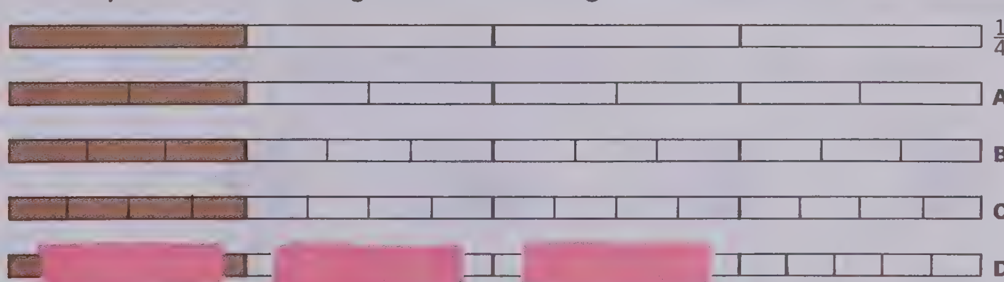


2. Give the next three fractions in each set.

A $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots\right\}$ **B** $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \ldots\right\}$

3. Study the chart and give the missing fractions.

4. Study the chart and give the missing fractions.

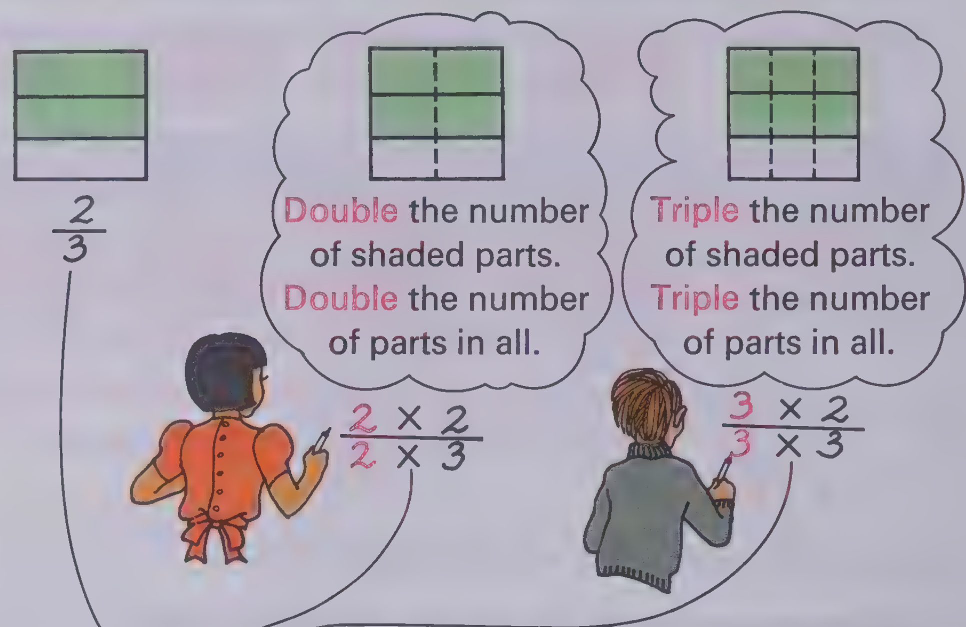


5. Give the next three fractions in each set.

A $\left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{20}{25}, \ldots\right\}$ **B** $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \ldots\right\}$

More practice, page A-18, Set 33

Investigating the Ideas

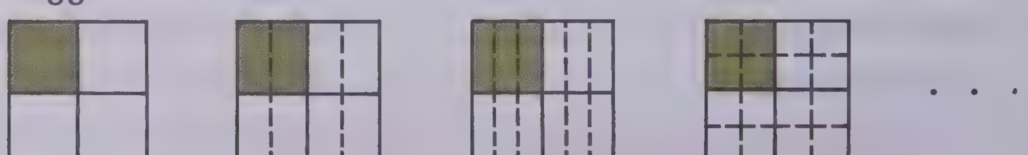


$$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \ldots\right\}$$

? Can you use this idea to give 5 more fractions for this set and to build a set of equivalent fractions starting with $\frac{4}{5}$?

Discussing the Ideas

1. What set of equivalent fractions does this set of pictures suggest?



2. Can you explain how to find some more fractions for this set of equivalent fractions?

$$\left\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \ldots\right\}$$

Give the missing fractions for exercises 1, 2, and 3.

1. $\frac{1\times 1}{1\times 5}, \frac{2\times 1}{2\times 5}, \frac{3\times 1}{3\times 5}, \ldots$

$\frac{1}{5}$, $\frac{2}{10}$, A, B, C, D, E, F

2. $\frac{1\times 3}{1\times 8}, \frac{2\times 3}{2\times 8}, \frac{3\times 3}{3\times 8}, \ldots$

$\frac{3}{8}$, $\frac{6}{16}$, A, B, C, D, E, F

3. $\frac{1\times 3}{1\times 4}, \frac{2\times 3}{2\times 4}, \frac{3\times 3}{3\times 4}, \ldots$

$\frac{3}{4}$, $\frac{6}{8}$, A, B, C, D, E, F

4. Give the next three fractions for each set.

A $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\right\}$

B $\left\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\right\}$

C $\left\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\right\}$

D $\left\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\right\}$

E $\left\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \ldots\right\}$

F $\left\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \ldots\right\}$

5. Give the missing numerators and denominators to form sets of equivalent fractions.

$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{A}{15}, \frac{6}{B}, \frac{C}{21}, \frac{D}{24}, \frac{9}{E}, \ldots\right\}$

$\left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{F}{25}, \frac{24}{G}, \frac{H}{35}, \frac{32}{I}, \frac{J}{45}, \ldots\right\}$

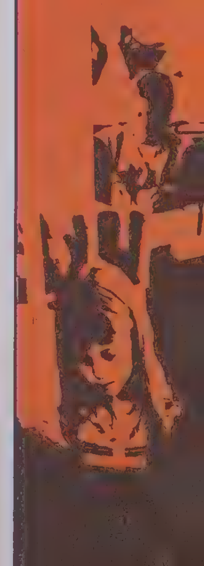
$\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \frac{K}{40}, \frac{L}{48}, \frac{7}{M}, \frac{8}{N}, \frac{O}{72}, \ldots\right\}$

$\left\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{P}, \frac{Q}{36}, \frac{R}{42}, \frac{40}{S}, \frac{T}{54}, \ldots\right\}$

★ 6. Give the missing numerator or denominator so that the fraction will belong to the set.

A $\frac{3}{10}$ $\frac{6}{20}$ $\frac{9}{30}$ … $\frac{300}{\square}$

B $\frac{3}{4}$ $\frac{6}{8}$ $\frac{9}{12}$ … $\frac{\square}{100}$



Investigating the Ideas

Pick any two fractions from a set of equivalent fractions.

$$\left\{ \frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \frac{7}{21}, \frac{8}{24}, \ldots \right\}$$

Find the product of the numbers in each ring.

$$\frac{2}{6} \quad \frac{7}{21}$$

? Can you pick two fractions from the set above so that the products of the numbers in the rings are different?

Discussing the Ideas

Explain each part of this poster by using another pair of equivalent fractions.

Two fractions (like $\frac{2}{6}$ and $\frac{7}{21}$) **are equivalent when they:**

1. **Show the same part of an object.**

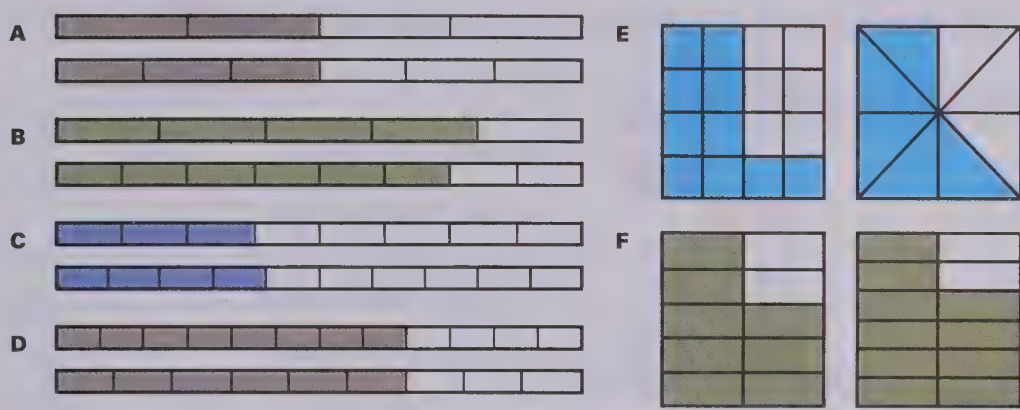
2. **Can be "built" from the same fraction.**

$$\frac{2 \times 1}{2 \times 3} \longrightarrow \frac{2}{6} \qquad \frac{7 \times 1}{7 \times 3} \longrightarrow \frac{7}{21}$$

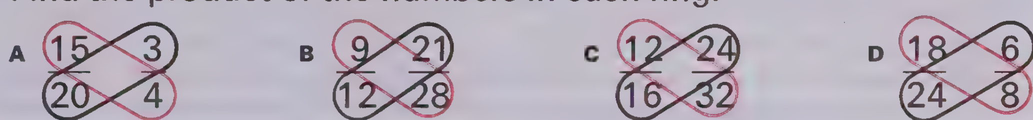
3. **Have the same cross products.**

$$\frac{2 \times 1}{2 \times 3} \quad \frac{7 \times 1}{7 \times 3} \qquad \frac{2}{6} \quad \frac{7}{21}$$

1. For each exercise, write the two fractions that are suggested by the shaded parts of the two regions. Then, by looking at the pictures, tell whether or not the two fractions are equivalent.



2. Find the product of the numbers in each ring.



Is each pair of fractions from this set of equivalent fractions?

$$\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}, \ldots\right\}$$

3. Which pairs are equivalent?

A $\frac{1}{2}, \frac{5}{10}$ E $\frac{5}{7}, \frac{10}{14}$ I $\frac{9}{9}, \frac{27}{27}$

B $\frac{3}{8}, \frac{9}{24}$ F $\frac{12}{5}, \frac{20}{8}$ J $\frac{6}{2}, \frac{24}{8}$

C $\frac{7}{5}, \frac{3}{2}$ G $\frac{8}{3}, \frac{6}{2}$ K $\frac{6}{15}, \frac{8}{20}$

D $\frac{3}{4}, \frac{2}{5}$ H $\frac{6}{9}, \frac{2}{3}$ L $\frac{8}{6}, \frac{28}{21}$

★ 4. Find the number for n so that the fractions will be equivalent.

A $\frac{3}{4}, \frac{n}{8}$ C $\frac{5}{10}, \frac{n}{2}$ E $\frac{10}{30}, \frac{n}{3}$

B $\frac{1}{2}, \frac{n}{6}$ D $\frac{12}{8}, \frac{3}{n}$ F $\frac{3}{6}, \frac{n}{42}$

think

The figure below shows a house floor plan. Draw a figure like this. Draw a path to show where you will be if you start inside and use each door exactly once. Is there an even or odd number of doors? Where would you end if the number of doors were even?

More practice, page A-18, Set 34

Investigating the Ideas

Fractions can be used to compare the lengths of strips. What is the missing fraction below? What are some fractions that compare the other strips to the light green strip?

The unit

$\frac{1}{3}$ as long as the unit

$\frac{2}{3}$ as long as the unit

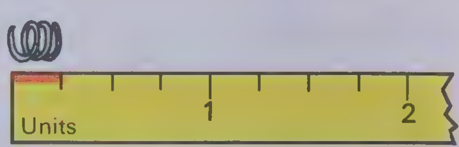
$\frac{3}{3}$ as long as the unit

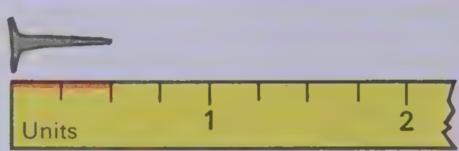
$\frac{\square}{\square}$ as long as the unit

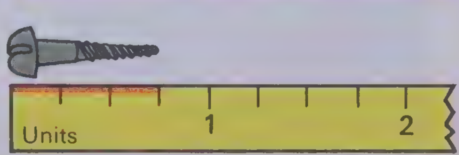
?	Can you use your strips to find fractions that compare each strip to the yellow strip?	Record the fractions that you find.

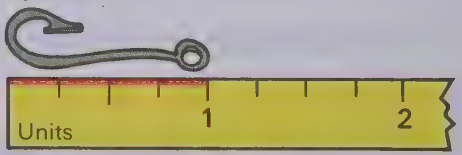
Discussing the Ideas

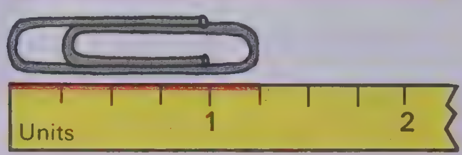
The spring is $\frac{1}{4}$ as long as the unit. Its length is $\frac{1}{4}$ unit.
Explain how to find a fraction for the length of each object.

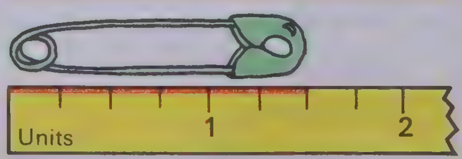
1. 

2. 

3. 

4. 

5. 

6. 

▶ Fractions that have the numerator equal to or greater than the denominator are sometimes called **improper fractions**.

1. Give the fraction for each exercise.

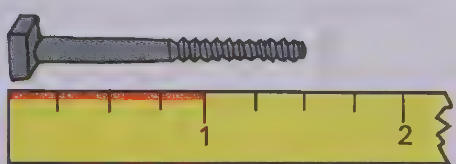
A The purple strip is |||| as long as the yellow strip.

B The blue strip is |||| as long as the red strip.

2. Give an improper fraction that compares each object with the unit. The denominator of each fraction is given.

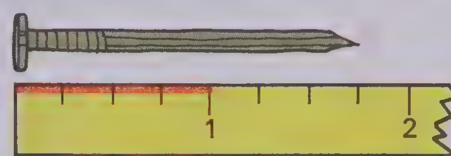
A

||||/4



B

||||/4

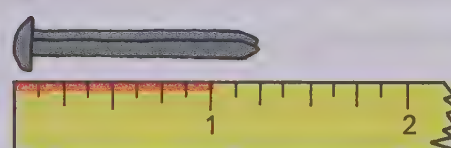


C

||||/2

D

||||/8



3. Give a fraction that compares the number of shaded parts to the total number of parts.

A

B

C

D

think

Guess which number below is greatest and which is smallest.

1. Number of seconds in an hour.
2. Number of hours in a year.
3. Number of minutes in a week.

Now find each number to check your guesses.

Investigating the Ideas

When you build a set of equivalent fractions, you start with a **lowest-terms fraction**.

LOWEST-TERMS FRACTION

$$\left\{\frac{3}{4},\quad \underset{\frac{6}{8}}{\frac{2\times 3}{2\times 4}},\quad \underset{\frac{9}{12}}{\frac{3\times 3}{3\times 4}},\quad \underset{\frac{12}{16}}{\frac{4\times 3}{4\times 4}},\quad \underset{\frac{15}{20}}{\frac{5\times 3}{5\times 4}},\quad \ldots\right\}$$

? Can you find the lowest-terms fraction that was used to "build" each of these fractions?

$$\underset{\frac{8}{12}}{\frac{4\times ?}{4\times ?}}\qquad \underset{\frac{14}{16}}{\frac{2\times ?}{2\times ?}}\qquad \underset{\frac{25}{35}}{\frac{5\times ?}{5\times ?}}\qquad \underset{\frac{4}{8}}{\frac{\square\times ?}{\square\times ?}}\qquad \underset{\frac{6}{9}}{\frac{\square\times ?}{\square\times ?}}\qquad \underset{\frac{20}{24}}{\frac{\square\times ?}{\square\times ?}}$$

Discussing the Ideas

1. Explain the method shown here for finding the lowest-terms fraction for $\frac{9}{15}$.

$$\frac{9}{15} \rightarrow \frac{9 \div 3}{15 \div 3} \rightarrow \frac{3}{5}$$

2. Sometimes you may need to use more than one step to find the lowest-terms fraction. Explain each step shown in finding the lowest-terms fraction for $\frac{90}{120}$.

$$\frac{90}{120} \rightarrow \frac{90 \div 10}{120 \div 10} \rightarrow \frac{9}{12} \rightarrow \frac{9 \div 3}{12 \div 3} \rightarrow \frac{3}{4}$$

3. Use some lowest-terms fractions and some fractions not in lowest terms to explain the following.

> If the greatest common factor of the numerator and denominator is 1, the fraction is in lowest terms.

1. Give the lowest-terms fraction for each fraction. The sets of equivalent fractions may help.

$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \frac{7}{21}, \frac{8}{24}, \ldots\}$

$\{\frac{3}{8}, \frac{6}{16}, \frac{9}{24}, \frac{12}{32}, \frac{15}{40}, \frac{18}{48}, \frac{21}{56}, \frac{24}{64}, \ldots\}$

$\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}, \frac{12}{30}, \frac{14}{35}, \frac{16}{40}, \ldots\}$

$\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \frac{35}{50}, \frac{42}{60}, \frac{49}{70}, \frac{56}{80}, \ldots\}$

$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \frac{16}{24}, \ldots\}$

A $\frac{12}{30}$ **B** $\frac{35}{50}$ **C** $\frac{14}{20}$ **D** $\frac{7}{21}$

E $\frac{4}{6}$ **F** $\frac{49}{70}$ **G** $\frac{9}{24}$ **H** $\frac{6}{15}$

I $\frac{56}{80}$ **J** $\frac{8}{24}$ **K** $\frac{5}{15}$ **L** $\frac{8}{12}$

M $\frac{42}{60}$ **N** $\frac{10}{25}$ **O** $\frac{15}{40}$ **P** $\frac{6}{18}$

Q $\frac{14}{21}$ **R** $\frac{21}{30}$ **S** $\frac{16}{40}$ **T** $\frac{28}{40}$

2. Tell whether or not the fraction is in lowest terms. All the factors of the numerator and denominator are given in red.

A $\dfrac{12\ \{1, 2, 3, 4, 6, 12\}}{15\ \{1, 3, 5, 15\}}$ **D** $\dfrac{11\ \{1, 11\}}{18\ \{1, 2, 3, 6, 9, 18\}}$

B $\dfrac{10\ \{1, 2, 5, 10\}}{9\ \{1, 3, 9\}}$ **E** $\dfrac{15\ \{1, 3, 5, 15\}}{16\ \{1, 2, 4, 8, 16\}}$

C $\dfrac{6\ \{1, 2, 3, 6\}}{35\ \{1, 5, 7, 35\}}$ **F** $\dfrac{24\ \{1, 2, 3, 4, 6, 8, 12, 24\}}{27\ \{1, 3, 9, 27\}}$

3. Give the lowest-terms fraction for each of the following.

A $\frac{4}{8}$ **B** $\frac{9}{30}$ **C** $\frac{15}{20}$ **D** $\frac{21}{10}$ **E** $\frac{6}{15}$ **F** $\frac{14}{5}$ **G** $\frac{27}{18}$ **H** $\frac{25}{35}$

I $\frac{6}{12}$ **J** $\frac{5}{10}$ **K** $\frac{1}{10}$ **L** $\frac{5}{15}$ **M** $\frac{2}{3}$ **N** $\frac{4}{6}$ **O** $\frac{4}{7}$ **P** $\frac{14}{7}$

Q $\frac{3}{2}$ **R** $\frac{8}{5}$ **S** $\frac{6}{4}$ **T** $\frac{7}{14}$ **U** $\frac{6}{8}$ **V** $\frac{25}{30}$ **W** $\frac{7}{20}$ **X** $\frac{26}{39}$

4. Find the greatest common factor of the numerator and denominator; then give the lowest-terms fraction.

A $\frac{7}{14}$ **B** $\frac{15}{10}$ **C** $\frac{3}{15}$ **D** $\frac{6}{8}$

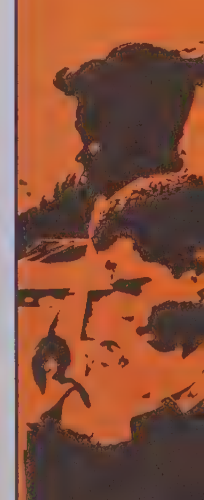
E $\frac{20}{15}$ **F** $\frac{50}{60}$ **G** $\frac{6}{15}$ **H** $\frac{75}{100}$

I $\frac{55}{20}$ **J** $\frac{50}{100}$ **K** $\frac{20}{50}$ **L** $\frac{56}{80}$

M $\frac{44}{28}$ **N** $\frac{5}{20}$ **O** $\frac{0}{7}$ **P** $\frac{6}{6}$

think

Find a fraction that is equivalent to $\frac{1}{2}$ **and** has a denominator that is 5 less than 3 times the numerator.



More practice, page A-19, Set 35

Fraction Short Stories

1 10 children. 8 girls.

A What fraction of the children are girls?

B Give a different fraction to tell what part of the children are girls.

2 2 home runs. 8 runs in all. Give two fractions to tell what part are home runs.

3 Expos, 7. White Sox, 5. What fraction of the runs did the White Sox score?

4 Pie cut into eighths. 6 pieces eaten. Give two fractions for the part that was eaten.

5 10-metre rope.

A Climb up $\frac{6}{10}$ of the way. How far?

B Climb up $\frac{9}{10}$ of the way. How far?

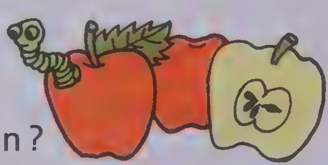
C Climb up $\frac{1}{2}$ of the way. How far?

D Climb up $\frac{2}{5}$ of the way. How far?

6 3 out of each 4 are blue. 5 groups of 4. How many are blue?

7 20 children. $\frac{3}{4}$ of these children are boys. How many are boys?

8 2 out of each 5 are missing. 7 groups of 5. How many are missing?



9 $\frac{2}{5}$ of the apples are rotten. 35 apples. How many are rotten?

10 $\frac{2}{3}$ of the children wear boots. Less than 6 children. How many wear boots?

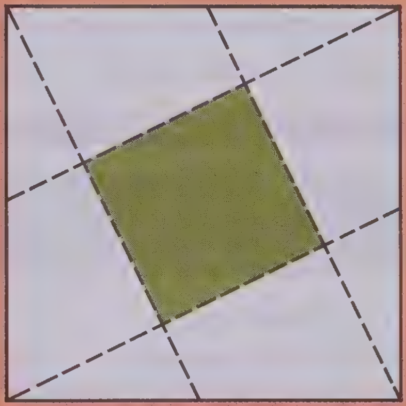
★ **11** 24 blocks to school. Run 2 blocks and then walk 1 block all the way to school. Give two fractions to tell what part of the way is covered by running.

1st Puzzler

This figure is $\frac{1}{4}$ of a square. Can you cut out 4 pieces exactly this shape and size and put them together to form the square?

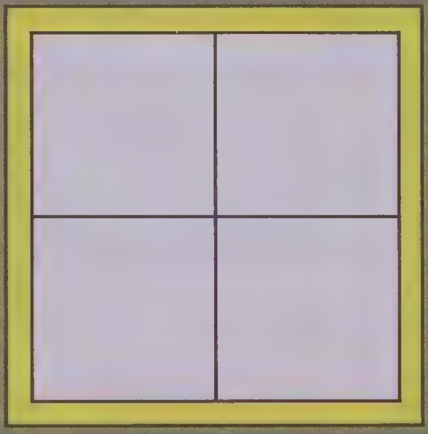
2nd Puzzler

The shaded square region is $\frac{1}{5}$ the size of the large square region. Trace the large square and cut on the dotted lines. Can you arrange the pieces to form 5 small squares?

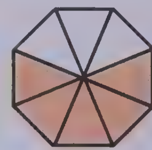


3rd Puzzler

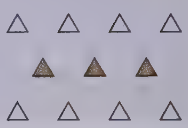
Can you trace this picture and color exactly half of this square window so that the uncolored half is still a perfect square?



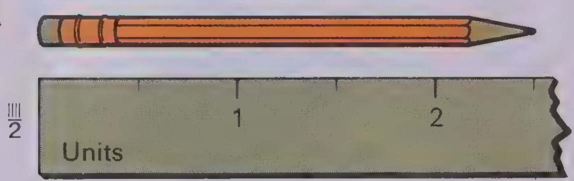
1. Give the fraction suggested by the shaded part of each region.

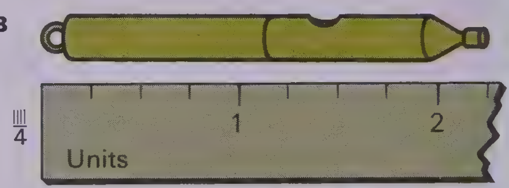
A  B  C 

2. Give the fraction suggested by the shaded part of each set.

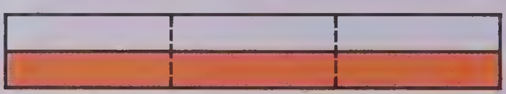
A  B  C 

3. Give an improper fraction that compares each object with the unit. The denominator of each fraction is given.

A 

B 

4. Each of these figures suggests two fractions. What are they?

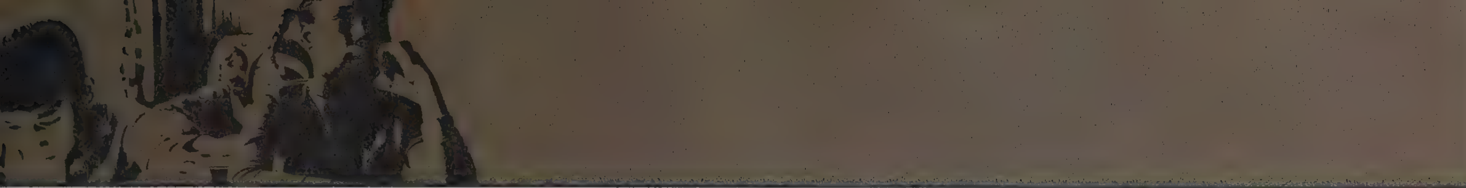
A  B 

5. Give the fraction suggested by the shaded divisions of each rod.

A
B
C
D
E

6. What can you say about the five fractions you wrote for exercise 5?

7. If a first fraction is equivalent to a second and the second is equivalent to a third, what can you say about the first and third fractions?



8. Give the missing fractions for each set.

$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \text{A}, \frac{10}{15}, \frac{12}{18}, \text{B}, \frac{16}{24}, \ldots\right\}$

$\left\{\frac{1}{8}, \text{C}, \frac{3}{24}, \frac{4}{32}, \text{D}, \frac{6}{48}, \frac{7}{56}, \text{E}, \ldots\right\}$

9. Tell whether or not the two fractions are equivalent.

A $\frac{5}{10}, \frac{48}{100}$ **B** $\frac{15}{20}, \frac{3}{4}$ **C** $\frac{6}{9}, \frac{8}{10}$ **D** $\frac{8}{20}, \frac{12}{25}$ **E** $\frac{36}{100}, \frac{18}{50}$

10. Build a set of equivalent fractions (about 8) for each of the following lowest-terms fractions.

A $\frac{1}{2}$ **B** $\frac{3}{4}$ **C** $\frac{1}{5}$ **D** $\frac{3}{10}$ **E** $\frac{7}{2}$ **F** $\frac{4}{7}$

11. Give the lowest-terms fraction for each of the following fractions. Some are in lowest terms already.

A $\frac{18}{20}$ **B** $\frac{5}{20}$ **C** $\frac{4}{10}$ **D** $\frac{10}{40}$ **E** $\frac{8}{9}$ **F** $\frac{12}{30}$ **G** $\frac{11}{100}$ **H** $\frac{12}{5}$

I $\frac{4}{7}$ **J** $\frac{8}{50}$ **K** $\frac{32}{20}$ **L** $\frac{16}{60}$ **M** $\frac{12}{10}$ **N** $\frac{65}{100}$ **O** $\frac{35}{50}$ **P** $\frac{10}{15}$

12. Jim said, "Exactly $\frac{3}{4}$ of the children in my class are girls."

A If there are only 4 children in Jim's class, how many of them are girls?

B How many are girls if there are just 8 children in the class?

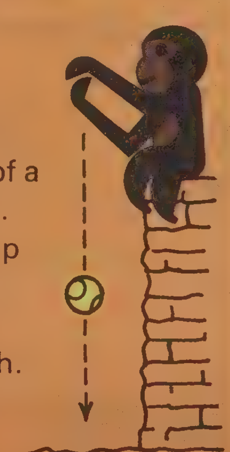
C How many are boys if there are 12 children in the class?

D Could there be 10 children in Jim's class?

E Could there be 10 girls in the class?

think

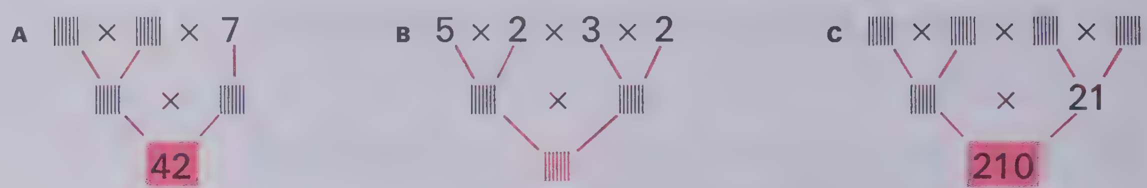
A special rubber ball is dropped from the top of a wall that is 16 m high. Each time it bounces up $\frac{1}{2}$ as high as it fell. The ball is caught when its bounce is 1 metre high. Find the total distance the ball travels.



1. Find the sums, products, differences, and quotients.

A $\begin{array}{r} 697 \\ +867 \\ \hline \end{array}$ B $\begin{array}{r} 642 \\ \times 9 \\ \hline \end{array}$ C $8\overline{)376}$ D $\begin{array}{r} 57 \\ \times 62 \\ \hline \end{array}$ E $19\overline{)114}$ F $\begin{array}{r} 6003 \\ -2764 \\ \hline \end{array}$

2. Copy and complete each factor tree.

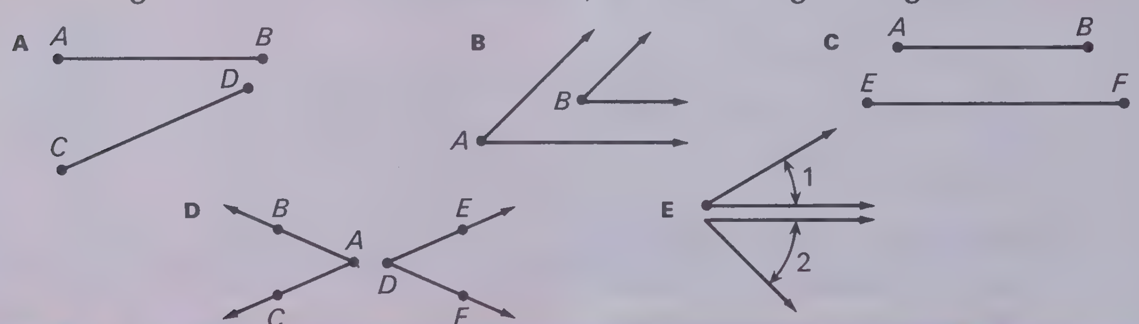


3. Write the numbers at the bottom of each tree in exercise 2 as the product of prime factors.

4. A List the multiples of 12 (up to 60).
   B List the multiples of 9 (up to 60).
   C List the common multiples of 12 and 9 (up to 60).
   D What is the least common multiple of 12 and 9?

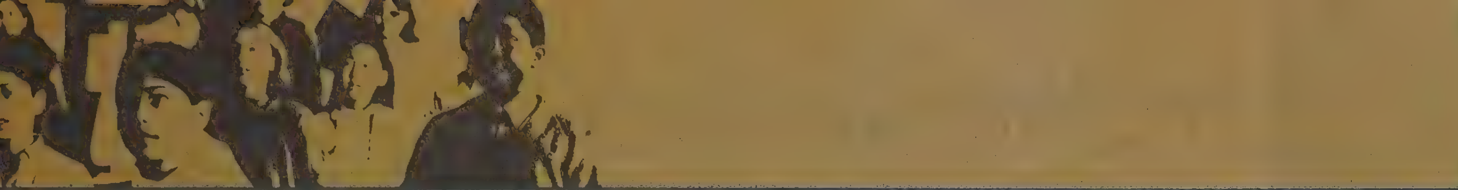
5. What is the least common multiple of 4 and 6?

6. Give the letters of the following exercises in which figures are congruent. In the other exercises, tell which figure is greater.



You are invited to explore

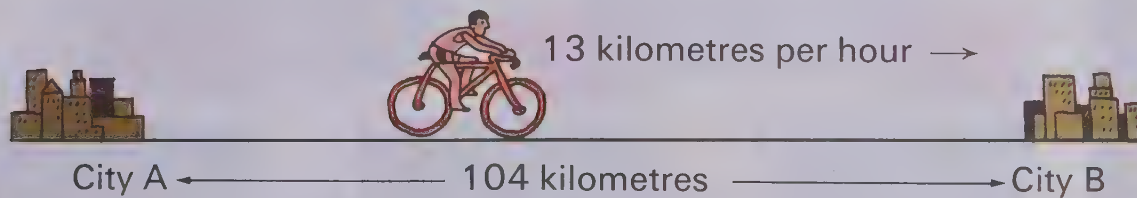
ACTIVITY CARD 9
Page 337



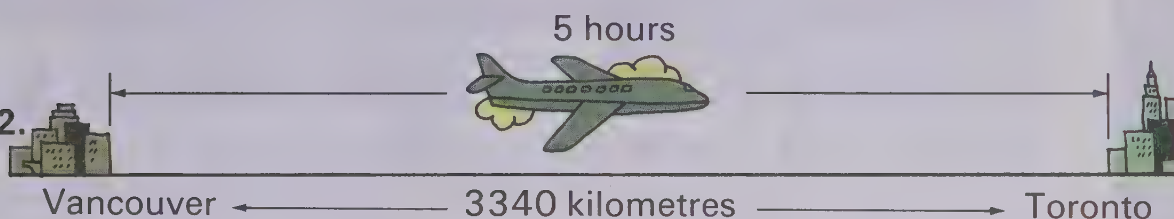
Writing Problems

Each picture below suggests a problem. Study the picture carefully; then write and solve your own problem for the picture.

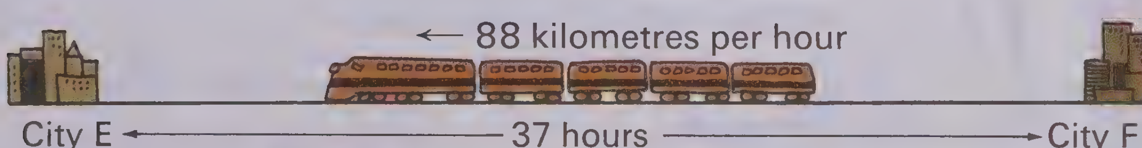
1.



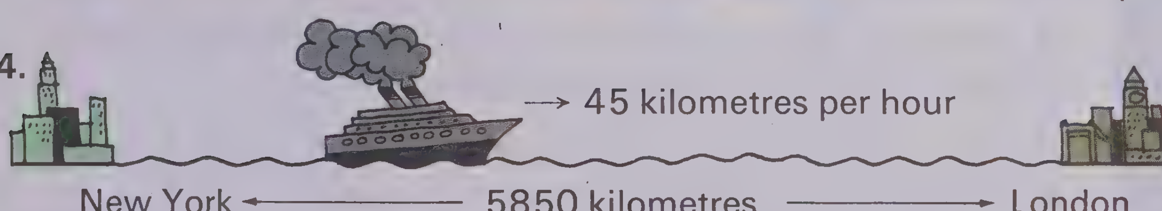
2.



3.



4.



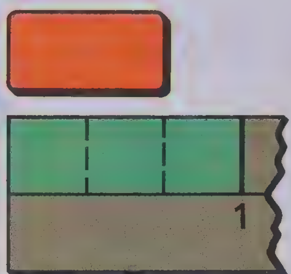
5.

10 Fractional Numbers

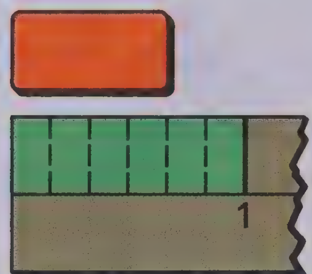
Can fractions be used to represent length?

Investigating the Ideas

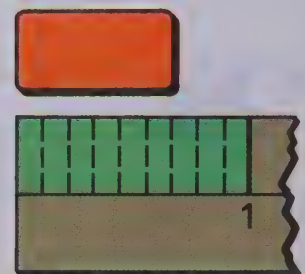
After a unit is chosen, **the length of the red strip does not change, but different fractions can be used to represent it.**



The length of the red strip is $\frac{2}{3}$



The length of the red strip is $\frac{4}{6}$



The length of the red strip is $\frac{6}{9}$

? Can you write five more fractions that represent the length of the red strip?

Discussing the Ideas

1. What can you say about the set of fractions that represent the length of the red strip in the Investigation?

2. After a unit is chosen, the length of a strip does not change. That length is a certain number.
   - A If the white strip is the unit, what might we write to represent the number that is the length of the yellow strip?
   - B If the purple strip is the unit, what might we write to represent the number that is the length of the dark green strip?

1. Give five equivalent fractions that can be used to represent the length of each strip. (Use the purple strip as the unit.)

2. Suppose the orange strip is the unit. Name the strip whose length is represented by the fractions in each set.

A $\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40} \ldots\}$ **C** $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8} \ldots\}$

B $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20} \ldots\}$ **D** $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20} \ldots\}$

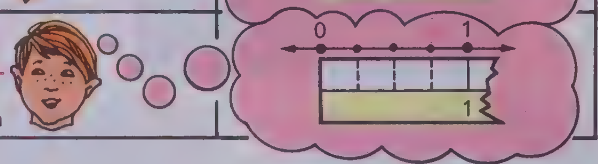
think

$\frac{a}{b}$	1	2	3	4	5	6	7
1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$
2	$\frac{2}{1}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	
3	$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	
4	$\frac{4}{1}$	$\frac{4}{2}$	$\frac{4}{3}$	$\frac{4}{4}$	$\frac{4}{5}$		
5	$\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$				
6	$\frac{6}{1}$	$\frac{6}{2}$					
7	$\frac{7}{1}$						

Think of the table as going on without end.

1. What fraction is in:
   (A) the 3rd row and the 5th column?
   (B) the 165th row and the 348th column?
2. Do you think every fraction (except those like $\frac{0}{7}$) is somewhere in this table?

Discussing the Ideas

For each set of equivalent fractions	we think of **one** fractional number	and **one** point on the number line.
$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \ldots\}$	Pat 	
$\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \ldots\}$	Jim 	

1. Pat is thinking of a fractional number. Explain how she might have found the point on the number line for her number.

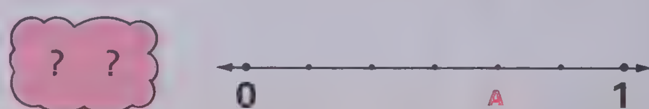
2. Jim is thinking of a fractional number. Explain how he might have found the point on the number line for his number.

3. How can you find which set of equivalent fractions goes with point A on the number line?

$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$

$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$

$\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \ldots\}$



For each set of equivalent fractions there is one fractional number. Give the point (A, B, or C) for the given fractional number in questions 4 through 7.

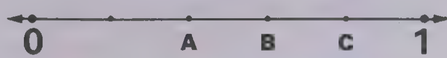
4. $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$

5. $\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\}$

6. $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\}$

7. $\{\frac{7}{8}, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \ldots\}$

Give the correct point for the fractional number that is indicated by the set of equivalent fractions.

1. 

$\left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \ldots\right\}$

2. 

$\left\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\right\}$

3. 

$\left\{\frac{2}{7}, \frac{4}{14}, \frac{6}{21}, \frac{8}{28}, \ldots\right\}$

4. 

$\left\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\right\}$

5. 

$\left\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \ldots\right\}$

6. 

$\left\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \ldots\right\}$

7. 

$\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\right\}$

8. 

$\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\right\}$

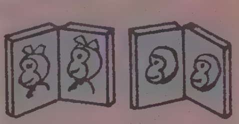
9. 

$\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\right\}$

10. 

$\left\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\right\}$

think



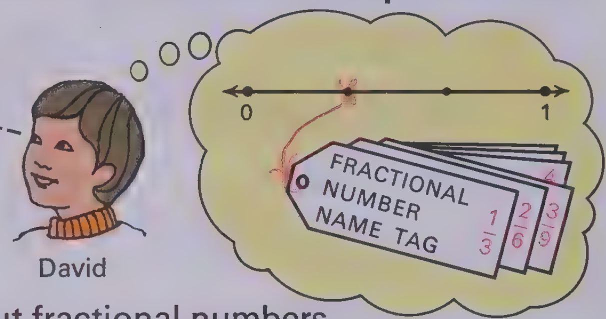
Jill and June are twins. Jim and John are twins. The girls are 2 years older than the boys. The sum of all their ages is 40. How old are they?

More practice, page A-19, Set 36

Discussing the Ideas

You have learned that **for each set of equivalent fractions there is exactly one fractional number and one point on the number line.**

$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \ldots\}$



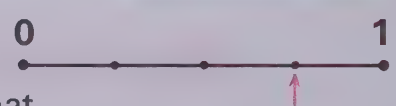
In order to talk and write about fractional numbers, we need names (symbols) for them. We agree that:

> Any fraction from a set of equivalent fractions can be used to name the fractional number for that set.

1. As you can see, **each fractional number has many names.** To show this, give six fractions for this sentence:
   "▮" names the fractional number that David is thinking about.

2. To show that two fractions name the same fractional number, we write an equation:
   $\frac{3}{9}$ names the fractional number for this set.
   $\frac{5}{15}$ names the fractional number for this set. $\rightarrow \{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \ldots\}$

   We write: $\frac{3}{9} = \frac{5}{15}$

   Write some equations using fractions that name the fractional number for the point above the red arrow.

3. The lowest-terms fraction is the most commonly used name for a fractional number. Choose some fractional numbers and give the lowest-terms fraction that names each.

1. Give fractions to name the fractional numbers for the points over the red arrows. The denominators are indicated.

	0																1
Fourths →	$\frac{0}{4}$				$\frac{1}{4}$				$\frac{2}{4}$				A				$\frac{4}{4}$
Eighths →	$\frac{0}{8}$		B		$\frac{2}{8}$		$\frac{3}{8}$		C		$\frac{5}{8}$		D		$\frac{7}{8}$		E
Sixteenths →	F	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	G	$\frac{5}{16}$	H	$\frac{7}{16}$	$\frac{8}{16}$	I	$\frac{10}{16}$	$\frac{11}{16}$	$\frac{12}{16}$	$\frac{13}{16}$	J	$\frac{15}{16}$	$\frac{16}{16}$

	0						1						2
Thirds →	$\frac{0}{3}$				K		$\frac{3}{3}$		$\frac{4}{3}$		$\frac{5}{3}$		L
Sixths →	M		$\frac{2}{6}$		$\frac{4}{6}$		N		O		$\frac{10}{6}$		$\frac{12}{6}$
Twelfths →	$\frac{0}{12}$	$\frac{2}{12}$	P	$\frac{6}{12}$	Q	R	$\frac{12}{12}$	$\frac{14}{12}$	S	$\frac{18}{12}$	T	$\frac{22}{12}$	$\frac{24}{12}$

2. Give three other names for each fractional number.

A $\frac{1}{5}$ **B** $\frac{3}{4}$ **C** $\frac{2}{8}$ **D** $\frac{6}{10}$ **E** $\frac{3}{7}$

3. Which fractions name the same fractional number? Write three equations.

$\frac{1}{6}$, $\frac{2}{8}$, $\frac{3}{18}$, $\frac{2}{16}$, $\frac{1}{4}$, $\frac{1}{8}$

4. Give the missing numerator or denominator.

A $\frac{1}{2} = \frac{\square}{4}$ **C** $\frac{2}{3} = \frac{4}{\square}$

B $\frac{1}{10} = \frac{\square}{100}$ **D** $\frac{3}{7} = \frac{9}{\square}$

think

If today is Tuesday, what day of the week is:

A 10 days from now?

B 50 days from now?

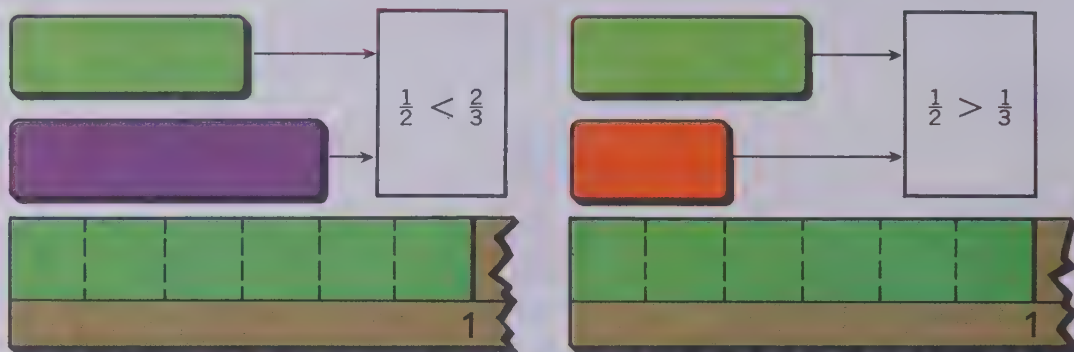
C 100 days from now?

More practice, page A-20, Set 37

Which of two fractional numbers is greater?

Investigating the Ideas

Here are two inequality statements:

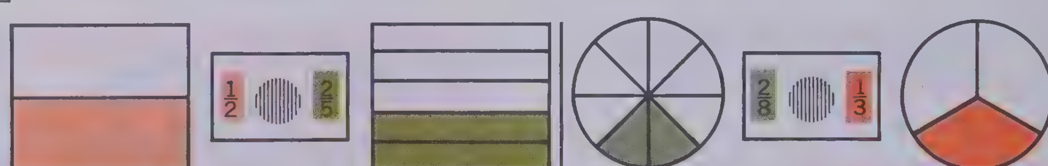


? Can you use the unit shown and other strips to write some more inequality statements? | List the strips used.

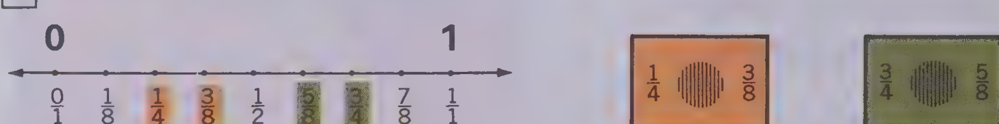
Discussing the Ideas

Here are three more ways to tell which of two fractional numbers is greater. Choose the correct symbol (>, <) and explain how you made your choice.

A Think about parts of a region:



B Think about the number line:



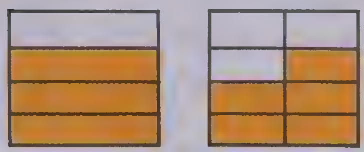
C Think about fractions with the same denominator.

$\frac{3}{5} \bigcirc \frac{1}{2}$ because $\frac{6}{10} > \frac{5}{10}$.

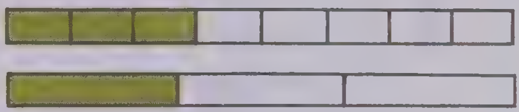
$\frac{1}{4} \bigcirc \frac{1}{3}$ because $\frac{3}{12} < \frac{4}{12}$.

1. Write an inequality for each exercise.

A



B



2. Use the number line to give the correct sign ($>$ or $<$) for each ◯.

$0 \quad \frac{1}{16} \quad \frac{1}{8} \quad \frac{3}{16} \quad \frac{1}{4} \quad \frac{5}{16} \quad \frac{3}{8} \quad \frac{7}{16} \quad \frac{1}{2} \quad \frac{9}{16} \quad \frac{5}{8} \quad \frac{11}{16} \quad \frac{3}{4} \quad \frac{13}{16} \quad \frac{7}{8} \quad \frac{15}{16} \quad 1$

A $\frac{5}{16}$ ◯ $\frac{3}{8}$ **B** $\frac{9}{16}$ ◯ $\frac{1}{2}$ **C** $\frac{5}{8}$ ◯ $\frac{7}{16}$ **D** $\frac{1}{2}$ ◯ $\frac{3}{16}$ **E** $\frac{15}{16}$ ◯ $\frac{7}{8}$

3. Give the numbers for ***a*** and ***b***. Then give the correct sign ($>$, $<$) for each ◯.

A $\frac{2}{3} = \frac{a}{6}$, $\frac{1}{2} = \frac{b}{6}$ → $\frac{2}{3}$ ◯ $\frac{1}{2}$

B $\frac{3}{4} = \frac{a}{20}$, $\frac{4}{5} = \frac{b}{20}$ → $\frac{3}{4}$ ◯ $\frac{4}{5}$

C $\frac{1}{6} = \frac{a}{30}$, $\frac{1}{5} = \frac{b}{30}$ → $\frac{1}{6}$ ◯ $\frac{1}{5}$

D $\frac{4}{7} = \frac{a}{21}$, $\frac{2}{3} = \frac{b}{21}$ → $\frac{4}{7}$ ◯ $\frac{2}{3}$

E $\frac{5}{8} = \frac{a}{56}$, $\frac{4}{7} = \frac{b}{56}$ → $\frac{5}{8}$ ◯ $\frac{4}{7}$

F $\frac{2}{5} = \frac{a}{45}$, $\frac{4}{9} = \frac{b}{45}$ → $\frac{2}{5}$ ◯ $\frac{4}{9}$

G $\frac{1}{2} = \frac{a}{6}$, $\frac{1}{3} = \frac{b}{6}$ → $\frac{1}{2}$ ◯ $\frac{1}{3}$

H $\frac{2}{3} = \frac{a}{12}$, $\frac{3}{4} = \frac{b}{12}$ → $\frac{2}{3}$ ◯ $\frac{3}{4}$

I $\frac{5}{6} = \frac{a}{12}$, $\frac{11}{12} = \frac{b}{12}$ → $\frac{5}{6}$ ◯ $\frac{11}{12}$

J $\frac{3}{8} = \frac{a}{8}$, $\frac{1}{2} = \frac{b}{8}$ → $\frac{3}{8}$ ◯ $\frac{1}{2}$

K $\frac{6}{7} = \frac{a}{21}$, $\frac{2}{3} = \frac{b}{21}$ → $\frac{6}{7}$ ◯ $\frac{2}{3}$

L $\frac{7}{8} = \frac{a}{24}$, $\frac{5}{6} = \frac{b}{24}$ → $\frac{7}{8}$ ◯ $\frac{5}{6}$

4. Give the correct sign ($>$, $=$, or $<$) for each ◯.

A $\frac{5}{6}$ ◯ $\frac{11}{12}$

B $\frac{2}{3}$ ◯ $\frac{3}{6}$

C $\frac{1}{12}$ ◯ $\frac{0}{12}$

D $\frac{1}{2}$ ◯ $\frac{6}{12}$

E $\frac{7}{12}$ ◯ $\frac{1}{2}$

F $\frac{7}{12}$ ◯ $\frac{2}{3}$

G $\frac{5}{12}$ ◯ $\frac{1}{2}$

H $\frac{3}{4}$ ◯ $\frac{5}{6}$

I $\frac{1}{2}$ ◯ $\frac{1}{3}$

J $\frac{1}{4}$ ◯ $\frac{1}{6}$

K $\frac{1}{4}$ ◯ $\frac{1}{3}$

L $\frac{1}{3}$ ◯ $\frac{4}{12}$

think

$\frac{5}{8} \quad \frac{?}{32} \quad \frac{?}{32} \quad \frac{?}{32} \quad \frac{3}{4}$

Find three fractions with denominator 32 so that the numbers they name are between $\frac{5}{8}$ and $\frac{3}{4}$.

More practice, page A-20, Set 38

Fractional-Number Short Stories

1 Jim ate $\frac{1}{4}$ of the pie.
Joe ate $\frac{1}{5}$ of the pie.
Who ate more pie?

2 Sale.
Store A: $\frac{1}{3}$ off.
Store B: $\frac{1}{2}$ off.
Which sale is better?

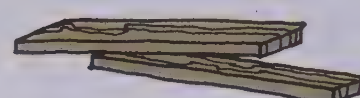
3 $\frac{3}{5}$ kilometre. $\frac{4}{10}$ kilometre.
Which is farther?

4 Halfway around the track.
Two thirds of the way around.
Which is less?

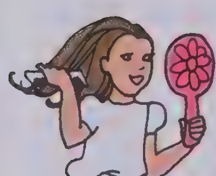
5 Car A: finished $\frac{3}{4}$ of the race.
Car B: finished $\frac{5}{8}$ of the race.
Which went farther?

6 Recipe 1: $\frac{7}{10}$ litre of milk.
Recipe 2: $\frac{3}{4}$ litre of milk.
Which recipe calls for more milk?

8 One board: 1 m 60 cm.
Other board: $\frac{15}{10}$ m.
Which board is longer?



7 Jane took $\frac{1}{6}$ of an hour to comb her hair.
Sue took 11 minutes.
Who took longer?



10 Orchard:
$\frac{1}{5}$ apple trees, $\frac{6}{20}$ peach trees, $\frac{4}{10}$ pear trees, $\frac{2}{20}$ apricot trees.

A Which kind of tree does the orchard have most of?

B Which kind of tree does the orchard have fewest of?

9 T-bone steak: 480 grams.
Rib steak: $\frac{4}{10}$ kilogram.
Which weighs more?

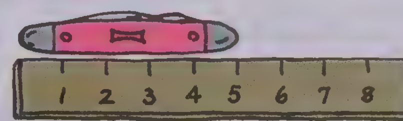
11 Number line.
Point A: $\frac{16}{40}$. Point B: $\frac{36}{80}$.
Which point is to the left of the other?

12 336-page book.
Pat read 150 pages.
Joe read $\frac{1}{2}$ of the book.
Who read more?

Comparing Fractional Numbers

For each exercise, give the missing number and answer the questions.

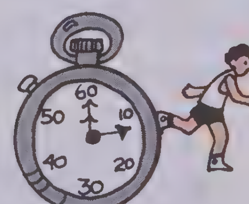
1. The length of the knife is $\frac{5}{9}$ of the ruler. The length of a second knife is $\frac{3}{5}$ of the ruler. Which knife is longer?



2. One trip took 20 minutes, or $\frac{\square}{60}$ hour. A second trip took $\frac{1}{4}$ hour. Which trip took more time?

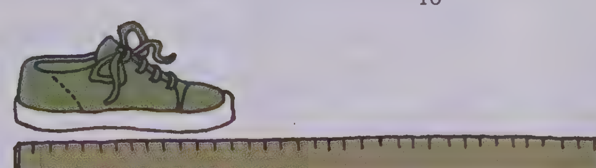
3. The sack of candy on the scale weighs $\frac{5}{10}$ kg. Is this more or less than $\frac{1}{4}$ kg?

4. Joe ran 50 metres in $\frac{1}{6}$ of a minute. Tom ran 50 metres in 12 seconds, or $\frac{\square}{60}$ of a minute. Which boy ran 50 metres in less time than the other?



5. This jug is $\frac{\square}{4}$ full. Would you have to add or pour out liquid to make the jug $\frac{5}{8}$ full?

6. The length of the shoe is $\square$ cm. Is the length of the shoe more or less than $\frac{1}{10}$ metre?


centimetres

7. A camel weighs $\frac{1}{2}$ tonne. A moose weighs 400 kg, or $\frac{\square}{1000}$ tonne. Which animal weighs more?

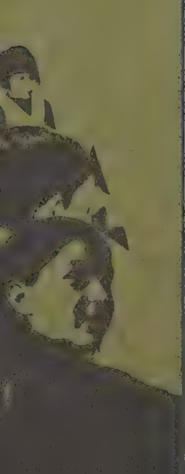
More practice, page A-21, Set 39

Investigating the Ideas

Since the purple strip is $\frac{1}{2}$ as long as the brown strip, we say the **ratio** of the length of the purple strip to the length of the brown strip is **1 to 2.**

Since the purple strip is 4 units and the brown strip is 8 units, we also say the ratio is **4 to 8.**

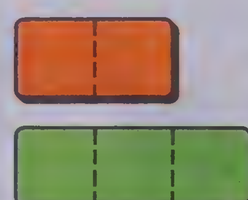
?	
How many other pairs of strips that have a ratio of 1 to 2 can you find?	Record your findings by giving the ratio suggested by the unit marks on the strips.



Discussing the Ideas

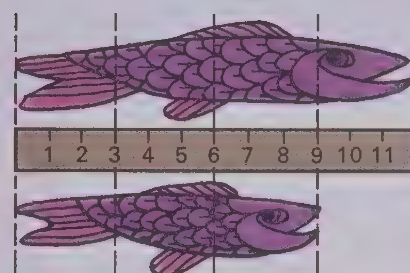
1. The ratio **4 to 8** is written as 4:8.
   - A How can you think about the strips to show a ratio of 8 to 4?
   - B How would you write the ratio **8 to 4**?

2. The red strip is $\frac{2}{3}$ as long as the light green strip. What ratio does this fact suggest for comparing the red strip to the light green strip?

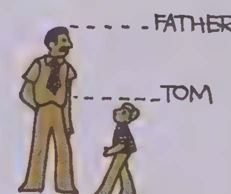


3. A What is the ratio of the number of boys to the number of girls in your room?
   - B What is the ratio of **girls to boys** in your room?

1. Copy the part in red and give the missing number.
   A The ratio of the length of the toy bat to that of the big bat is 2 to ▯.
   B The ratio of the length of the big bat to that of the toy bat is ▯ to ▯.
   C The ratio of the length of the big bat to that of the toy bat is 36 to ▯.
   D The ratio of the length of the large fish to the length of the small fish is 4 to ▯.
   E The ratio of the length of the small fish to the length of the large fish is ▯ to ▯.

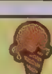
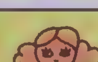
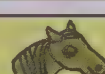


2. The ratio of Father's height to Tom's is 2 to 1. Father is 180 cm tall. How tall is Tom?



3. We can use ratio to compare two sets. There is 1 tent for every 3 boys. The **ratio** of the tents to boys is **1 :3**. If there are 4 tents, how many boys are there?

4. Give the missing numbers:

A	Ratio of 's to 's is 1:2.	6 's, ? 's
B	Ratio of 's to 's is 1:10.	3 's, ? 's
C	Ratio of 's to 's is 2:3.	? 's, 9 's
D	Ratio of 's to 's is 4:1.	40 's, ? 's

More practice, page A-21, Set 40

Let's explore map scales using fractional numbers.

Investigating the Ideas

? Can you figure out what the "scale" means and use your ruler to find these distances?

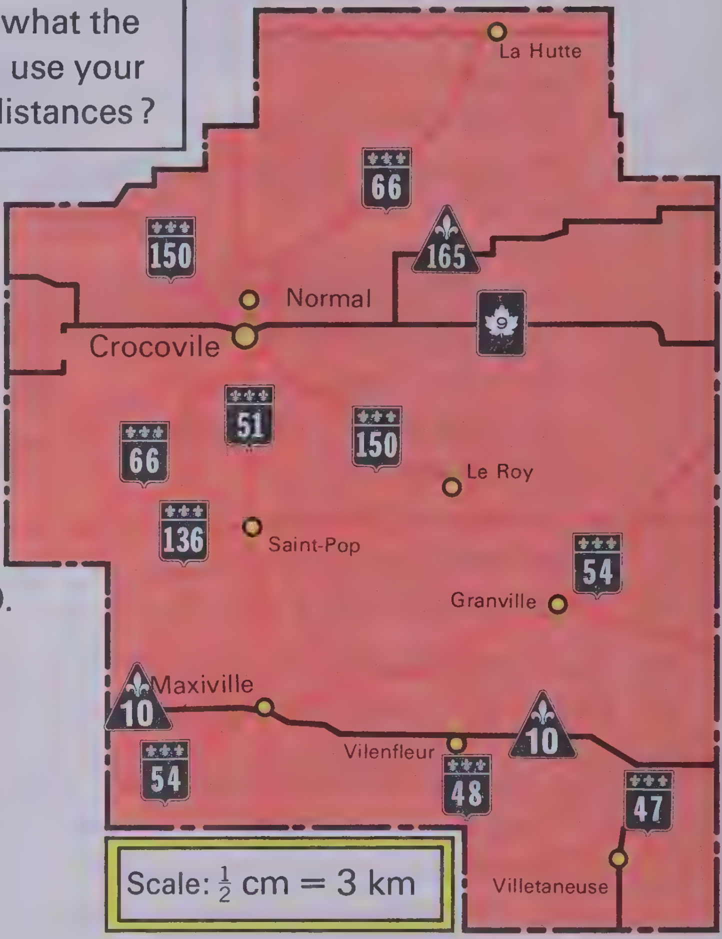
A From Maxiville to Crocovile along route 51.

B From Normal to Vilenfleur, along routes 51 and 10.

C From Crocovile to LeRoy, along route 150.

D From Crocovile to Vilenfleur, "as the crow flies."

E From Villetaneuse to La Hutte "as the crow flies."



Scale: $\frac{1}{2}$ cm = 3 km

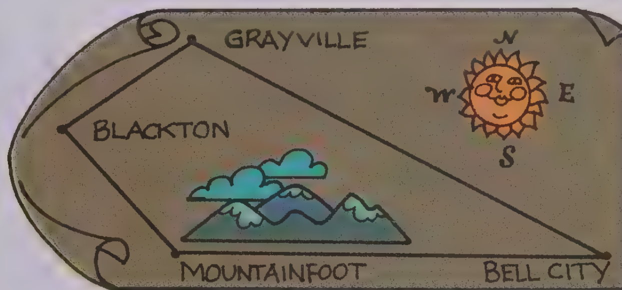
Discussing the Ideas

1. To show that $\frac{1}{2}$ centimetre on the map represents 3 kilometres on the earth, we wrote: $\frac{1}{2}$ cm = 3 km
   Complete the following to show other ways to indicate the scale for the map.
   A 1 cm = ▒ km B ▒ cm = 12 km

2. What scale would you choose if you wanted to draw a map of your community on the chalkboard? on your paper?

1. Since each segment on the map represents a certain distance on land, this map is drawn to scale. Use your centimetre ruler and give the missing numbers.

   A If the distance from Blackton to Grayville is 2 kilometres, then the distance from Grayville to Bell City is |||| kilometres.



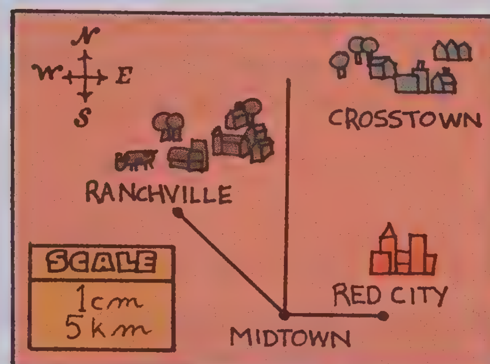
   B According to exercise A, one centimetre on the map means |||| kilometres on land.

   C If the map scale is $\frac{1}{2}$ cm = 3 kilometres, what is the distance from Blackton through each city, and back to Blackton?

2. A How far is it from Red City to Midtown?

   B How far is it from Midtown to Ranchville?

   C It is 20 kilometres due north from Midtown to Crosstown. How far above the dot for Midtown should the dot for Crosstown be placed?

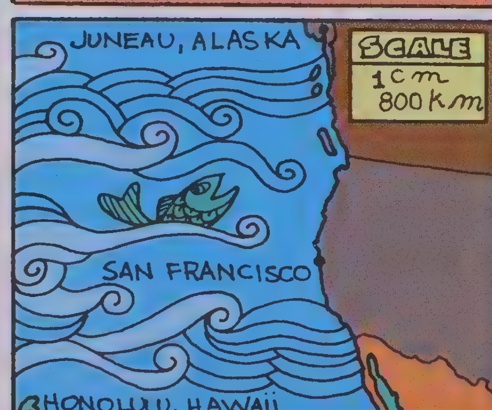


3. Use the scale shown on the map to answer these questions.

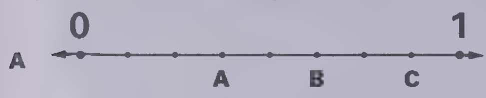
   A About how far is it from San Francisco to Juneau?

   B About how far is it from Juneau to Honolulu?

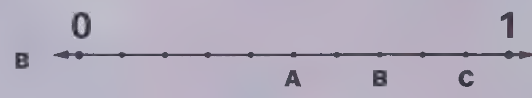
   C About how far is it from Honolulu to San Francisco?



1. Give the correct point for the fractional number indicated by each set of equivalent fractions.

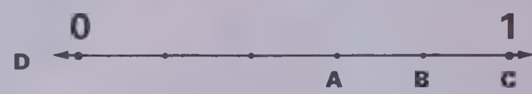


$\left\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\right\}$

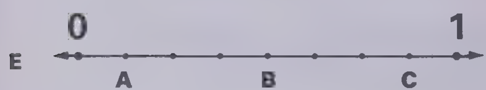


$\left\{\frac{9}{10}, \frac{18}{20}, \frac{27}{30}, \frac{36}{40}, \ldots\right\}$

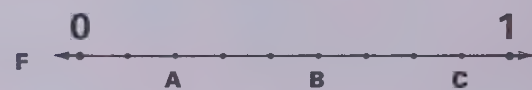
$\left\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \ldots\right\}$



$\left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \ldots\right\}$

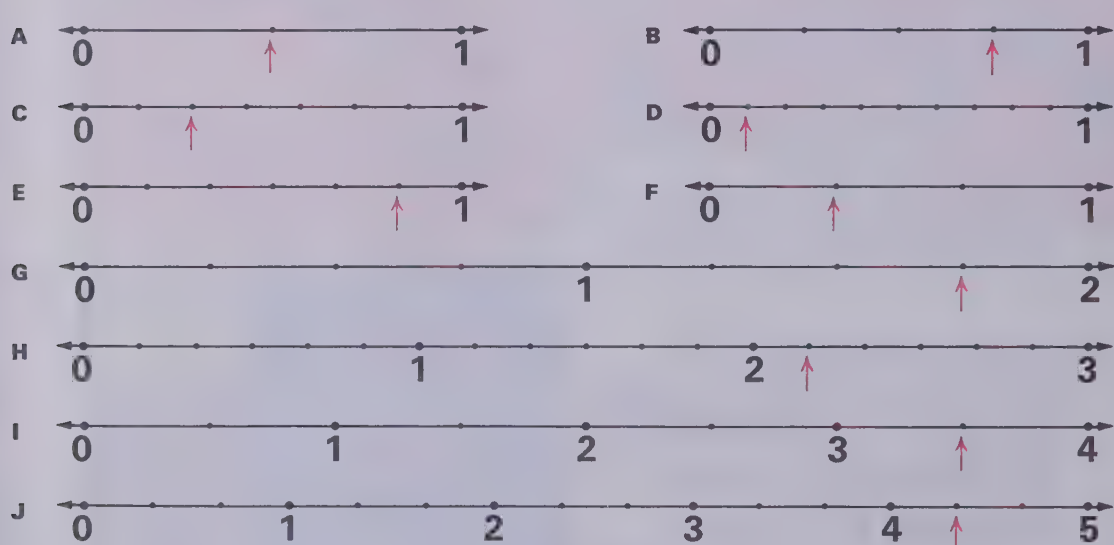


$\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\right\}$



$\left\{\frac{5}{9}, \frac{10}{18}, \frac{15}{27}, \frac{20}{36}, \ldots\right\}$

2. Use the lowest-terms fraction to name the fractional number for the point over the red arrow.



3. Give two other names for each fractional number.

A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{4}$ D $\frac{2}{3}$ E $\frac{3}{4}$ F $\frac{5}{7}$ G $\frac{3}{10}$ H $\frac{4}{5}$

4. Give the correct sign (=, ≠) for each ▒.

A $\frac{3}{4}$ ▒ $\frac{75}{100}$　　C $\frac{4}{6}$ ▒ $\frac{12}{15}$　　E $\frac{5}{7}$ ▒ $\frac{70}{80}$　　G $\frac{30}{5}$ ▒ $\frac{6}{1}$

B $\frac{15}{20}$ ▒ $\frac{3}{4}$　　D $\frac{11}{15}$ ▒ $\frac{20}{30}$　　F $\frac{56}{80}$ ▒ $\frac{60}{90}$　　H $\frac{0}{7}$ ▒ $\frac{0}{2}$

5. Give the missing numerator or denominator.

A $\frac{4}{10} = \frac{▒}{20}$　　C $\frac{3}{4} = \frac{▒}{8}$　　E $\frac{5}{2} = \frac{▒}{8}$　　G $\frac{60}{60} = \frac{5}{▒}$

B $\frac{15}{6} = \frac{5}{▒}$　　D $\frac{4}{6} = \frac{▒}{18}$　　F $\frac{7}{10} = \frac{▒}{100}$　　H $\frac{0}{8} = \frac{▒}{7}$

6. Give the correct sign (< or >) for each ▒.

A $\frac{1}{2}$ ▒ $\frac{2}{3}$　　C $\frac{1}{8}$ ▒ $\frac{1}{4}$　　E $\frac{3}{5}$ ▒ $\frac{2}{5}$　　G $\frac{4}{5}$ ▒ $\frac{7}{10}$

B $\frac{1}{2}$ ▒ $\frac{1}{4}$　　D $\frac{6}{10}$ ▒ $\frac{11}{20}$　　F $\frac{5}{8}$ ▒ $\frac{5}{6}$　　H $\frac{4}{6}$ ▒ $\frac{3}{4}$

7. In each exercise, list the numbers in order from smallest to largest.

A $\frac{7}{2}, \frac{4}{2}, \frac{3}{2}, \frac{5}{2}, \frac{8}{2}, \frac{12}{2}, \frac{9}{2}, \frac{1}{2}$

B $\frac{5}{6}, \frac{1}{3}, \frac{7}{6}, \frac{2}{3}, \frac{5}{3}, \frac{4}{3}, \frac{0}{3}$

C $\frac{8}{8}, \frac{1}{2}, \frac{3}{8}, \frac{3}{4}, \frac{5}{8}, \frac{1}{8}, \frac{1}{4}, \frac{0}{8}$

D $\frac{2}{3}, \frac{1}{2}, \frac{5}{6}, \frac{1}{3}, \frac{1}{6}, \frac{1}{4}, \frac{3}{4}$

8. Each whole number is also a fractional number. Some fractional numbers are whole numbers. Give the whole number for each of the following fractional numbers.

A $\frac{12}{3}$　　D $\frac{28}{4}$　　G $\frac{56}{8}$

B $\frac{25}{5}$　　E $\frac{14}{7}$　　H $\frac{80}{10}$

C $\frac{16}{2}$　　F $\frac{36}{4}$　　I $\frac{40}{5}$

9. Write a fraction for each of the following whole numbers.

A 2　　C 3　　E 4　　G 5

B 1　　D 0　　F 6　　H 7

think

Give the lowest-terms improper fractions for points **A**, **B**, and **C**.

Keeping in Touch with Computing, Place value, Measurement, Number theory, Geometry

1. A List the multiples of 4 (less than 50).
   B List the multiples of 6 (less than 50).
   C List the common multiples of 6 and 4.
   D What is the least common multiple of 6 and 4?

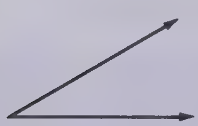
2. Find the least common multiple of the two numbers given.
   A 3, 4 B 6, 9 C 5, 3 D 3, 8 E 12, 20

3. Solve the equations.
   A $42 = n + 8$
   B $100 - 49 = n$
   C $720 = n \times 90$
   D $270 \div 3 = n$
   E $352 = n + 50 + 2$
   F $486 = 400 + n + 6$
   G $974 = (n \times 100) + (7 \times 10) + 4$
   H $6037 = (6 \times n) + 30 + 7$
   I $8 \div 8 = n$
   J $93 \times 0 = n$
   K $17 \times 1 = n$
   L $0 \div 9 = n$

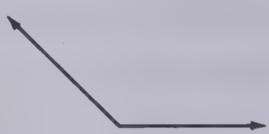
4. Find the sum, difference, product, and quotient.
   A $\begin{array}{r} 9659 \\ +7838 \\ \hline \end{array}$ B $\begin{array}{r} 683 \\ \times 246 \\ \hline \end{array}$ C $\begin{array}{r} 8002 \\ -4526 \\ \hline \end{array}$ D $63\overline{)3030}$

5. Use your protractor to find the degree measure of each angle.

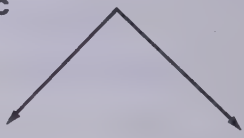
A



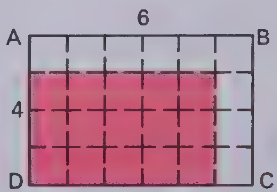
B



C



6. A What is the area of rectangle *ABCD*?
   B What fraction of the rectangular region is shaded pink?
   C What is the area of the pink region?

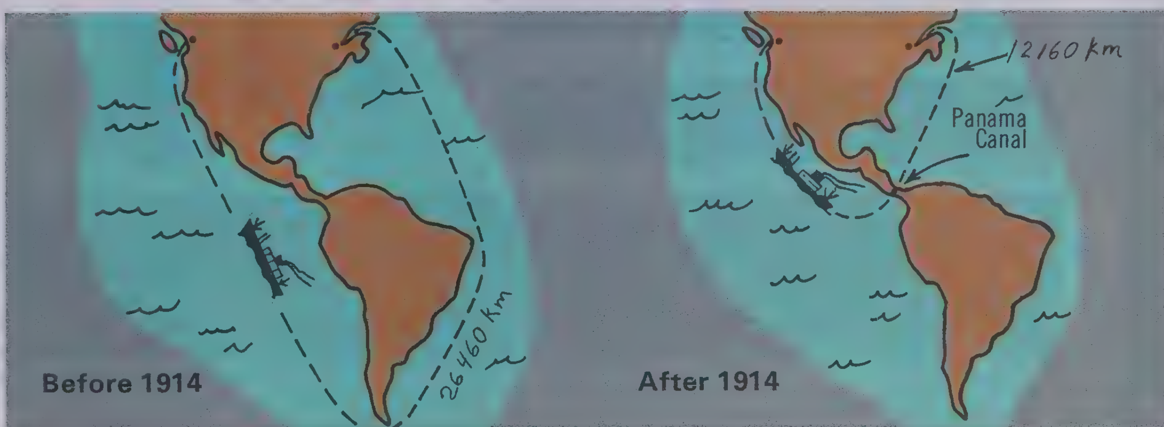


You are invited to explore

ACTIVITY CARD 10
Page 338

THE PANAMA CANAL

The Panama Canal is a waterway that crosses Central America to connect the Atlantic and Pacific oceans. Before it was completed in 1914, a ship sailing from Montreal to Vancouver had to travel about 24 640 kilometres. After 1914, the distance was cut to 12 160 kilometres.



A The Panama Canal is about 80 kilometres long.

B Average time for passing through is about 7 hours.

C 1962 — 10 866 commercial vessels passed through.
1969 — 14 606 commercial vessels passed through.

1. How many kilometres are saved by passing through the canal between Montreal and Vancouver ?

2. If a ship makes 6 round trips from Montreal to Vancouver, how many kilometres does the canal save the ship ?

3. How many more ships went through the canal in 1969 than in 1962 ?

4. If the average amount of cargo carried by each ship in 1969 was 7000 tonnes, what was the total tonnage carried by the ships ?

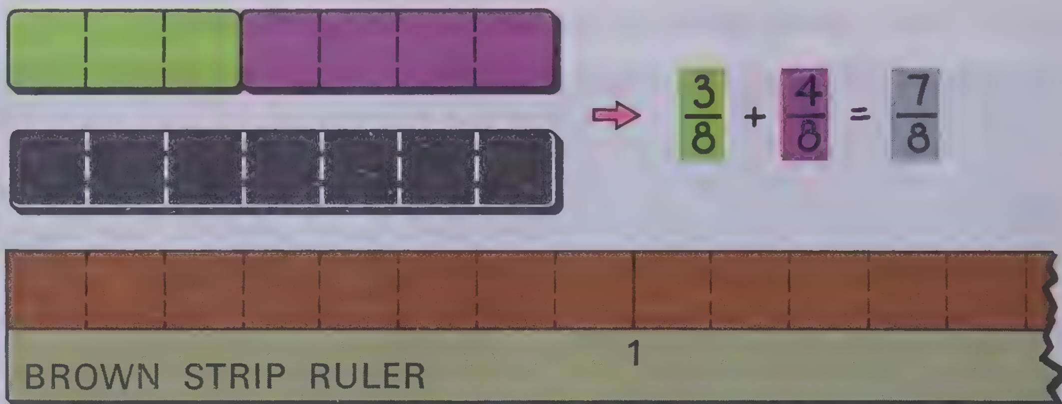
★ 5. About what is the average speed (kilometres per hour) of a ship moving through the canal ?

11 Addition and Subtraction of Fractional Numbers

Can fractional numbers be added and subtracted?

Investigating the Ideas

Think of a ruler with the brown strip as the unit.



$$\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

? Can you use your strips to help you write some other equations about eighths?

Discussing the Ideas

1. Here is one subtraction equation related to the addition equation above.

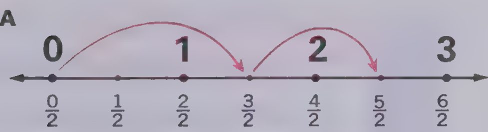
   $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$ Can you find another one?

2. Give some subtraction equations related to some of the addition equations you found with your strips.

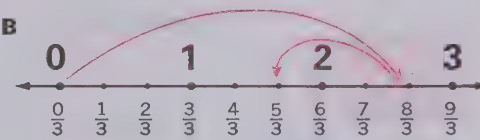
3. Do you know a rule for adding and subtracting when the fractions have the same denominator?

1. Write an equation for each number-line picture.

A

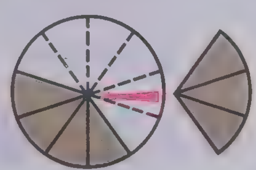


B



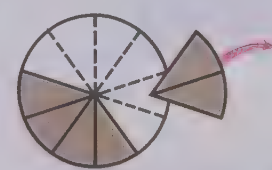
2. Find the sum and difference. The red arrows help you think about putting things together or taking them away.

A



$\frac{4}{10} + \frac{3}{10} = n$

B



$\frac{6}{10} - \frac{2}{10} = a$

3. Find the sums and differences.

A $\frac{3}{8} + \frac{1}{8}$	D $\frac{8}{3} + \frac{9}{3}$	G $\frac{2}{5} + \frac{2}{5}$	J $\frac{3}{7} - \frac{1}{7}$
B $\frac{6}{7} + \frac{5}{7}$	E $\frac{10}{20} - \frac{1}{20}$	H $\frac{7}{3} - \frac{7}{3}$	K $\frac{7}{50} - \frac{6}{50}$
C $\frac{15}{100} - \frac{9}{100}$	F $\frac{7}{10} - \frac{2}{10}$	I $\frac{3}{4} + \frac{0}{4}$	L $\frac{3}{10} + \frac{7}{10}$

Short Stories

1. Diane ate $\frac{3}{12}$ of the cookies. Paula ate $\frac{4}{12}$ of them. What fraction of the cookies were eaten?

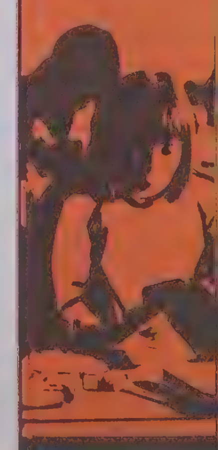
2. Mother had $\frac{1}{6}$ of a pie left after we ate $\frac{4}{6}$ of the pie. She started with what part of a pie?

3. Jack grew $\frac{7}{10}$ cm while Bob grew $\frac{3}{10}$ cm. How much more did Jack grow?

4. Dennis painted $\frac{4}{10}$ of the fence, while Craig painted $\frac{3}{10}$ of it. What part of the fence did the boys paint?

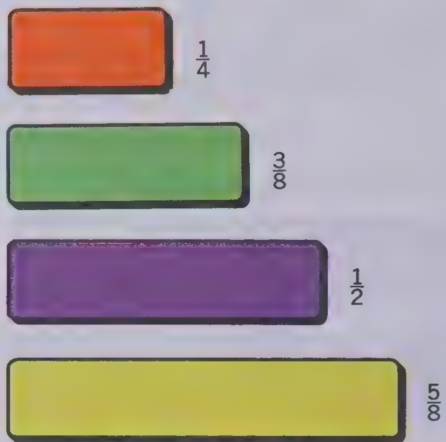
5. Kurt mowed $\frac{3}{8}$ of the yard in the morning and $\frac{5}{8}$ in the afternoon. Did he mow the whole yard?

6. Susan lives $\frac{9}{10}$ km from school. Carol lives only $\frac{6}{10}$ km away. How much farther does Susan live?



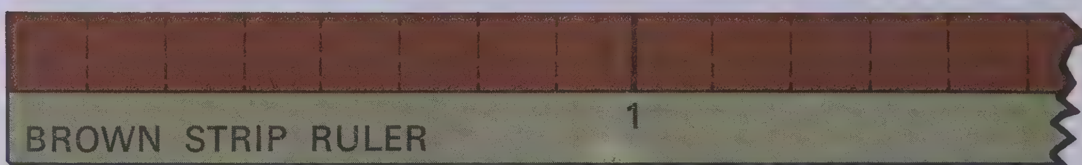
More practice, page A-22, Set 41

Investigating the Ideas



? Can you use your strips to help you write some equations using these lowest-terms fractions for addends?

Example: $\frac{1}{2} + \frac{3}{8} = \frac{7}{8}$



Discussing the Ideas

1. Why do you think these two sums are the same? $\frac{4}{8} + \frac{3}{8}$ | $\frac{1}{2} + \frac{3}{8}$

2. Can you find another pair of fractions that have the same sum as $\frac{1}{2} + \frac{1}{4}$?

3. Explain each example below and give the sum or difference.

A To find $\frac{1}{4} + \frac{1}{3}$, we think $\frac{3}{12} + \frac{4}{12} = ▦$.

$\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$ $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$

B To find $\frac{1}{2} - \frac{1}{5}$, we think $\frac{5}{10} - \frac{2}{10} = ▦$.

$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots\}$ $\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \ldots\}$

1. For each exercise, write an equation using fractions that have the same denominator.

A To find $\frac{1}{2} + \frac{1}{3}$, we think ____ .
$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$
$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$

B To find $\frac{3}{4} - \frac{1}{6}$, we think ____ .
$\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$
$\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$

C To find $\frac{2}{3} + \frac{1}{4}$, we think ____ .
$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$
$\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$

D To find $\frac{5}{6} - \frac{1}{4}$, we think ____ .
$\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\}$
$\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$

2. Find the sums and differences.

A Since $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$, we know that $\frac{1}{2} + \frac{1}{3} = a$.

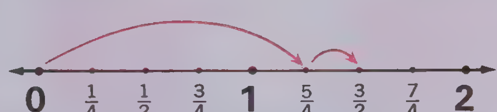
B Since $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{6} = \frac{2}{12}$, we know that $\frac{3}{4} - \frac{1}{6} = c$.

C Since $\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$, we know that $\frac{2}{3} + \frac{1}{4} = x$.

D Since $\frac{5}{6} = \frac{10}{12}$ and $\frac{1}{4} = \frac{3}{12}$, we know that $\frac{5}{6} - \frac{1}{4} = b$.

3. For each number-line picture, write an addition or subtraction equation.

A



B

★ **4.** Give the numbers for *a*, *b*, and *c*.

A To find $\frac{1}{2} - \frac{1}{8}$, we think $\frac{a}{8} - \frac{b}{8} = \frac{c}{8}$.

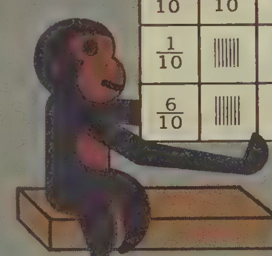
B To find $\frac{3}{8} - \frac{1}{4}$, we think $\frac{a}{8} - \frac{b}{8} = \frac{c}{8}$.

C To find $\frac{1}{3} - \frac{1}{6}$, we think $\frac{a}{6} - \frac{b}{6} = \frac{c}{6}$.

D To find $\frac{3}{4} - \frac{2}{3}$, we think $\frac{a}{12} - \frac{b}{12} = \frac{c}{12}$.

think

$\frac{8}{10}$	$\frac{3}{10}$	____
$\frac{1}{10}$	____	____
$\frac{6}{10}$	____	$\frac{2}{10}$



Give the missing numbers so that this will be a magic square.

Discussing the Ideas

1. For each example below, give the missing number for the $\square$. Then solve for n. Explain the steps used in finding the number for n.

A

$$\begin{array}{lll} \frac{1}{2} & \left\{\frac{1}{2}, \frac{2}{4}, \quad , \frac{4}{8}, \ldots\right\} & \frac{3}{6} \\ +\frac{1}{3} & \left\{\frac{1}{3}, \quad , \frac{3}{9}, \frac{4}{12}, \ldots\right\} & +\frac{2}{6} \\ \hline n & \longleftarrow & \square \end{array}$$

B

$$\begin{array}{lll} \frac{1}{8} & \left\{\quad , \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\right\} & \frac{1}{8} \\ +\frac{1}{2} & \left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \quad , \ldots\right\} & +\frac{4}{8} \\ \hline n & \longleftarrow & \square \end{array}$$

C

$\frac{1}{4}$ +	$\frac{3}{10}$
$\frac{2}{8}$	
$\frac{3}{12}$	$\frac{9}{30}$
$\frac{4}{16}$	$\frac{12}{40}$
	$\frac{15}{50}$
$\vdots$	$\vdots$

$\frac{1}{4} + \frac{3}{10} = n$

$\frac{5}{20} + \frac{6}{20} = \square$

2. Study the following examples and give the missing numbers. Explain how the process of finding a difference is much the same as finding a sum.

A

$$\begin{array}{lll} \frac{1}{2} & \left\{\frac{1}{2}, \frac{2}{4}, \quad , \frac{4}{8}, \ldots\right\} & \frac{3}{6} \\ -\frac{1}{3} & \left\{\frac{1}{3}, \quad , \frac{3}{9}, \frac{4}{12}, \ldots\right\} & -\frac{2}{6} \\ \hline n & \longleftarrow & \square \end{array}$$

B

$$\begin{array}{lll} \frac{5}{6} & \left\{\frac{5}{6}, \quad , \frac{15}{18}, \frac{20}{24}, \ldots\right\} & \frac{10}{12} \\ -\frac{1}{4} & \left\{\frac{1}{4}, \frac{2}{8}, \quad , \frac{4}{16}, \ldots\right\} & -\frac{3}{12} \\ \hline n & \longleftarrow & \square \end{array}$$

C

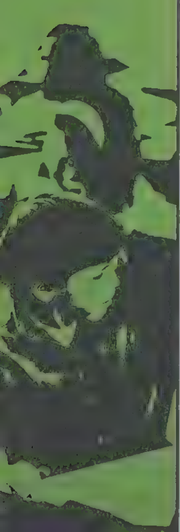
$\frac{3}{10}$ −	$\frac{1}{4}$
	$\frac{2}{8}$
$\frac{9}{30}$	$\frac{3}{12}$
$\frac{12}{40}$	$\frac{4}{16}$
$\frac{15}{50}$	
$\vdots$	$\vdots$

$\frac{3}{10} - \frac{1}{4} = n$

$\frac{6}{20} - \frac{5}{20} = \square$

3. For each part, make your own lists of equivalent fractions until you find two fractions with the same denominator, one for each fraction given. Then give the sum or difference.

A $\frac{1}{2} + \frac{1}{8}$ **B** $\frac{1}{4} - \frac{1}{10}$ **C** $\frac{2}{5} + \frac{1}{4}$ **D** $\frac{1}{3} + \frac{3}{4}$ **E** $\frac{3}{4} - \frac{1}{10}$



1. Copy each exercise (without the arrows) on your paper and give the fraction, in the order shown, for ***a***, ***b***, and ***c***.

A

$$\frac{3}{4} + \frac{2}{3} = (c)$$

$\frac{6}{8}$	$\frac{4}{6}$
$\frac{9}{12}$	$\frac{8}{12}$
$\vdots$	$\vdots$

$$(a) + \frac{8}{12} = (b)$$

B

$$\begin{array}{rll} \frac{3}{5} & \rightarrow \{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \ldots\} & \rightarrow (a) \\ -\frac{1}{10} & \rightarrow \{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \ldots\} & \rightarrow (b) \\ \hline (d) & \longleftarrow & (c) \end{array}$$

C

$$\begin{array}{rll} \frac{1}{2} & \rightarrow \{\frac{1}{2}, (a), (b), \ldots\} & \rightarrow (c) \\ +\frac{2}{3} & \rightarrow \{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \ldots\} & \rightarrow (d) \\ \hline (f) & \longleftarrow & (e) \end{array}$$

2. Make your own lists of equivalent fractions until you find two with the same denominator. Then find the sum or difference.

A $\frac{1}{4} + \frac{1}{10}$ **B** $\frac{3}{4} - \frac{1}{2}$ **C** $\frac{5}{6} - \frac{3}{8}$ **D** $\frac{1}{2} + \frac{3}{10}$

E $\frac{1}{4} - \frac{1}{10}$ **F** $\frac{1}{4} + \frac{1}{5}$ **G** $\frac{3}{4} + \frac{2}{5}$ **H** $\frac{3}{4} - \frac{1}{10}$

I $\frac{3}{4} - \frac{1}{3}$ **J** $\frac{3}{5} + \frac{1}{10}$ **K** $\frac{2}{5} + \frac{1}{3}$ **L** $\frac{1}{3} + \frac{1}{4}$ **M** $\frac{2}{3} - \frac{1}{4}$ **N** $\frac{1}{4} + \frac{1}{5}$

O $\frac{63}{100} + \frac{1}{5}$ **P** $\frac{11}{50} - \frac{1}{10}$ **Q** $\frac{3}{2} + \frac{1}{4}$ **R** $\frac{6}{5} - \frac{3}{10}$ **S** $\frac{1}{3} + \frac{7}{10}$ **T** $\frac{83}{100} + \frac{1}{2}$

3. Copy each exercise on your paper and give the numbers for ***a*** and ***b***. Then find the sum or difference of the two numbers.

A $\frac{1}{2} + \frac{1}{3} \rightarrow \frac{a}{6} + \frac{b}{6}$

B $\frac{2}{3} + \frac{1}{6} \rightarrow \frac{a}{6} + \frac{b}{6}$

C $\frac{1}{8} + \frac{1}{4} \rightarrow \frac{a}{8} + \frac{b}{8}$

D $\frac{1}{4} - \frac{1}{6} \rightarrow \frac{a}{12} - \frac{b}{12}$

E $\frac{5}{8} - \frac{1}{6} \rightarrow \frac{a}{24} - \frac{b}{24}$

F $\frac{8}{15} - \frac{1}{5} \rightarrow \frac{a}{15} - \frac{b}{15}$

think

Find fractional numbers for ***a*** and ***b*** so that

$a + b = 1$

and

$a - b = \frac{1}{2}$.

More practice, page A-22, Set 42

Discussing the Ideas

1. Find each sum, A, B, and C.

$$\frac{3}{4} \to \left\{\frac{3}{4}, \frac{6}{8}, \boxed{\frac{9}{12}}, \frac{12}{16}, \frac{15}{20}, \boxed{\frac{18}{24}}, \frac{21}{28}, \frac{24}{32}, \boxed{\frac{27}{36}}, \ldots\right\}$$

$$+\frac{1}{6} \to \left\{\frac{1}{6}, \boxed{\frac{2}{12}}, \frac{3}{18}, \boxed{\frac{4}{24}}, \frac{5}{30}, \boxed{\frac{6}{36}}, \frac{7}{42}, \frac{8}{48}, \frac{9}{54}, \ldots\right\}$$

A $\frac{9}{12} + \frac{2}{12} =$ |||||

B $\frac{18}{24} + \frac{4}{24} =$ |||||

C $\frac{27}{36} + \frac{6}{36} =$ |||||

2. Do you think each sum you found is the sum of $\frac{3}{4}$ and $\frac{1}{6}$?

3. Which of the additions would you be most likely to use to find $\frac{3}{4} + \frac{1}{6}$?

4. The denominator 12 is the least common denominator for $\frac{3}{4}$ and $\frac{1}{6}$. You can find the least common denominator for $\frac{3}{4}$ and $\frac{1}{6}$ without writing out the sets of equivalent fractions.

> When you find the **least common multiple** of **4** and **6**, you have found the **least common denominator** for $\frac{3}{4}$ and $\frac{1}{6}$.

Why would the least common denominator for $\frac{1}{4}$ and $\frac{5}{6}$ also be 12?

5. A What is the least common multiple of 5 and 3?
B What is the least common denominator for $\frac{1}{5}$ and $\frac{1}{3}$?

6. Give two fractions that have the denominator you found in exercise 5B and could be used to find the sum $\frac{1}{5} + \frac{1}{3}$.

7. Find the sum. $\frac{1}{5} + \frac{1}{3} = n$

1. A What is the least common multiple of 2 and 5?
   B What is the least common denominator for $\frac{2}{5}$ and $\frac{1}{2}$?
   C Give the numerators. $\frac{2}{5} = \frac{|||||}{10}$ $\frac{1}{2} = \frac{|||||}{10}$
   D Find the sum. $\frac{4}{10} + \frac{5}{10} = a$ E Find the sum. $\frac{2}{5} + \frac{1}{2} = b$

2. A What is the least common multiple of 2 and 3?
   B What is the least common denominator for $\frac{1}{2}$ and $\frac{1}{3}$?
   C Give the numerators. $\frac{1}{2} = \frac{|||||}{6}$ $\frac{1}{3} = \frac{|||||}{6}$
   D Find the sum. $\frac{3}{6} + \frac{2}{6} = r$ E Find the sum. $\frac{1}{2} + \frac{1}{3} = s$

3. A What is the least common multiple of 8 and 4?
   B What is the least common denominator for $\frac{3}{8}$ and $\frac{1}{4}$?
   C Give the numerators. $\frac{3}{8} = \frac{|||||}{8}$ $\frac{1}{4} = \frac{|||||}{8}$
   D Find the sum. $\frac{3}{8} + \frac{2}{8} = c$ E Find the sum. $\frac{3}{8} + \frac{1}{4} = d$

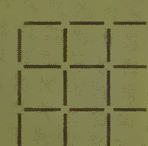
4. A What is the least common multiple of 9 and 6?
   B What is the least common denominator for $\frac{4}{9}$ and $\frac{1}{6}$?
   C Give the numerators. $\frac{4}{9} = \frac{|||||}{18}$ $\frac{1}{6} = \frac{|||||}{18}$
   D Find the difference. $\frac{8}{18} - \frac{3}{18} = m$
   E Find the difference. $\frac{4}{9} - \frac{1}{6} = n$

★ 5. Find the sums and differences.

A $\frac{1}{3} + \frac{1}{4}$	G $\frac{5}{8} + \frac{1}{4}$
B $\frac{1}{3} - \frac{1}{4}$	H $\frac{5}{8} - \frac{1}{4}$
C $\frac{2}{5} - \frac{1}{4}$	I $\frac{7}{8} - \frac{1}{4}$
D $\frac{1}{2} + \frac{2}{3}$	J $\frac{2}{3} + \frac{1}{6}$
E $\frac{2}{3} - \frac{1}{2}$	K $\frac{2}{3} - \frac{1}{6}$
F $\frac{7}{6} - \frac{1}{2}$	L $\frac{7}{10} - \frac{1}{6}$

think

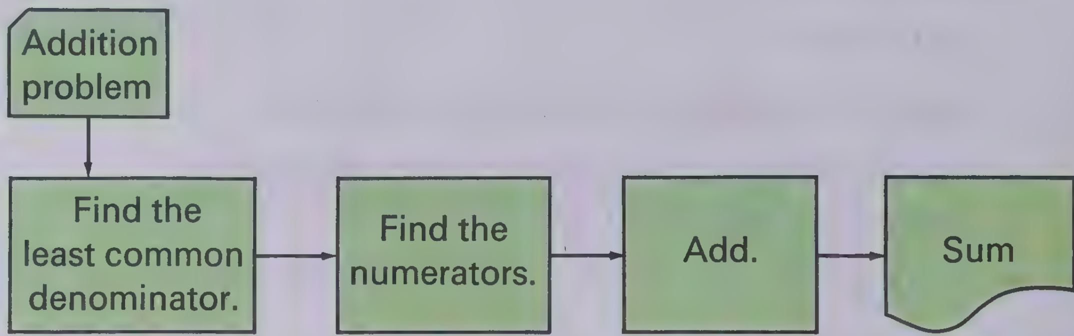
Arrange 24 matches or sticks into 9 small squares as in the figure.



1. Remove 8 matches so you have 5 small squares left.
2. Try forming 5 small squares by removing only 4 matches.

Investigating the Ideas

Study the flow chart for finding $\frac{3}{4} + \frac{1}{6}$.



$$\frac{3}{4} = \frac{}{12} \qquad + \frac{1}{6} = \frac{}{12}$$

$$\frac{3}{4} = \frac{9}{12} \qquad + \frac{1}{6} = \frac{2}{12}$$

$$\frac{3}{4} = \frac{9}{12} \qquad + \frac{1}{6} = \frac{2}{12}$$

$$\frac{3}{4} = \frac{9}{12} \qquad + \frac{1}{6} = \frac{2}{12} \qquad \frac{11}{12}$$

? Can you follow the flow chart to find the sum of $\frac{2}{3}$ and $\frac{1}{2}$?

Discussing the Ideas

Explain each step in the examples below.
Then find the sum or difference.

A

$\frac{5}{8} + \frac{1}{6}$

Think: The least common denominator is 24.

Write: $\frac{5}{8} = \frac{}{24}$, $+\frac{1}{6} = \frac{}{24}$

Think: $\frac{5}{8} = \frac{15}{24}$, $\frac{1}{6} = \frac{4}{24}$

Write: $\frac{5}{8} = \frac{15}{24}$, $+\frac{1}{6} = \frac{4}{24}$, $\boxed{}$

B

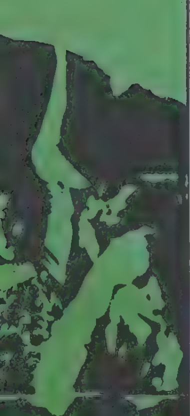
$\frac{5}{4} - \frac{1}{6}$

Think: The least common denominator is 12.

Write: $\frac{5}{4} - \frac{1}{6}$, $\frac{}{12} - \frac{}{12}$

Think: $\frac{5}{4} = \frac{15}{12}$, $\frac{1}{6} = \frac{2}{12}$

Write: $\frac{5}{4} - \frac{1}{6}$, $\frac{15}{12} - \frac{2}{12} = \boxed{}$



1. Find the least common multiple of the numbers shown in red. This is the least common denominator of the two fractions.

A $\frac{1}{2}, \frac{1}{3}$ B $\frac{1}{6}, \frac{5}{9}$ C $\frac{1}{8}, \frac{1}{4}$ D $\frac{3}{4}, \frac{5}{6}$ E $\frac{3}{8}, \frac{1}{2}$ F $\frac{1}{10}, \frac{1}{4}$ G $\frac{5}{4}, \frac{7}{10}$ H $\frac{11}{20}, \frac{1}{4}$ I $\frac{2}{3}, \frac{3}{7}$ J $\frac{1}{5}, \frac{7}{10}$

2. Give the missing numerators.

A $\frac{1}{3} = \frac{\square}{6}$ B $\frac{1}{2} = \frac{\square}{6}$ C $\frac{5}{10} = \frac{\square}{30}$ D $\frac{5}{8} = \frac{\square}{8}$ E $\frac{1}{4} = \frac{\square}{100}$ F $\frac{7}{8} = \frac{\square}{8}$ G $\frac{1}{2} = \frac{\square}{8}$ H $\frac{3}{5} = \frac{\square}{20}$ I $\frac{1}{3} = \frac{\square}{21}$ J $\frac{1}{7} = \frac{\square}{21}$ K $\frac{7}{10} = \frac{\square}{100}$ L $\frac{21}{50} = \frac{\square}{100}$

3. Find the sums and differences.

A $\frac{1}{3} + \frac{1}{2}$ B $\frac{1}{2} + \frac{1}{4}$ C $\frac{1}{2} - \frac{1}{3}$ D $\frac{1}{2} - \frac{1}{4}$ E $\frac{2}{5} - \frac{1}{10}$ F $\frac{2}{3} - \frac{1}{4}$ G $\frac{5}{7} + \frac{1}{4}$

H $\frac{3}{2} + \frac{1}{3}$ I $\frac{3}{2} - \frac{1}{3}$ J $\frac{7}{10} - \frac{1}{2}$ K $\frac{7}{10} + \frac{1}{2}$ L $\frac{3}{2} - \frac{1}{4}$ M $\frac{3}{4} + \frac{2}{5}$ N $\frac{3}{4} - \frac{2}{5}$

4. Find the sums and differences.

A $\frac{1}{3} + \frac{1}{6}$ B $\frac{1}{8} + \frac{1}{4}$ C $\frac{2}{3} + \frac{1}{6}$ D $\frac{1}{3} - \frac{1}{6}$ E $\frac{1}{4} - \frac{1}{8}$ F $\frac{2}{3} - \frac{1}{6}$ G $\frac{1}{3} + \frac{1}{7}$ H $\frac{1}{5} + \frac{7}{10}$ I $\frac{7}{10} - \frac{5}{100}$ J $\frac{1}{3} - \frac{1}{7}$ K $\frac{1}{2} - \frac{1}{10}$ L $\frac{5}{10} + \frac{1}{5}$ M $\frac{7}{10} - \frac{3}{5}$ N $\frac{9}{10} - \frac{25}{100}$ O $\frac{3}{10} + \frac{18}{100}$

Short Stories

1 Walked $\frac{7}{10}$ km. Ran $\frac{1}{2}$ km How far in all?

3 First box: $\frac{2}{3}$ full. Second box: $\frac{3}{4}$ full. How much more is in the second box?

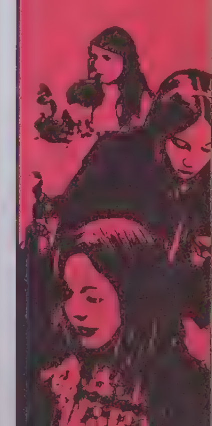
2 $\frac{3}{10}$ centimetre of rain. Another $\frac{1}{4}$ centimetre of rain. How much rain?

4 Joe ate $\frac{1}{2}$ of the candy. Jim ate $\frac{1}{3}$ of it. How much was left for Tom?

5 Jack's step: $\frac{7}{10}$ metre. Sue's step: $\frac{3}{5}$ metre.

A How much longer is Jack's step?

B How long are two of Jack's steps?

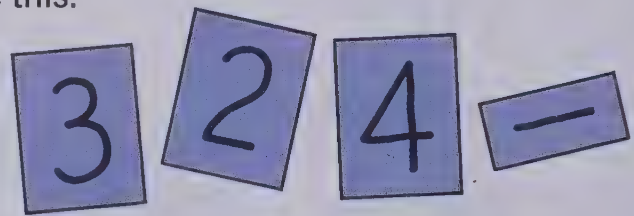


More practice, page A-23, Set 43

What are some other names for fractional numbers?

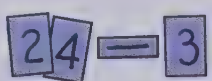
Investigating the Ideas

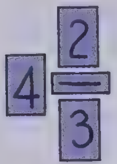
Cut out 4 slips of paper and label them like this.

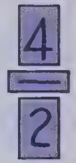


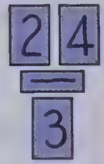
?	How many different number symbols can you make with these slips?	Record your findings.

Examples:









Discussing the Ideas

1. Numerals such as $4\frac{2}{3}$ and $5\frac{3}{8}$ are called mixed numerals $4\frac{2}{3}$ means $4 + \frac{2}{3}$. Which of your numerals in the Investigation were mixed numerals?

2. Fractions such as $\frac{4}{2}$ and $\frac{5}{1}$ are names for whole numbers. What whole numbers do these numerals name?

3. Were any of your fraction symbols in the Investigation names for whole numbers?

4. Give the whole number for each set of equivalent fractions.

   A $\{\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \ldots\}$ C $\{\frac{10}{1}, \frac{20}{2}, \frac{30}{3}, \ldots\}$ E $\{\frac{9}{1}, \frac{18}{2}, \frac{27}{3}, \ldots\}$

   B $\{\frac{6}{1}, \frac{12}{2}, \frac{18}{3}, \ldots\}$ D $\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots\}$ F $\{\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \ldots\}$

1. Each of these fractions names a fractional number that you can think of as a whole number. Give the whole number for each fraction.

   A $\frac{2}{1}$ B $\frac{8}{2}$ C $\frac{3}{1}$ D $\frac{14}{2}$ E $\frac{20}{2}$ F $\frac{15}{3}$ G $\frac{12}{3}$ H $\frac{6}{3}$

   I $\frac{6}{1}$ J $\frac{24}{2}$ K $\frac{24}{3}$ L $\frac{18}{3}$ M $\frac{0}{7}$ N $\frac{0}{1}$ O $\frac{1}{1}$ P $\frac{7}{7}$

2. Each of these sums is a whole number. Give the whole number for each sum.

   A $\frac{2}{3}+\frac{4}{3}$ C $\frac{2}{3}+\frac{1}{3}$ E $\frac{7}{5}+\frac{8}{5}$ G $\begin{array}{r}\frac{10}{4}\\ +\frac{2}{4}\\ \hline\end{array}$ H $\begin{array}{r}\frac{7}{6}\\ +\frac{11}{6}\\ \hline\end{array}$ I $\begin{array}{r}\frac{10}{8}\\ +\frac{6}{8}\\ \hline\end{array}$

   B $\frac{5}{2}+\frac{3}{2}$ D $\frac{1}{2}+\frac{1}{2}$ F $\frac{7}{2}+\frac{9}{2}$

3. Give the correct mixed numeral for each sum.

   A $1+\frac{1}{3}$ C $1+\frac{1}{5}$ E $4+\frac{2}{3}$ G $5+\frac{1}{4}$ I $3+\frac{1}{10}$ K $1+\frac{1}{2}$

   B $\frac{3}{3}+\frac{1}{3}$ D $\frac{5}{5}+\frac{1}{5}$ F $\frac{12}{3}+\frac{2}{3}$ H $\frac{20}{4}+\frac{1}{4}$ J $\frac{30}{10}+\frac{1}{10}$ L $\frac{100}{100}+\frac{3}{100}$

4. Give the correct whole number for n.

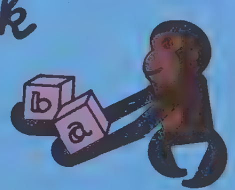
   A $1=\frac{n}{2}$ C $2=\frac{n}{2}$ E $3=\frac{n}{1}$ G $3=\frac{n}{7}$ I $6=\frac{n}{2}$

   B $1=\frac{n}{5}$ D $2=\frac{n}{3}$ F $3=\frac{n}{2}$ H $4=\frac{n}{3}$ J $7=\frac{n}{5}$

5. In each exercise, give the correct mixed numeral for n.

   A Since $\frac{8}{4}=2$, we know that $\frac{9}{4}=n$.
   B Since $\frac{6}{2}=3$, we know that $\frac{7}{2}=n$.
   C Since $\frac{5}{5}=1$, we know that $\frac{7}{5}=n$.
   D Since $\frac{10}{2}=5$, we know that $\frac{11}{2}=n$.
   E Since $\frac{12}{3}=4$, we know that $\frac{14}{3}=n$.
   F Since $\frac{12}{4}=3$, we know that $\frac{15}{4}=n$.
   G Since $\frac{18}{3}=6$, we know that $\frac{19}{3}=n$.
   H Since $\frac{20}{10}=2$, we know that $\frac{29}{10}=n$.
   I Since $\frac{16}{4}=4$, we know that $\frac{19}{4}=n$.

think



Find the whole numbers for a and b that make the sentences true.

$\frac{a}{8}=\frac{2}{a}$ $\frac{b}{16}=\frac{4}{b}$

Discussing the Ideas

1. Give the whole number for ***a***. Then give the whole number for ***b***.

A $1\frac{1}{2} = \frac{a}{2} + \frac{1}{2} \rightarrow 1\frac{1}{2} = \frac{b}{2}$
B $2\frac{1}{3} = \frac{a}{3} + \frac{1}{3} \rightarrow 2\frac{1}{3} = \frac{b}{3}$
C $5\frac{1}{4} = \frac{a}{4} + \frac{1}{4} \rightarrow 5\frac{1}{4} = \frac{b}{4}$
D $6\frac{1}{2} = \frac{a}{2} + \frac{1}{2} \rightarrow 6\frac{1}{2} = \frac{b}{2}$

2. Study this flow chart: **Mixed Numerals to Improper Fractions**

Any mixed numeral → Multiply whole number by the denominator. → Add the numerator. → Write the sum over denominator.

$2\frac{3}{4}$ | $2 \times 4 = 8$ | $8 + 3 = 11$ | $\frac{11}{4}$

Now find improper fractions for the following mixed numerals.

A $3\frac{1}{5}$ B $7\frac{1}{2}$ C $3\frac{2}{3}$ D $4\frac{1}{4}$ E $8\frac{7}{10}$ F $9\frac{3}{4}$

3. Give the whole number for ***a*** and the whole number for ***b***.
Then write a mixed numeral for each improper fraction.

A $\frac{5}{2} = \frac{a}{2} + \frac{1}{2} \rightarrow \frac{5}{2} = b + \frac{1}{2} \rightarrow \frac{5}{2} = \square$

Answer: $a = 4$, $b = 2$, $\frac{5}{2} = 2\frac{1}{2}$

B $\frac{10}{3} = \frac{a}{3} + \frac{1}{3} \rightarrow \frac{10}{3} = b + \frac{1}{3} \rightarrow \frac{10}{3} = \square$
C $\frac{15}{4} = \frac{a}{4} + \frac{3}{4} \rightarrow \frac{15}{4} = b + \frac{3}{4} \rightarrow \frac{15}{4} = \square$

4. Study this flow chart: **Improper Fractions to Mixed Numerals**

Any improper fraction → Divide numerator by denominator. → Use quotient as whole number part. → Write remainder over denominator.

$\frac{17}{5}$ | $5\overline{)17}$ = 3 R2 | $\frac{17}{5} = 3\frac{\square}{\square}$ | $\frac{17}{5} = 3\frac{2}{5}$

Now find mixed numerals for the following improper fractions.

A $\frac{9}{2}$ B $\frac{13}{4}$ C $\frac{22}{7}$ D $\frac{33}{10}$ E $\frac{250}{100}$ F $\frac{43}{5}$ G $\frac{29}{3}$

1. Write an improper fraction for each mixed numeral.

A $1\frac{2}{3}$ B $3\frac{4}{5}$ C $2\frac{1}{2}$ D $1\frac{3}{10}$ E $3\frac{4}{5}$ F $4\frac{3}{4}$ G $5\frac{1}{8}$

H $2\frac{9}{10}$ I $6\frac{7}{10}$ J $1\frac{75}{100}$ K $4\frac{1}{10}$ L $7\frac{2}{5}$ M $6\frac{50}{100}$ N $9\frac{7}{10}$

2. Write a mixed numeral for each improper fraction.

A $\frac{10}{3}$ B $\frac{26}{3}$ C $\frac{34}{10}$ D $\frac{12}{7}$ E $\frac{15}{4}$ F $\frac{13}{4}$ G $\frac{8}{3}$

H $\frac{5}{2}$ I $\frac{17}{4}$ J $\frac{27}{5}$ K $\frac{16}{10}$ L $\frac{11}{10}$ M $\frac{115}{100}$ N $\frac{264}{100}$

Give your answer to each exercise as a mixed numeral.

3. Janet and Rita walked to their grandfather's house. It took them $\frac{3}{4}$ of an hour to get there and $\frac{1}{2}$ hour to get back home.
   A How much time did they spend going both ways?
   B If they spent $\frac{1}{3}$ of an hour visiting, how long were they gone?

4. If butter and margarine come in $\frac{1}{4}$-kilogram sticks
   A How much would 10 sticks of butter weigh?
   B How much would 17 sticks of butter weigh?

5. A If a car race is 14 laps around a $\frac{1}{4}$-km track, how many km long is the race?
   B How many km long is a race that is 5 laps around a $\frac{1}{2}$-km track?

6. One bunch of grapes weighs $\frac{3}{10}$ of a kilogram. Add another, and the scale shows $1\frac{3}{4}$ kg. How much does the second bunch weigh?

think

A boy spends $\frac{1}{2}$ of his money in a store. Then he goes to another store and spends $\frac{1}{2}$ of what is left. After that he has 24 cents. With how much did he start?

More practice, page A-24, Set 44

Investigating the Ideas

How long (in purple units) will the "train" be if you place these strips end-to-end?

? How many addition equations can you write by using all the strips above?

Discussing the Ideas

1. Does the length of your train depend upon the order in which you place your strips?

2. What basic principle does this equation show?

$$\tfrac{1}{5} + \tfrac{3}{5} = \tfrac{3}{5} + \tfrac{1}{5}$$

3. What principle does this equation show?

$$(\tfrac{1}{10} + \tfrac{3}{10}) + \tfrac{5}{10} = \tfrac{1}{10} + (\tfrac{3}{10} + \tfrac{5}{10})$$

4. Can you state a principle illustrated by this equation?

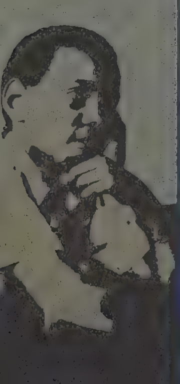
$$\tfrac{2}{3} + 0 = \tfrac{2}{3}$$

5. Explain the steps in each example. Give the sum.

A $2\frac{1}{4} + 1\frac{1}{2} = (2 + 1) + (\frac{1}{4} + \frac{1}{2}) = 3 + \frac{3}{4} = \text{▯}$

B $4\frac{1}{4} + 1\frac{1}{3} = (4 + 1) + (\frac{3}{12} + \frac{4}{12}) = 5 + \frac{7}{12} = \text{▯}$

C
$$\begin{array}{rcl} 4\frac{1}{2} & = & 4\frac{3}{6} \\ \underline{+7\frac{1}{3}} & = & \underline{7\frac{2}{6}} \\ & & \text{▯} \end{array}$$



1. Copy each exercise and give the numbers for *a*, *b*, and *c*.

A $1\frac{1}{7} + 5\frac{3}{7} = (a + b) + (\frac{1}{7} + \frac{3}{7}) = 6 + \frac{4}{7} = c$

B $5\frac{1}{2} + 3\frac{1}{4} = (5 + a) + (\frac{2}{4} + b) = 8 + c = 8\frac{3}{4}$

C $6\frac{1}{5} + 1\frac{7}{10} = (6 + 1) + (a + \frac{7}{10}) = b + \frac{9}{10} = c$

2. Copy and complete each exercise.

A $\begin{array}{r} 2\frac{1}{3} = 2\frac{\square}{6} \\ +5\frac{1}{6} = 5\frac{\square}{6} \\ \hline \square\frac{\square}{6} \end{array}$ B $\begin{array}{r} 8\frac{1}{3} = 8\frac{\square}{12} \\ +7\frac{1}{4} = 7\frac{\square}{12} \\ \hline \square\frac{\square}{12} \end{array}$ C $\begin{array}{r} 2\frac{1}{3} = 2\frac{\square}{6} \\ +5\frac{1}{2} = 5\frac{\square}{6} \\ \hline \square\frac{\square}{6} \end{array}$

D $\begin{array}{r} 3\frac{1}{4} = 3\frac{\square}{4} \\ +9\frac{1}{2} = 9\frac{\square}{4} \\ \hline \square\frac{\square}{4} \end{array}$ E $\begin{array}{r} 5\frac{1}{8} = 5\frac{\square}{24} \\ +7\frac{1}{6} = 7\frac{\square}{24} \\ \hline \square\frac{\square}{24} \end{array}$ F $\begin{array}{r} 3\frac{2}{3} = 3\frac{\square}{12} \\ +1\frac{1}{4} = 1\frac{\square}{12} \\ \hline \square\frac{\square}{12} \end{array}$

3. Find the sums.

A $\begin{array}{r} 6\frac{1}{5} \\ +1\frac{7}{10} \\ \hline \end{array}$ B $\begin{array}{r} 5\frac{1}{2} \\ +3\frac{3}{10} \\ \hline \end{array}$ C $\begin{array}{r} 1\frac{1}{7} \\ +5\frac{3}{7} \\ \hline \end{array}$ D $\begin{array}{r} 5\frac{1}{2} \\ +2\frac{1}{3} \\ \hline \end{array}$ E $\begin{array}{r} 8\frac{1}{3} \\ +7\frac{1}{4} \\ \hline \end{array}$ F $\begin{array}{r} 3\frac{1}{2} \\ +4\frac{1}{4} \\ \hline \end{array}$ G $\begin{array}{r} 6\frac{3}{4} \\ +2\frac{1}{8} \\ \hline \end{array}$

H $\begin{array}{r} 5\frac{3}{10} \\ +3\frac{23}{50} \\ \hline \end{array}$ I $\begin{array}{r} 8\frac{7}{10} \\ +1\frac{1}{5} \\ \hline \end{array}$ J $\begin{array}{r} 7\frac{1}{5} \\ +8\frac{1}{4} \\ \hline \end{array}$ K $\begin{array}{r} 9\frac{1}{2} \\ +\frac{1}{5} \\ \hline \end{array}$ L $\begin{array}{r} \frac{1}{10} \\ +6\,\frac{1}{5} \\ \hline \end{array}$ M $\begin{array}{r} 9\frac{7}{10} \\ +8\frac{2}{30} \\ \hline \end{array}$ N $\begin{array}{r} 6\frac{3}{5} \\ +7\frac{1}{10} \\ \hline \end{array}$

★ **4.** Solve the equations.

A $3\frac{1}{2} + 5\frac{3}{4} = 9 + n$

B $6\frac{5}{8} + 7\frac{3}{4} = 14 + n$

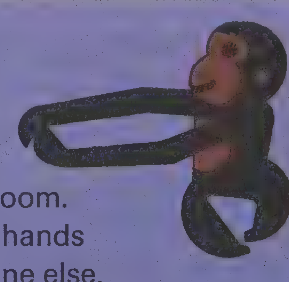
C $9\frac{2}{3} + 7\frac{1}{2} = n + \frac{1}{6}$

D $3\frac{7}{8} + 1\frac{3}{4} = 5 + n$

E $9\frac{5}{6} + 7\frac{1}{4} = n + \frac{1}{12}$

F $15\frac{7}{10} + 29\frac{2}{3} = 45 + n$

think



Five people in a room.
Everyone shakes hands
once with everyone else.
How many handshakes?

More practice, page A-24, Set 45

Changing Numerals

1. Give the missing numerators.

A $\frac{5}{3} = \frac{3}{3} + \frac{\square}{3}$

B $\frac{6}{5} = \frac{5}{5} + \frac{\square}{5}$

C $\frac{7}{4} = \frac{4}{4} + \frac{\square}{4}$

D $\frac{8}{5} = \frac{5}{5} + \frac{\square}{5}$

E $\frac{9}{7} = \frac{7}{7} + \frac{\square}{7}$

F $\frac{12}{7} = \frac{7}{7} + \frac{\square}{7}$

G $\frac{17}{10} = \frac{10}{10} + \frac{\square}{10}$

H $\frac{12}{10} = \frac{10}{10} + \frac{\square}{10}$

I $\frac{3}{2} = \frac{2}{2} + \frac{\square}{2}$

J $\frac{10}{8} = \frac{8}{8} + \frac{\square}{8}$

K $\frac{6}{4} = \frac{4}{4} + \frac{\square}{4}$

L $\frac{10}{6} = \frac{6}{6} + \frac{\square}{6}$

M $\frac{11}{10} = \frac{10}{10} + \frac{\square}{10}$

N $\frac{15}{10} = \frac{10}{10} + \frac{\square}{10}$

O $\frac{16}{10} = \frac{10}{10} + \frac{\square}{10}$

P $\frac{120}{100} = \frac{100}{100} + \frac{\square}{100}$

2. Give the missing numerators.

A $\frac{7}{4} = 1 + \frac{\square}{4}$

B $\frac{17}{10} = 1 + \frac{\square}{10}$

C $\frac{9}{6} = 1 + \frac{\square}{2}$

D $\frac{16}{10} = 1 + \frac{\square}{5}$

E $\frac{14}{10} = 1 + \frac{\square}{5}$

F $\frac{8}{6} = 1 + \frac{\square}{3}$

G $\frac{8}{5} = 1 + \frac{\square}{5}$

H $\frac{6}{5} = 1 + \frac{\square}{5}$

I $\frac{13}{10} = 1 + \frac{\square}{10}$

J $\frac{6}{4} = 1 + \frac{\square}{2}$

K $\frac{15}{10} = 1 + \frac{\square}{2}$

L $\frac{9}{8} = 1 + \frac{\square}{8}$

M $\frac{9}{7} = 1 + \frac{\square}{7}$

N $\frac{5}{3} = 1 + \frac{\square}{3}$

O $\frac{12}{7} = 1 + \frac{\square}{7}$

P $\frac{10}{8} = 1 + \frac{\square}{4}$

3. Solve the equations. (All fractions should be in lowest terms.)

A $3\frac{9}{6} = 4 + n$

B $5\frac{8}{5} = n + \frac{3}{5}$

C $7\frac{15}{10} = 8 + n$

D $9\frac{12}{7} = n + \frac{5}{7}$

E $15\frac{5}{3} = 16 + n$

F $4\frac{15}{10} = 5 + n$

G $16\frac{9}{5} = n + \frac{4}{5}$

H $7\frac{17}{10} = 8 + n$

I $8\frac{16}{10} = 9 + n$

J $3\frac{6}{5} = 4 + n$

K $2\frac{10}{6} = n + \frac{2}{3}$

L $1\frac{10}{8} = 2 + n$

M $4\frac{9}{7} = 5 + n$

N $6\frac{6}{4} = n + \frac{1}{2}$

O $19\frac{8}{6} = n + \frac{1}{3}$

P $33\frac{7}{4} = n + \frac{3}{4}$

Q $41\frac{14}{10} = 42 + n$

R $27\frac{13}{10} = 28 + n$

S $84\frac{9}{8} = n + \frac{1}{8}$

T $65\frac{12}{8} = 66 + n$

4. Solve the equations. (All fractions should be in lowest terms.)

A $\frac{9}{5} = 1 + a$

B $2\frac{7}{6} = a + \frac{1}{6}$

C $4\frac{6}{4} = 5 + a$

D $2\frac{10}{3} = a + \frac{1}{3}$

E $5\frac{13}{10} = 6 + a$

F $75\frac{6}{5} = 76 + a$

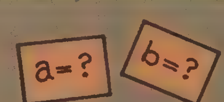
G $56\frac{3}{2} = 57 + a$

H $83\frac{11}{10} = 84 + a$

I $96\frac{12}{10} = 97 + a$

J $38\frac{7}{4} = 39 + a$

think



Find the whole numbers for a and b that make the sentences true.

$$\frac{a}{3} = \frac{4}{a+4} \qquad \frac{b}{4} = \frac{9}{b+9}$$

Adding and Renaming

1. Give each sum in simplest form.

A $\begin{array}{r} \frac{1}{2} = \frac{2}{4} \\ +\frac{3}{4} = \frac{3}{4} \\ \hline \frac{5}{4} = \square \end{array}$ **B** $\begin{array}{r} 7\frac{5}{8} = 7\frac{15}{24} \\ +9\frac{2}{3} = 9\frac{16}{24} \\ \hline 16\frac{31}{24} = \square \end{array}$ **C** $\begin{array}{r} 4\frac{1}{2} = 4\frac{3}{6} \\ +2\frac{5}{6} = 2\frac{5}{6} \\ \hline 6\frac{8}{6} = 7\frac{2}{6} = \square \end{array}$

2. Find the sums. Use mixed numerals for your answers.

A $\frac{2}{3} + \frac{2}{3}$ **B** $\frac{5}{6} + \frac{2}{3}$ **C** $\frac{7}{10} + \frac{3}{5}$ **D** $\frac{7}{10} + \frac{2}{3}$ **E** $\frac{7}{10} + \frac{5}{4}$ **F** $\frac{7}{8} + \frac{1}{4}$ **G** $\frac{3}{10} + \frac{4}{5}$

3. Find the sums. Use mixed numerals for your answers.

A $7\frac{2}{3} + 8\frac{2}{3}$ **B** $6\frac{5}{6} + 9\frac{2}{3}$ **C** $4\frac{7}{10} + 8\frac{4}{5}$ **D** $6\frac{7}{10} + 7\frac{2}{3}$ **E** $19\frac{7}{10} + 2\frac{3}{4}$ **F** $27\frac{7}{10} + 8\frac{3}{5}$

G $17\frac{7}{8} + 26\frac{1}{4}$ **H** $54\frac{3}{10} + 37\frac{4}{5}$ **I** $85\frac{1}{2} + 61\frac{5}{6}$ **J** $37\frac{3}{5} + 84\frac{3}{4}$ **K** $86\frac{7}{10} + 25\frac{11}{20}$ **L** $92\frac{3}{5} + 88\frac{5}{7}$

4. Find the sums. Use mixed numerals for your answers.

A $5\frac{5}{8} + 6\frac{3}{4}$ **B** $8\frac{1}{8} + 7\frac{9}{10}$ **C** $9\frac{4}{5} + 2\frac{13}{15}$ **D** $13\frac{9}{10} + 6\frac{1}{2}$ **E** $27\frac{3}{4} + 8\frac{5}{7}$ **F** $36\frac{95}{100} + 28\frac{1}{5}$

Short Stories

1 Live $1\frac{9}{10}$ km from school.
Walked both ways.
Walked how far?

2 Normal body temperature: 37 °C.
Caught the flu: up $1\frac{1}{2}$ C°.
What was the temperature?

3 Height in June: $124\frac{1}{2}$ cm.
Grew $3\frac{2}{10}$ cm during summer.
How tall in September?

4 Music lesson: $\frac{1}{3}$ hour.
Homework: $\frac{1}{2}$ hour.
How long for both?

5 Pork chops: $2\frac{3}{4}$ kilograms.
Hamburger: $3\frac{1}{2}$ kilograms.
How much is total weight?

6 Recipe: Need $1\frac{7}{10}$ litres milk. Double batch. How much milk for both?

More practice, page A-25, Set 46

Subtracting with Renaming

1. Some of the numerators are covered by screens.
Copy each exercise on your paper and give the missing numerators.

A $6\frac{1}{2} = 6\frac{\square}{4} = 5\frac{6}{4}$

B $7\frac{1}{4} = 7\frac{5}{20} = 6\frac{\square}{20}$

C $3\frac{1}{3} = 3\frac{3}{9} = 2\frac{\square}{9}$

D $5 = 5\frac{0}{8} = 4\frac{\square}{8}$

E $9\frac{1}{2} = 9\frac{5}{10} = 8\frac{\square}{10}$

F $7\frac{1}{4} = 7\frac{\square}{8} = 6\frac{\square}{8}$

G $8\frac{3}{5} = 8\frac{\square}{10} = 7\frac{\square}{10}$

H $1 = 1\frac{\square}{10} = \frac{\square}{10}$

I $4\frac{3}{10} = 4\frac{\square}{100} = 3\frac{\square}{100}$

J $2\frac{1}{2} = 2\frac{\square}{4} = 1\frac{\square}{4}$

K $5\frac{1}{2} = 5\frac{\square}{6} = 4\frac{\square}{6}$

L $3\frac{3}{5} = 3\frac{\square}{15} = 2\frac{\square}{15}$

Study these examples.

$\boxed{A}$
$$\begin{array}{rcrcr} 7\frac{1}{8} & = & 7\frac{1}{8} & = & 6\frac{9}{8} \\ \underline{-2\frac{1}{4}} & = & \underline{2\frac{2}{8}} & = & \underline{2\frac{2}{8}} \\ & & & & 4\frac{7}{8} \end{array}$$

$\boxed{B}$
$$\begin{array}{rcrcr} 4\frac{1}{2} & = & 4\frac{3}{6} & = & 3\frac{9}{6} \\ \underline{-1\frac{5}{6}} & = & \underline{1\frac{5}{6}} & = & \underline{1\frac{5}{6}} \\ & & & & 2\frac{4}{6} = 2\frac{2}{3} \end{array}$$

2. Copy and complete each exercise.

A
$$\begin{array}{rcrcr} 6\frac{1}{2} & = & 6\frac{2}{4} & = & 5\frac{\square}{4} \\ \underline{-1\frac{3}{4}} & = & \underline{1\frac{3}{4}} & = & \underline{1\frac{\square}{4}} \\ & & & & 4\frac{\square}{4} \end{array}$$

B
$$\begin{array}{rcrcr} 7\frac{1}{4} & = & 7\frac{3}{12} & = & 6\frac{\square}{\square} \\ \underline{-1\frac{1}{3}} & = & \underline{1\frac{4}{12}} & = & \underline{1\frac{\square}{\square}} \\ & & & & 5\frac{\square}{\square} \end{array}$$

C
$$\begin{array}{rcrcr} 4\frac{1}{3} & = & 4\frac{3}{9} & = & \square\frac{\square}{9} \\ \underline{-2\frac{5}{9}} & = & \underline{2\frac{5}{9}} & = & \underline{\square\frac{\square}{9}} \\ & & & & \square\frac{\square}{9} \end{array}$$

3. Find the differences. Give the differences in lowest terms.

A $\begin{array}{r} 8\frac{1}{4} \\ -2\frac{1}{2} \\ \hline \end{array}$ B $\begin{array}{r} 9\frac{2}{5} \\ -6\frac{7}{10} \\ \hline \end{array}$ C $\begin{array}{r} 8\frac{1}{2} \\ -4\frac{3}{4} \\ \hline \end{array}$ D $\begin{array}{r} 7\frac{3}{5} \\ -6\frac{2}{3} \\ \hline \end{array}$ E $\begin{array}{r} 19\frac{7}{10} \\ -2\frac{3}{4} \\ \hline \end{array}$ F $\begin{array}{r} 28\frac{7}{10} \\ -7\frac{5}{8} \\ \hline \end{array}$ G $\begin{array}{r} 26 \\ -17\frac{7}{8} \\ \hline \end{array}$

H $\begin{array}{r} 54\frac{3}{10} \\ -37\frac{4}{5} \\ \hline \end{array}$ I $\begin{array}{r} 6\frac{3}{10} \\ -5 \\ \hline \end{array}$ J $\begin{array}{r} 8\frac{3}{4} \\ -7\frac{4}{5} \\ \hline \end{array}$ K $\begin{array}{r} 9\frac{7}{10} \\ -2\frac{11}{15} \\ \hline \end{array}$ L $\begin{array}{r} 13\frac{3}{5} \\ -6\frac{13}{15} \\ \hline \end{array}$ M $\begin{array}{r} 85\frac{5}{8} \\ -61\frac{3}{4} \\ \hline \end{array}$ N $\begin{array}{r} 84\frac{1}{8} \\ -37 \\ \hline \end{array}$

O $\begin{array}{r} 91\frac{1}{2} \\ -88\frac{3}{5} \\ \hline \end{array}$ P $\begin{array}{r} 19\frac{2}{3} \\ -5\frac{9}{10} \\ \hline \end{array}$ Q $\begin{array}{r} 36\frac{1}{3} \\ -29\frac{4}{5} \\ \hline \end{array}$ R $\begin{array}{r} 83 \\ -42\frac{7}{8} \\ \hline \end{array}$ S $\begin{array}{r} 61\frac{4}{5} \\ -39\frac{9}{10} \\ \hline \end{array}$ T $\begin{array}{r} 86\frac{7}{15} \\ -59\frac{9}{10} \\ \hline \end{array}$ U $\begin{array}{r} 93 \\ -39\frac{9}{10} \\ \hline \end{array}$

You are invited to explore

ACTIVITY CARD 11
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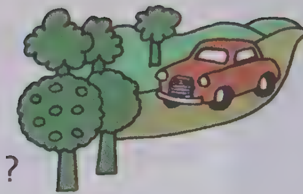
Short Stories

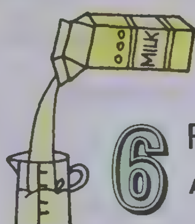


1 $6\frac{1}{2}$ km going. $9\frac{7}{10}$ km returning a different way. How far ?

2 New candle: 20 cm. Burns $2\frac{1}{4}$ centimetres. How long now ?

3 Normal body temperature: 37 °C. Caught the flu: $39\frac{3}{4}$ °C. Temperature was how many degrees above normal ?

4 Average yearly rainfall: $35\frac{1}{2}$ cm. This year: $29\frac{3}{4}$ cm. How much is this below average ?



5 Jim: $34\frac{2}{10}$ kg. Joe: $36\frac{3}{4}$ kg. On the scales together. How many kg ?

6 Recipe A: $1\frac{1}{5}$ litres milk. Recipe B: $\frac{3}{4}$ litre milk.

A How much more milk for A than for B ?

B How much milk is needed for both recipes ?

7 15 pies for the party. Eat $12\frac{5}{6}$ of the pies. How much pie is left ?

9 Triangular area: $17\frac{1}{2}$ units. Rectangular area: $15\frac{7}{8}$ units.

A How much smaller is the rectangle ?

B What is the total area of the two figures together ?

8 Number-line point A: $7\frac{5}{6}$. Point B is $3\frac{1}{3}$ units to the right of A. Where is B ?

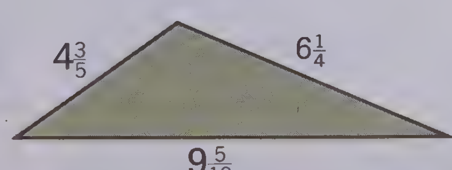
10 $6\frac{7}{12}$ dozen eggs. $5\frac{1}{2}$ dozen eggs. How many dozen eggs ?

11 Flight time: $5\frac{3}{4}$ hours. How long for two such trips ?

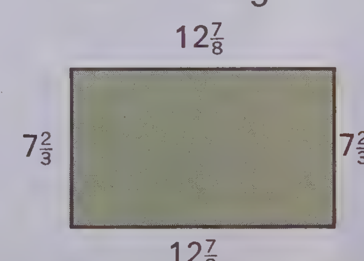
12 One box of Brand X: $376\frac{3}{4}$ grams. One box of Brand Y: $348\frac{1}{2}$ grams. How much more does a box of Brand X weigh than a box of Brand Y ?

★ **13** Give the perimeter for each figure.

A



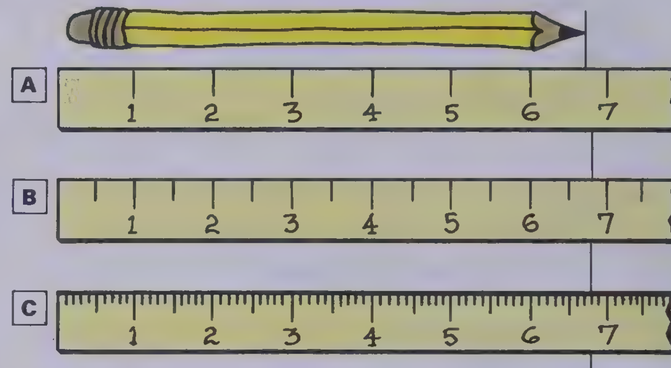
B



More practice, page A-25, Set 47

How can you use fractional numbers in measurement?

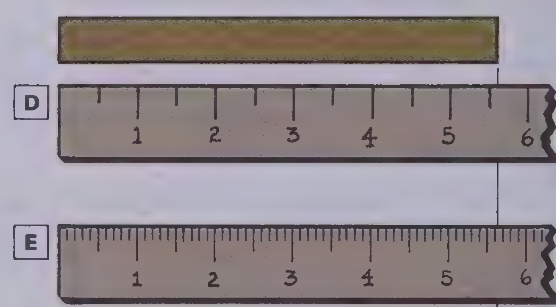
Investigating the Ideas



Using ruler A, the length of the pencil is closer to 7 centimetres than to 6 centimetres. The length of the pencil to the nearest centimetre is 7 centimetres. Using ruler B, the length of the pencil to the nearest half centimetre is still 7 centimetres. What does ruler C show?

? Can you measure some objects and record their lengths to the nearest cm, nearest $\frac{1}{2}$ cm, and nearest $\frac{1}{10}$ cm ?

Discussing the Ideas

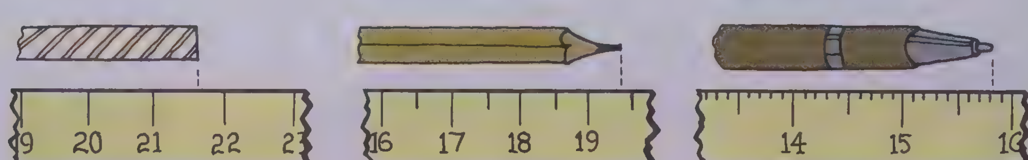


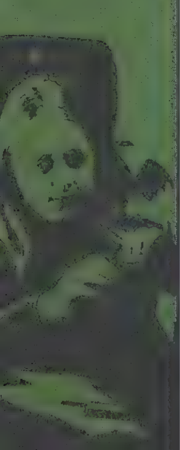
1. Use ruler D to explain how to find the length of the bar to the nearest half centimetre.

2. Use ruler E to find the length of the bar to the nearest tenth of a centimetre.

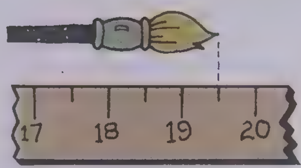
3. Give the length of each object, according to the directions.

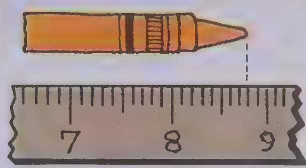
   A nearest centimetre B nearest $\frac{1}{2}$ centimetre C nearest $\frac{1}{10}$ centimetre

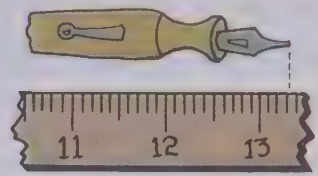




1. Give the length of each object according to the directions.
   A nearest $\frac{1}{2}$ cm B nearest $\frac{1}{10}$ cm C nearest $\frac{1}{10}$ cm







2. Find the length of the segment to the nearest $\frac{1}{2}$ centimetre.

   A ______________________ B ______________________

3. A Draw a segment that is as long as 2A and 2B together.
   B What is the sum of their lengths to the nearest $\frac{1}{10}$ centimetre ?

4. Draw segments of the following lengths.
   A $2\frac{1}{2}$ cm C $3\frac{8}{10}$ cm E $\frac{15}{10}$ cm G $9\frac{1}{2}$ cm
   B $7\frac{1}{2}$ cm D 6 cm F $4\frac{4}{10}$ cm H $1\frac{5}{10}$ cm

5. Use the segments in exercise 4. Find the sums and differences of each pair of lengths.
   A A and C
   B B and G
   C C and F
   D F and H

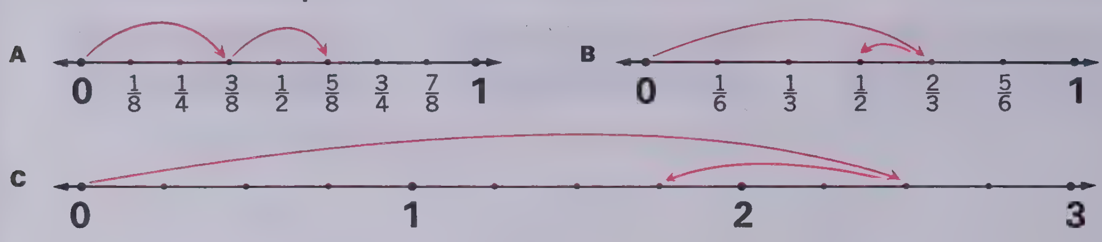
6. Tell which is longer.
   A $\frac{3}{2}$ cm, $\frac{5}{10}$ cm
   B $\frac{7}{10}$ cm, $\frac{1}{2}$ cm
   C $1\frac{19}{10}$ cm, $1\frac{1}{2}$ cm
   D $\frac{21}{10}$ cm, 2 cm

think

There are 16 girls and 14 boys in Miss Gray's class. 20 of the children ride the bus.

1. Is it possible that all the girls ride the bus ?
2. What is the least number of girls that might ride the bus ?
3. What is the least number of boys that might ride the bus ?

1. Write the addition or subtraction equation suggested by each number-line picture.



2. Give the correct whole number for each exercise.

A $\frac{9}{3}$ B $\frac{12}{3}$ C $\frac{16}{2}$ D $\frac{10}{6} + \frac{8}{6}$ E $\frac{12}{5} + \frac{8}{5}$

3. Solve the equations.

A $2 + \frac{1}{3} = n$ B $3 + n = 3\frac{3}{4}$ C $n + \frac{6}{7} = 4\frac{6}{7}$ D $5\frac{7}{8} = 5 + n$

4. Find the sums and differences. Give your answers in lowest terms.

A $\frac{2}{5} + \frac{1}{5}$ B $\frac{3}{7} - \frac{2}{7}$ C $\frac{3}{10} + \frac{7}{10}$ D $\frac{9}{10} - \frac{3}{10}$ E $\frac{5}{8} + \frac{1}{4}$ F $\frac{3}{4} - \frac{1}{2}$ G $\frac{5}{6} - \frac{1}{4}$

H $\frac{3}{10} + \frac{1}{10}$ I $\frac{8}{15} - \frac{2}{15}$ J $\frac{7}{20} + \frac{3}{20}$ K $\frac{7}{20} - \frac{3}{20}$

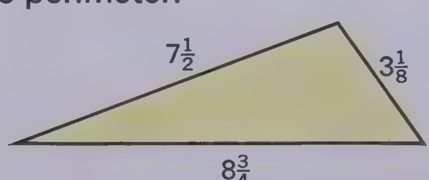
L $\frac{3}{8} + \frac{1}{3}$ M $\frac{3}{8} - \frac{1}{3}$ N $\frac{4}{5} - \frac{1}{3}$ O $\frac{9}{10} + \frac{1}{4}$

5. Find the sums and differences. Give your answers in lowest terms.

A $7\frac{1}{2} + 3\frac{1}{4}$ B $8\frac{1}{3} - 4\frac{1}{6}$ C $3\frac{7}{10} + 9\frac{4}{5}$

D $15\frac{4}{5} - 8\frac{7}{10}$ E $13\frac{3}{5} + 9\frac{9}{10}$ F $58\frac{1}{4} - 12\frac{5}{6}$

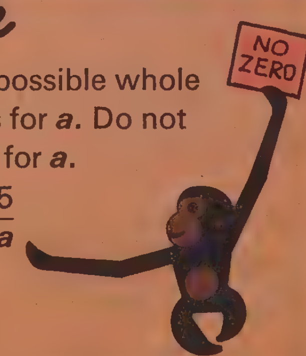
★ 6. Give the perimeter.

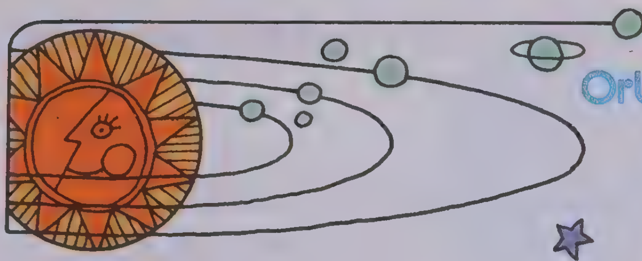


think

Give all possible whole numbers for *a*. Do not use zero for *a*.

$\frac{a}{5} < \frac{5}{a}$





Orbital Speeds of the Planets

The table shows the symbols for the planets and the speed at which some of the planets go around the sun. The picture above shows the orbital paths of some of the planets around the sun. The sun is much larger than any of the planets. If you think of Jupiter, the largest planet, as being the size of a pea, then the sun would be about the size of a softball and Earth would be only a tiny speck.

Planet	Symbol	Approximate orbital speed (Km per second)
Mercury	☿	[illegible]
Venus	♀	$34\frac{4}{5}$
Earth	⊕	$29\frac{3}{5}$
Mars	♂	24
Jupiter	♃	$12\frac{9}{10}$
Saturn	♄	$9\frac{3}{5}$
Uranus	♅	$6\frac{3}{4}$
Neptune	♆	$5\frac{2}{5}$
Pluto	♇	[illegible]

1. About how many kilometres per second faster is:
   - A Mars than Saturn ?
   - B Venus than Earth ?
   - C Mars than Jupiter ?
   - D Saturn than Uranus ?
   - E Uranus than Neptune ?
   - F Jupiter than Neptune ?

2. Mercury is about $40\frac{4}{5}$ kilometres per second faster than Uranus. About what is the orbital speed of Mercury ?

3. The moon orbits Earth at about $1\frac{1}{5}$ kilometres per second. The speed of Pluto is about $3\frac{3}{5}$ kilometres per second more than that. About what is the orbital speed of Pluto ?

4. Manned satellites orbit Earth at about 27 200 kilometres per hour, or $2\frac{3}{20}$ kilometres per second faster than Neptune orbits the sun. About what is the speed of these satellites in kilometres per second ?

1. For the numeral 96 321 587, tell which digit is in each of these places.

A thousands' C hundred thousands' E millions'
B ten millions' D ten thousands' F hundreds'

2. Copy each sentence. Give the missing words and numbers.

6 709 483 875

A The 3 in the __?__ place means 3 × ▒.
B The 9 in the __?__ place means 9 × ▒.
C The 6 in the __?__ place means 6 × ▒.

3. Give the correct sign (<, =, >) for each ▒.

A 53 680 ▒ 50 000 + 3000 + 500 + 80
B 657 009 ▒ 600 000 + 50 000 + 7000 + 90

4. Each book costs $2.49. Bought 6 books. Paid how much for books?

5. Think about the function machine and complete these tables.

Function Rule: $(2 \times n) + 8$

n	Output
4	A
24	B
359	C
0	D
E	18

Function Rule: $(n \times n) - n$

n	Output
0	F
1	G
10	H
99	I
325	J

6. Solve each equation.

A $\frac{1}{10} = \frac{n}{100}$ D $\frac{1}{10} = \frac{n}{1000}$
B $\frac{3}{10} = \frac{n}{100}$ E $\frac{1}{100} = \frac{n}{1000}$
C $\frac{6}{10} = \frac{n}{100}$ F $\frac{5}{100} = \frac{n}{1000}$

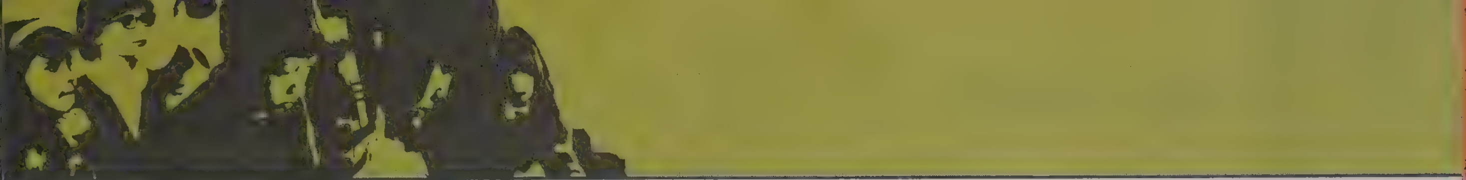
7. Find the length (to the nearest centimetre) of each segment.

A ________________________________ *B*

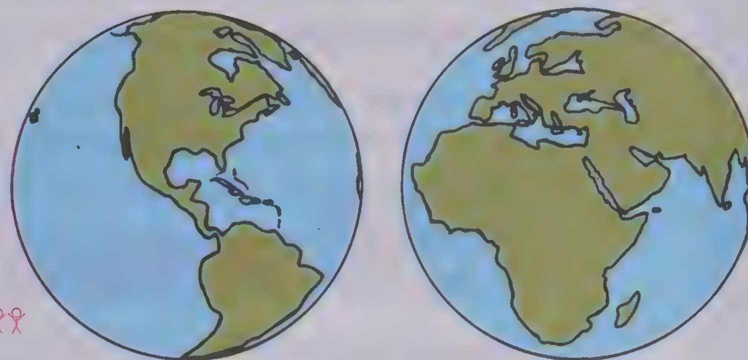
C ____________________ *D*

E __________________________ *F*

You are invited to explore **ACTIVITY CARD 12** Page 339



THE WORLD



This **pictograph** shows the population of five of the seven large blocks of land (continents) on the earth's surface. Antarctica is the only continent that is not populated. The population of Australia and the islands of the Pacific Ocean (Oceania) is about nineteen million and is too small to show on the pictograph.

1. List the continents given in the pictograph. Beside each continent give the approximate population.

2. Use the pictograph to tell how many times as many people live in Asia as in North America.

3. Use the pictograph to tell how many more people live in Asia than on the other four continents combined.

4. The total population of the world was about three billion six hundred thirty-two million people in 1970. Write the numeral for this population.

★ 5. The approximate area (in square kilometres) of each continent is given in the table. Let the symbol ■ represent 1 million square kilometres and make a pictograph that shows the sizes of the continents.

Continent	Area
N. America	24 500 000
S. America	17 800 000
Europe	10 500 000
Asia	44 600 000
Africa	30 300 000
Australia	7 700 000
Antarctica	13 300 000

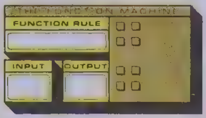
12 Decimals

What is a decimal numeral?

Investigating the Ideas

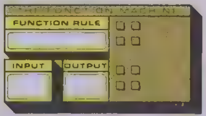
Each output from function machine A becomes an input for function machine B.

A



n	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{7}{10}$	$\frac{3}{5}$	$\frac{2}{10}$	$2\frac{1}{2}$	$7\frac{1}{5}$	$35\frac{1}{10}$	$\frac{3}{2}$	$\frac{13}{5}$
	$\frac{5}{10}$	$\frac{2}{10}$	$\frac{7}{10}$	$\frac{6}{10}$	$\frac{2}{10}$	$2\frac{5}{10}$	$7\frac{2}{10}$	$35\frac{1}{10}$	$1\frac{5}{10}$	$2\frac{6}{10}$

B



n	$\frac{5}{10}$	$\frac{2}{10}$	$\frac{7}{10}$	$\frac{6}{10}$	$\frac{2}{10}$	$2\frac{5}{10}$	$7\frac{2}{10}$	$35\frac{1}{10}$	$1\frac{5}{10}$	$2\frac{6}{10}$
	0.5	0.2	0.7	0.6	0.2	2.5	7.2	35.1	1.5	2.6

? Can you figure out how these two function machines work? **Show you know** by giving output B for each of these inputs for A. $\frac{2}{5}$, $\frac{4}{5}$, $3\frac{1}{2}$, $4\frac{1}{5}$, $\frac{8}{10}$, $6\frac{1}{10}$, $7\frac{2}{5}$

Discussing the Ideas

1. Explain each function rule in the Investigation.
2. Study the table and give the missing **decimals**.

	We see	We think	We write: fractions	We write: decimals	We say
A		2 tens, 3 ones, and 8 tenths	$23\frac{8}{10}$	23.8	twenty-three and eight tenths
B		1 ten, 8 ones, and 4 tenths	$18\frac{4}{10}$	18.4	eighteen and four tenths
C		1 and 2 tenths	$1\frac{2}{10}$		one and two tenths
D		7 tenths	$\frac{7}{10}$		seven tenths

1. For each exercise, give the correct mixed numeral and the correct decimal.

A For 8 tens, 3 ones, and 6 tenths, we write $\square$ or $\square$.
Answer: $83\frac{6}{10}$, 83.6

B For 1 ten, 2 ones, and 5 tenths, we write $\square$ or $\square$.

C For 3 tens, 0 ones, and 7 tenths, we write $\square$ or $\square$.

D For 9 tens, 7 ones, and 1 tenth, we write $\square$ or $\square$.

E For 8 tens, 3 ones, and 9 tenths, we write $\square$ or $\square$.

F For 1 ten, 4 ones, and 3 tenths, we write $\square$ or $\square$.

G For 5 tens, 1 one, and 8 tenths, we write $\square$ or $\square$.

H For 6 tens, 2 ones, and 4 tenths, we write $\square$ or $\square$.

2. Copy each exercise and give the missing numerator or denominator.

A $6.8 = 6 + \frac{\square}{10}$

B $17.5 = 17 + \frac{5}{\square}$

C $0.6 = \frac{\square}{10}$

D $23.9 = 23 + \frac{\square}{10}$

E $74.6 = 74 + \frac{6}{\square}$

F $18.1 = 18 + \frac{\square}{10}$

G $2.2 = 2 + \frac{\square}{10}$

H $3.7 = 3 + \frac{\square}{10}$

I $37.4 = 37 + \frac{4}{\square}$

3. Give the correct decimal for each sum.

A $7 + \frac{4}{10}$

B $23 + \frac{8}{10}$

C $80 + \frac{9}{10}$

D $66 + \frac{1}{10}$

E $1 + \frac{1}{10}$

F $237 + \frac{9}{10}$

G $53 + \frac{8}{10}$

H $60 + \frac{7}{10}$

I $48 + \frac{2}{10}$

★ **4.** Give the missing numerator and then give the decimal for the sum ***n***.

A $4 + \frac{1}{2} = 4 + \frac{\square}{10} = n$

B $6 + \frac{1}{5} = 6 + \frac{\square}{10} = n$

C $23 + \frac{1}{2} = 23 + \frac{\square}{10} = n$

D $10 + \frac{4}{5} = 10 + \frac{\square}{10} = n$

E $36 + \frac{2}{5} = 36 + \frac{\square}{10} = n$

F $126 + \frac{1}{2} = 126 + \frac{\square}{10} = n$

G $74 + \frac{3}{5} = 74 + \frac{\square}{10} = n$

H $81 + \frac{1}{5} = 81 + \frac{\square}{10} = n$

More practice, page A-26, Set 48

Discussing the Ideas

1. Study the decimal for $9876 + \frac{4}{10} + \frac{3}{100} + \frac{2}{1000}$. Then give the missing word and number in each exercise that follows.

Thousands | Hundreds | Tens | Ones | Tenths | Hundredths | Thousandths

9 8 7 6 . 4 3 2

The 4 in the **tenths'** place means $\frac{4}{10}$.
The 3 in the **hundredths'** place means $\frac{3}{100}$.
The 2 in the **thousandths'** place means $\frac{2}{1000}$.

A 28.45: The 2 in the __?__ place means |||||.
B 28.45: The 4 in the __?__ place means |||||.
C 531.64: The 5 in the __?__ place means |||||.
D 531.64: The 4 in the __?__ place means |||||.
E 2876.354: The 2 in the __?__ place means |||||.
F 2876.354: The 4 in the __?__ place means |||||.
G 5.04: The 4 in the __?__ place means |||||.
H 26.008: The 8 in the __?__ place means |||||.
I 0.326: The 2 in the __?__ place means |||||.
J 0.605: The 5 in the __?__ place means |||||.

2. Study examples A and B below. Then give the missing numerator in each exercise.

$\boxed{A}$ $8.25 = 8 + \frac{2}{10} + \frac{5}{100} = 8 + \frac{20}{100} + \frac{5}{100} = 8\frac{25}{100}$

We read "eight and twenty-five hundredths" for 8.25.

$\boxed{B}$ $8.396 = 8 + \frac{3}{10} + \frac{9}{100} + \frac{6}{1000} = 8 + \frac{300}{1000} + \frac{90}{1000} + \frac{6}{1000} = 8\frac{396}{1000}$

We read "eight and three hundred ninety-six thousandths" for 8.396.

A $7.36 = 7 + \frac{3}{10} + \frac{6}{100} = 7 + \frac{|||}{100} + \frac{6}{100} = 7\frac{|||}{100}$
B $2.94 = 2 + \frac{9}{10} + \frac{4}{100} = 2 + \frac{|||}{100} + \frac{|||}{100} = 2\frac{|||}{100}$

3. Read each decimal in question 1 of the Discussion.

1. Copy each exercise on your paper and give the missing numerator.

A $8.6 = 8\frac{\square}{10}$
B $7.9 = 7\frac{\square}{10}$
C $0.4 = \frac{\square}{10}$
D $0.05 = \frac{\square}{100}$
E $20.7 = 20\frac{\square}{10}$
F $56.8 = 56\frac{\square}{10}$
G $17.6 = 17\frac{\square}{10}$
H $156.4 = 156\frac{\square}{10}$
I $15.04 = 15\frac{\square}{100}$
J $9.07 = 9\frac{\square}{100}$
K $1.007 = 1\frac{\square}{1000}$
L $0.008 = \frac{\square}{1000}$

2. Write each fractional number as in the examples.

Example 1: $13.28 = 13 + \frac{2}{10} + \frac{8}{100}$

Example 2: $4.725 = 4 + \frac{7}{10} + \frac{2}{100} + \frac{5}{1000}$

A 4.62
B 7.83
C 9.25
D 3.14
E 8.62
F 86.2
G 7.32
H 73.2
I 7.81
J 7.812
K 7.846
L 70.84
M 926.4
N 92.64
O 9.264
P 9.064
Q 76.8
R 7.68
S 0.768
T 0.076
U 43.4
V 43.04
W 4.304
X 4.004

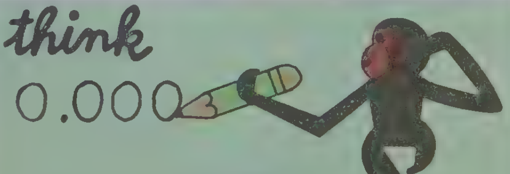
3. Give the correct decimal for each sum.

A $7 + \frac{2}{10}$
B $7 + \frac{2}{10} + \frac{6}{100}$
C $7 + \frac{2}{10} + \frac{6}{100} + \frac{7}{1000}$
D $3 + \frac{5}{10}$
E $3 + \frac{5}{10} + \frac{1}{100}$
F $3 + \frac{5}{10} + \frac{1}{100} + \frac{9}{1000}$
G $6 + \frac{7}{10} + \frac{8}{100}$
H $6 + \frac{7}{10} + \frac{8}{100} + \frac{1}{1000}$
I $8 + \frac{2}{10} + \frac{3}{100} + \frac{6}{1000}$
J $8 + \frac{2}{10} + \frac{0}{100} + \frac{6}{1000}$
K $8 + \frac{2}{10} + \frac{6}{1000}$
L $8 + \frac{0}{10} + \frac{0}{100} + \frac{6}{1000}$
M $8 + \frac{6}{1000}$
N $9 + \frac{6}{100}$
O $7 + \frac{8}{1000}$
P $4 + \frac{1}{10} + \frac{5}{1000}$
Q $765 + \frac{2}{10}$
R $76 + \frac{5}{10} + \frac{2}{100}$
S $7 + \frac{6}{10} + \frac{5}{100} + \frac{2}{1000}$
T $7 + \frac{5}{100} + \frac{2}{1000}$
U $7 + \frac{5}{10} + \frac{2}{1000}$

★ **4.** Give a decimal for each sum.

A $\frac{2}{10} + \frac{3}{100} + \frac{6}{1000} + \frac{7}{10\,000}$
B $\frac{5}{10} + \frac{3}{100} + \frac{8}{10\,000} + \frac{9}{100\,000}$
C $100\,000 + \frac{1}{100\,000}$
D $1000 + \frac{1}{1000} + \frac{1}{1\,000\,000}$

think



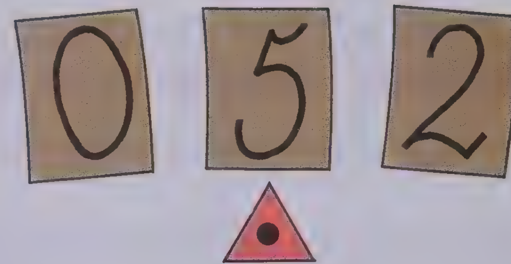
Write a decimal for $\frac{1}{25}$.
Write a fraction for 0.000001.

More practice, page A-26, Set 49

● *Can we think of decimals in the same way we think of fractions?*

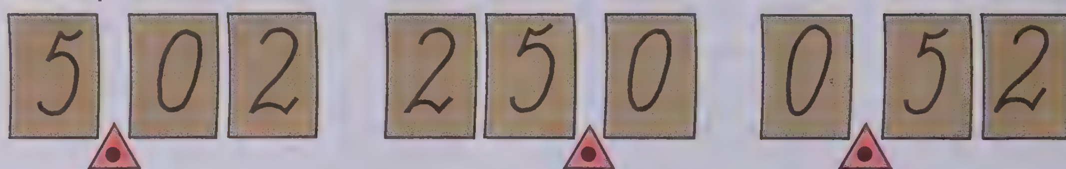
Investigating the Ideas

Cut out 4 slips of paper and label them like this.



?	Using all your slips, how many decimals can you form, if the decimal point is always between two digits?	Record your decimals.

Examples:



Discussing the Ideas

1. A Which of your decimals represent numbers greater than 1?
   B Which are less than 1?
   C Can you match any of your decimals with these numbers?
   $2\frac{1}{2}$, $5\frac{2}{10}$, 25, $50\frac{1}{5}$

2. For each pair, tell which is greater.

   A 25.0 or 2.50
   B 25.0 or 5.20
   C 25.0 or 52.0
   D 0.052 or 0.52
   E 0.052 or 0.025
   F 52.0 or 50.2

3. A Solve the equation: $\frac{2}{10} + \frac{5}{10} = n$
   B What numeral should go on the blank slip?

1. Give the correct sign ($<$, $=$, or $>$) for each $\bigcirc$.

A $75.6 \bigcirc 75.3$	E $53.2 \bigcirc 52.9$	I $6.231 \bigcirc 6.232$
B $84.6 \bigcirc 84.9$	F $8.75 \bigcirc 8.76$	J $6.451 \bigcirc 6.351$
C $67.2 \bigcirc 68.2$	G $8.74 \bigcirc 8.64$	K $7.213 \bigcirc 6.987$
D $61.6 \bigcirc 62.1$	H $3.05 \bigcirc 3.04$	L $5.607 \bigcirc 5.617$

2. Find the sums and differences.

A $\frac{6}{10}-\frac{3}{10}=n$
$0.6-0.3=m$

B $\frac{2}{10}+\frac{5}{10}=n$
$0.2+0.5=m$

C $\frac{23}{100}+\frac{34}{100}=n$
$0.23+0.34=m$

D $\frac{15}{100}+\frac{63}{100}=n$
$0.15+0.63=m$

E $\frac{25}{100}-\frac{15}{100}=n$
$0.25-0.15=m$

F $\frac{38}{100}+\frac{15}{100}=n$
$0.38+0.15=m$

G $\frac{37}{100}-\frac{19}{100}=n$
$0.37-0.19=m$

H $\frac{56}{100}+\frac{24}{100}=n$
$0.56+0.24=m$

I $\begin{array}{r} \frac{3}{10} \\ +\frac{5}{10} \\ \hline \end{array}$ $\begin{array}{r} 0.3 \\ +0.5 \\ \hline \end{array}$

J $\begin{array}{r} \frac{7}{10} \\ -\frac{2}{10} \\ \hline \end{array}$ $\begin{array}{r} 0.7 \\ -0.2 \\ \hline \end{array}$

K $\begin{array}{r} \frac{16}{100} \\ +\frac{42}{100} \\ \hline \end{array}$ $\begin{array}{r} 0.16 \\ +0.42 \\ \hline \end{array}$

L $\begin{array}{r} \frac{82}{100} \\ -\frac{26}{100} \\ \hline \end{array}$ $\begin{array}{r} 0.82 \\ -0.26 \\ \hline \end{array}$

M $\begin{array}{r} \frac{29}{100} \\ +\frac{17}{100} \\ \hline \end{array}$ $\begin{array}{r} 0.29 \\ +0.17 \\ \hline \end{array}$

N $\begin{array}{r} \frac{58}{100} \\ -\frac{28}{100} \\ \hline \end{array}$ $\begin{array}{r} 0.58 \\ -0.28 \\ \hline \end{array}$

O $\begin{array}{r} \frac{27}{100} \\ +\frac{7}{10} \\ \hline \end{array}$ $\begin{array}{r} 0.27 \\ +0.70 \\ \hline \end{array}$

P $\begin{array}{r} \frac{5}{10} \\ +\frac{32}{100} \\ \hline \end{array}$ $\begin{array}{r} 0.50 \\ +0.32 \\ \hline \end{array}$

3. In each exercise, copy the first equation and give the missing numerators. Then give the correct decimal for the sum in the second equation.

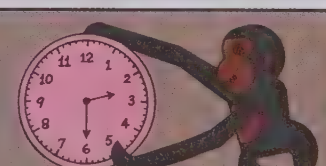
A $\frac{6}{10}+\frac{7}{10}=\frac{10}{10}+\frac{\square}{10}=1+\frac{\square}{10}$
$0.6+0.7=n$

B $\frac{5}{10}+\frac{9}{10}=\frac{10}{10}+\frac{\square}{10}=1+\frac{\square}{10}$
$0.5+0.9=n$

C $\frac{8}{10}+\frac{6}{10}=\frac{10}{10}+\frac{\square}{10}=1+\frac{\square}{10}$
$0.8+0.6=n$

D $\frac{4}{10}+\frac{7}{10}=\frac{10}{10}+\frac{\square}{10}=1+\frac{\square}{10}$
$0.4+0.7=n$

think



When will the two hands of the clock next come together?

A Between 3:12 and 3:15

B Between 3:15 and 3:18

C Between 3:18 and 3:21

More practice, page A-27, Set 50

Investigating the Ideas

Marty was usually a careful mathematics student. But on this paper she missed every answer because she forgot all of the decimal points.

Marty

$$\begin{array}{r} 2.3 \\ +4.5 \\ \hline 68 \end{array} \times \quad \begin{array}{r} 0.7 \\ +0.6 \\ \hline 13 \end{array} \times \quad \begin{array}{r} 4.8 \\ +3.7 \\ \hline 85 \end{array} \times$$

(1), (2), (3)

$$\begin{array}{r} 6.9 \\ -2.7 \\ \hline 42 \end{array} \times \quad \begin{array}{r} 1.4 \\ -0.6 \\ \hline 8 \end{array} \times \quad \begin{array}{r} 7.2 \\ -1.5 \\ \hline 57 \end{array} \times$$

(4), (5), (6)

? Can you copy Marty's problems and correct them for her? Be careful!

Discussing the Ideas

Explain each step in the two examples.

1. Step 1	Step 2	Step 3
$\begin{array}{r} {}^{1} \\ 7.84 \\ +\ 6.79 \\ \hline 3 \end{array}$	$\begin{array}{r} {}^{1\ \ 1} \\ 7.84 \\ +\ 6.79 \\ \hline .63 \end{array}$	$\begin{array}{r} {}^{1\ \ 1} \\ 7.84 \\ +\ 6.79 \\ \hline 14.63 \end{array}$
$\frac{4}{100} + \frac{9}{100} = \frac{13}{100}$	$\frac{1}{10} + \frac{8}{10} + \frac{7}{10} = 1\frac{6}{10}$	$1 + 7 + 6 = 14$

2. Step 1	Step 2	Step 3
$\begin{array}{r} {}^{5} \\ \cancel{6}.{}^{1}2 \\ -\ 3.8 \\ \hline \end{array}$	$\begin{array}{r} {}^{5} \\ \cancel{6}.{}^{1}2 \\ -\ 3.8 \\ \hline .4 \end{array}$	$\begin{array}{r} {}^{5} \\ \cancel{6}.{}^{1}2 \\ -\ 3.8 \\ \hline 2.4 \end{array}$
$6 + \frac{2}{10} = 5 + \frac{12}{10}$	$\frac{12}{10} - \frac{8}{10} = \frac{4}{10}$	$5 - 3 = 2$

1. Each example is worked correctly except for the decimal point. Copy the answers and place the decimal point correctly in each sum or difference.

A $\begin{array}{r} 7.6 \\ +8.9 \\ \hline 165 \end{array}$ B $\begin{array}{r} 3.69 \\ +1.54 \\ \hline 523 \end{array}$ C $\begin{array}{r} 0.632 \\ +0.819 \\ \hline 1451 \end{array}$

D $\begin{array}{r} 5.1 \\ -1.7 \\ \hline 34 \end{array}$ E $\begin{array}{r} 0.62 \\ -0.43 \\ \hline 19 \end{array}$ F $\begin{array}{r} 7.06 \\ -2.19 \\ \hline 487 \end{array}$

2. Find the sums.

A $\begin{array}{r} 8.38 \\ +6.75 \\ \hline \end{array}$ B $\begin{array}{r} 0.927 \\ +0.846 \\ \hline \end{array}$ C $\begin{array}{r} 32.8 \\ +65.4 \\ \hline \end{array}$ D $\begin{array}{r} 9.762 \\ +8.431 \\ \hline \end{array}$ E $\begin{array}{r} 64.35 \\ +74.69 \\ \hline \end{array}$

F $\begin{array}{r} 72.80 \\ +97.54 \\ \hline \end{array}$ G $\begin{array}{r} 8.346 \\ +7.52 \\ \hline \end{array}$ H $\begin{array}{r} 92.6 \\ +87.59 \\ \hline \end{array}$ I $\begin{array}{r} 600.4 \\ +738.7 \\ \hline \end{array}$ J $\begin{array}{r} 92.65 \\ +34.71 \\ \hline \end{array}$

K $\begin{array}{r} 87.4 \\ 65.2 \\ +93.1 \\ \hline \end{array}$ L $\begin{array}{r} 64.3 \\ 2.74 \\ +84.5 \\ \hline \end{array}$ M $\begin{array}{r} 52.74 \\ 6.5 \\ +23.88 \\ \hline \end{array}$

N $4.58 + 7.6 + 25.8$
O $0.832 + 5.26 + 39.1$
P $9.642 + 376 + 84.75$

3. Find the differences. Check each of your answers by addition.

A $\begin{array}{r} 6.4 \\ -2.8 \\ \hline \end{array}$ B $\begin{array}{r} 2.3 \\ -0.9 \\ \hline \end{array}$ C $\begin{array}{r} 8.6 \\ -1.9 \\ \hline \end{array}$ D $\begin{array}{r} 0.92 \\ -0.65 \\ \hline \end{array}$ E $\begin{array}{r} 0.83 \\ -0.26 \\ \hline \end{array}$

F $\begin{array}{r} 0.76 \\ -0.09 \\ \hline \end{array}$ G $\begin{array}{r} 6.82 \\ -0.63 \\ \hline \end{array}$ H $27.9 - 8.6$ I $0.832 - 0.57$

J $\begin{array}{r} 7.41 \\ -5.07 \\ \hline \end{array}$ K $\begin{array}{r} 6.95 \\ -2.9 \\ \hline \end{array}$

L $\begin{array}{r} 8.07 \\ -1.58 \\ \hline \end{array}$ M $\begin{array}{r} 0.930 \\ -0.307 \\ \hline \end{array}$

N $\begin{array}{r} 7.064 \\ -1.255 \\ \hline \end{array}$ O $\begin{array}{r} 7.602 \\ -0.009 \\ \hline \end{array}$

think

This is April for a certain year.

APRIL

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

May has 31 days and June has 30 days. In this year, on what day of the week is the last day of May? of June?

More practice, page A-27, Set 51

Let's solve some decimal problems.

Investigating the Ideas

Jan Jones kept a record of the number of litres and the cost of the gasoline she used in her car in one week.

	Litres	Cost
Mon.	7.8	$2.57
Wed.	13.5	$4.44
Sat.	16.6	$5.46

? Can you write and solve one addition and one subtraction problem from the chart?

Discussing the Ideas

1. Explain how decimals are used in symbols for money, such as $7.54.

2. What are some other uses of decimals?

3. Give the missing amounts and explain your answers.
   - A Since $6.59+8.23=14.82$, we know that $6.59 and $8.23 is |||||.
   - B Since $3.65+6.25=9.90$, we know that $3.65 and $6.25 is |||||.
   - C Since $4.87+7.38=12.25$, we know that $4.87 and $7.38 is |||||.
   - D Since $5.95-3.49=2.46$, we know that $5.95 less $3.49 is |||||.
   - E Since $10.00-6.98=3.02$, we know that $10.00 less $6.98 is |||||.

1. Find the total amounts.

A	B	C	D
$8.53	$0.96	$10.90	$34.59
4.27	4.34	5.38	86.79

2. Find the difference in the amounts.

A	B	C	D
$9.67	$6.75	$10.00	$50.00
4.83	0.86	3.67	46.72

3. A gasoline pump shows these amounts after 9.3 litres of gasoline have been put into a gas tank.

1.30	9.3
Dollars	Litres

A After 0.9 of a litre more goes into the tank, how many litres will the pump show ?

B If gasoline costs $0.14 per litre, how much will it cost to fill the tank with 10 litres ?

4. The picture shows the odometer reading before and after a trip. How many kilometres were travelled ?

5 9 4 6 8   6 0 3 2 5

5. A litre of pure water weighs 1 kilogram. A litre of gasoline weighs about 0.62 kilograms. Which weighs more ? How much more ?

6. Oil is lighter than water but heavier than gasoline.

A litre of oil weighs about 0.81 kg.

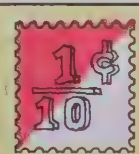
A How much more is the weight of a litre of water than a litre of oil ?

B How much more is the weight of a litre of oil than a litre of gasoline ?

C By how much does a litre of water, a litre of oil, and a litre of gasoline increase the weight of a car ?

★**7.** Mrs. Gomez spent $2.75 on fresh fruits and vegetables, $7.58 on meat, and $9.24 on other items. How much discount did she get ?

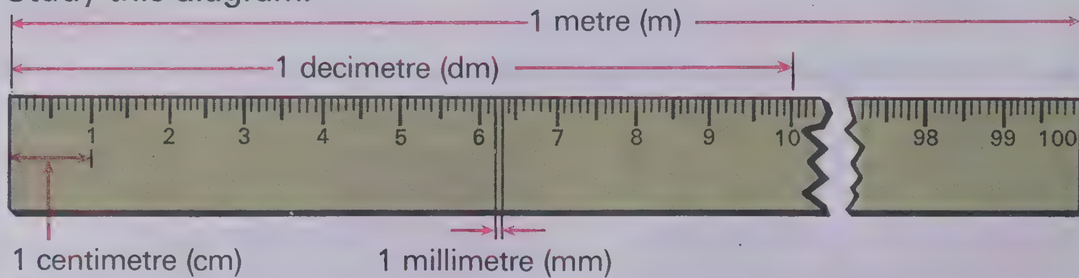
The store gives 1/10¢ discount for each 10-cent purchase.



More practice, page A-28, Set 52

Investigating the Ideas

Study this diagram.



How many of these measurements can you find?

- A Your height in metres.
- B The length of your shoe in decimetres.
- C The length and width of your mathematics book in centimetres.
- D The thickness of your pencil in millimetres.

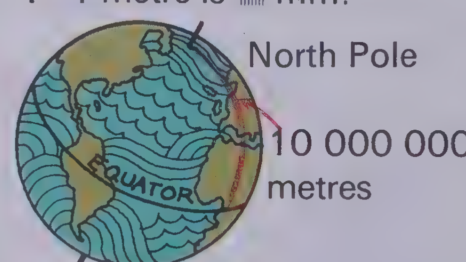
Discussing the Ideas

1. Janet found that her height was 147 centimetres. How could Janet write a decimal that would give her height in metres?

2. Use the diagram above to help you give the missing numbers.

   A 1 metre is |||| cm.
   B 1 decimetre is |||| cm.
   C 1 metre is |||| dm.
   D 1 centimetre is |||| mm.
   E 1 decimetre is |||| mm.
   F 1 metre is |||| mm.

3. A metre is one ten millionth of the distance from the North Pole to the equator.



   A About how many metres is it from the North Pole to the South Pole?
   B About how many metres is it all the way around the world?

1. Give the missing fractions.
   - A 1 dm is ||||| of a metre.
   - B 1 cm is ||||| of a metre.
   - C 1 mm is ||||| of a metre.

2. Give the correct decimal for each exercise.
   - A $\frac{1}{10}$ m = ||||| metre
     Answer: 0.1
   - B 1 dm = ||||| metre
   - C $\frac{1}{100}$ m = ||||| metre
   - D 1 cm = ||||| metre
   - E $\frac{1}{1000}$ m = ||||| metre
   - F 1 mm = ||||| metre
   - G $\frac{25}{100}$ m = ||||| metre
   - H 25 cm = ||||| metre

think

Ellen spent half her money at the drug store. She then spent half of what was left at the candy store.

1. What fractional part of her money has she spent?
2. If she has 20¢ left, how much did she start with?

3. Give the missing numbers.
   - A 0.001 metre = ||||| millimetre
   - B 0.01 metre = ||||| centimetre
   - C 0.1 metre = ||||| decimetre
   - D 0.007 metre = ||||| millimetre
   - E 0.06 metre = ||||| centimetre
   - F 0.8 metre = ||||| decimetre

4. Study the example. Then copy the sentences on your paper and give the missing numbers.
   Example: 6.254 m = 6 m, 2 dm, 5 cm, and 4 mm
   - A 7.834 m = ||||| m, ||||| dm, ||||| cm, and ||||| mm
   - B 8.203 m = ||||| m, ||||| dm, ||||| cm, and ||||| mm
   - C 9.640 m = ||||| m, ||||| dm, ||||| cm, and ||||| mm
   - D 7.023 m = ||||| m, ||||| dm, ||||| cm, and ||||| mm

★ 5. Find the totals so the number of millimetres, centimetres, and decimetres is less than 10.

   A
   2 m 5 dm 1 cm 8 mm
   4 m 7 dm 2 cm 6 mm
   ___

   B
   6 m 7 dm 9 cm 4 mm
   4 m 4 dm 8 cm 7 mm
   ___

1. Copy each exercise and give the missing numerators.

A $6.79 = 6 + \frac{\square}{10} + \frac{\square}{100}$

B $8.07 = 8 + \frac{\square}{10} + \frac{\square}{100}$

C $9.263 = 9 + \frac{\square}{10} + \frac{\square}{100} + \frac{\square}{1000}$

D $8.407 = 8 + \frac{\square}{10} + \frac{\square}{1000}$

2. Give the correct decimal for each sum.

A $2 + \frac{3}{10} + \frac{6}{100}$

B $75 + \frac{7}{10} + \frac{9}{100}$

C $1 + \frac{4}{10} + \frac{6}{100} + \frac{7}{1000}$

D $\frac{8}{10} + \frac{7}{100} + \frac{6}{1000}$

E $18 + \frac{6}{10} + \frac{0}{100} + \frac{7}{1000}$

F $18 + \frac{6}{10} + \frac{7}{1000}$

3. Give the fraction suggested by each decimal.

A 0.7 B 0.07 C 0.007 D 0.06 E 0.76 F 0.076

4. Give a mixed numeral for each decimal. Use 10, 100, or 1000 for your denominators.

A 17.6 B 38.23 C 29.07 D 6.124 E 18.062

5. Give the correct sign ($<$, $=$, $>$) for each $\square$.

A $7.8 \ \square \ 7.7$

B $6.4 \ \square \ 6.40$

C $0.32 \ \square \ 3.2$

D $0.67 \ \square \ 0.68$

E $8.32 \ \square \ 8.30$

F $83.2 \ \square \ 8.40$

G $0.832 \ \square \ 8.30$

H $832 \ \square \ 840$

I $0.005 \ \square \ 0.05$

6. Find the sums and differences.

A $\begin{array}{r} 6.4 \\ +8.7 \\ \hline \end{array}$ B $\begin{array}{r} 9.2 \\ +27.5 \\ \hline \end{array}$ C $\begin{array}{r} 27.5 \\ -9.2 \\ \hline \end{array}$ D $\begin{array}{r} 35.6 \\ +8.92 \\ \hline \end{array}$ E $\begin{array}{r} 72.4 \\ -26.7 \\ \hline \end{array}$ F $\begin{array}{r} 0.83 \\ +0.969 \\ \hline \end{array}$

G $5.34 + 23.7 + 58.60$

H $0.586 + 4.9 + 23.64$

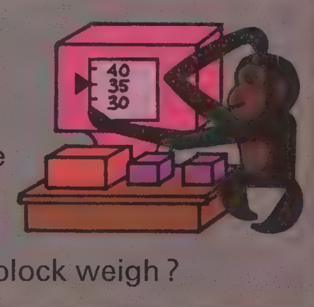
I $8.64 + 39.5 + 0.807$

★ **7.** Give a decimal for each fraction.

A $\frac{1}{2}$ B $\frac{1}{5}$ C $\frac{4}{5}$ D $\frac{1}{4}$

★ **8.** Give the lowest-terms fraction for 0.750.

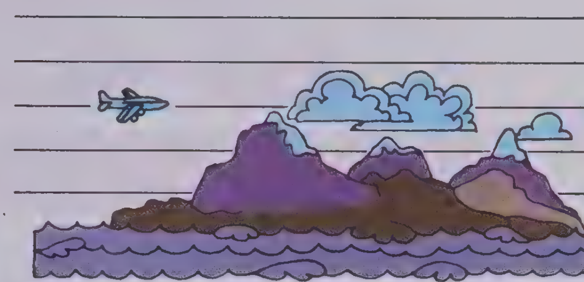
think



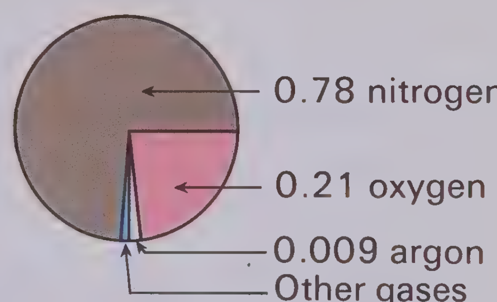
The large block weighs 3 kg more than the 2 small blocks together. What does each block weigh?

Land, Sea, and Air

The chart below shows how North America's land area and coastline is divided among 10 countries.



1. Canada and the United States make up what part of North America's land area?
2. If Mexico's land area is added to that of the U.S. and Canada, what part remains?
3. What part of North America's coastline do the 9 countries other than Canada share?
4. What part of North America's coastline do El Salvador, Guatemala, and the Honduras have?
5. Air surrounds us in a thin layer above the land and sea. Air is made up mostly of nitrogen and oxygen. The circle graph shows what part is nitrogen and what part is oxygen.



   A Nitrogen and oxygen together make up what part of the air?

   ★ B What part of the air is made up by other gases?

REGION	AREA	COASTLINE	REGION	AREA	COASTLINE
British Honduras	—	—	Honduras	0.01	—
Canada	0.45	0.73	Mexico	0.09	0.08
Costa Rica	—	0.01	Nicaragua	0.01	0.01
El Salvador	—	—	Panama	—	0.02
Guatemala	0.01	—	United States	0.42	0.14

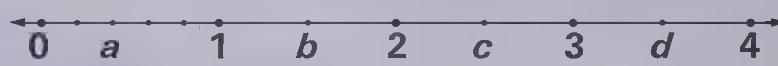
*No entry means that this part of the whole is less than 0.01.

1. Find the product, difference, and quotients.

A $\begin{array}{r} 5905 \\ -1767 \\ \hline \end{array}$ B $\begin{array}{r} 376 \\ \times 27 \\ \hline \end{array}$ C $7\overline{)36\,574}$ D $65\overline{)3445}$ E $87\overline{)4567}$

2. Choose the point on the number line for the fractional number.

A $\{\frac{5}{2}, \frac{10}{4}, \frac{15}{6}, \frac{20}{8}, \ldots\}$



B $\{\frac{7}{3}, \frac{14}{6}, \frac{21}{9}, \frac{28}{12}, \ldots\}$

3. Mark true (T) or false (F) for each exercise.

A $\frac{3}{4} = \frac{6}{8}$ D $\frac{0}{5} > \frac{0}{2}$ G $\frac{5}{5} < \frac{8}{8}$ J $\frac{1}{4} < \frac{4}{10}$ M $\frac{12}{4} = 3$

B $\frac{1}{4} = \frac{2}{6}$ E $3\frac{1}{7} < 4$ H $3\frac{1}{2} = \frac{7}{2}$ K $\frac{1}{5} < \frac{1}{6}$ N $\frac{15}{3} > 5$

C $\frac{3}{2} = 1\frac{1}{2}$ F $\frac{5}{2} = \frac{10}{2}$ I $\frac{3}{4} = \frac{4}{5}$ L $\frac{3}{7} = \frac{4}{10}$ O $4\frac{1}{8} > 4$

4. Find the sums and differences.

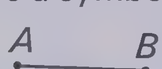
A $\frac{3}{4} + \frac{4}{5}$ C $\frac{12}{7} - \frac{6}{7}$ E $\frac{4}{5} + \frac{3}{10}$ G $\frac{4}{5} + \frac{1}{10}$

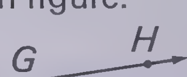
B $2\frac{1}{6} + 3$ D $5\frac{3}{4} - 2\frac{1}{4}$ F $\frac{1}{4} - \frac{1}{5}$ H $6\frac{1}{5} + 8\frac{3}{4}$

5. Find the totals. Write the answer so that you have the greatest possible number of the larger unit.

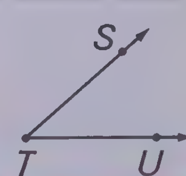
A $\begin{array}{r} \text{6 days 12 h} \\ \text{5 days 18 h} \\ \hline \end{array}$ B $\begin{array}{r} \text{24 wk 9 days} \\ \text{8 wk 12 days} \\ \hline \end{array}$ C $\begin{array}{r} \text{7 h 51 min} \\ \text{24 h 36 min} \\ \hline \end{array}$

6. Give a symbol for each figure.

A 

B 

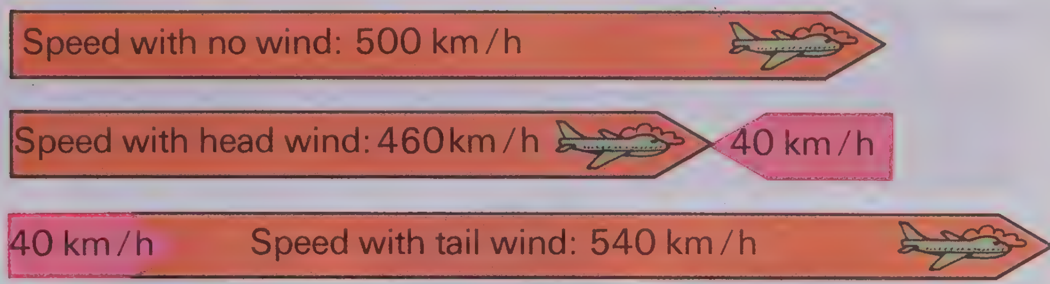
C 

D 

You are invited to explore

ACTIVITY CARD 13
Page 339

AIRPLANE SPEEDS



Each of the three airplanes would travel at the rate of 500 kilometres per hour if there were no wind. The airplane that has a 40-kilometre-per-hour head wind travels only 460 kilometres in one hour. The airplane that has a 40-kilometre-per-hour tail wind travels 540 kilometres in one hour.

1. Give the distance that each plane travels in one hour.

Plane	Tail wind	Speed without wind	Head wind
A	—	600	30
B	47	595	—
C	—	320	—
D	75	580	—
E	—	310	57

2. If an airplane travels 620 kilometres in one hour and would travel 580 kilometres per hour if there were no wind, is there a head wind or a tail wind? How fast is the wind?

3. Give the missing numbers in this chart.

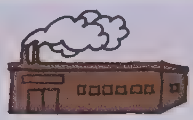
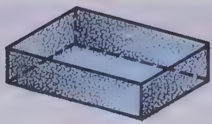
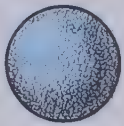
	Kilometres per hour	Hours	Kilometres
A	286	12	▯
B	▯	14	8022
C	▯	5	7835
★ D	586	▯	9962

★ 4. An airplane flies 600 km / h with no wind. If it flies $\frac{1}{2}$ hour with no wind, then flies 2 hours with a 50-km / h head wind, and finally flies $1\frac{1}{2}$ hours with a 20 km / h tail wind, how far does it fly?

13 Geometry and Measurement II

What are some basic space figures?

Investigating the Ideas

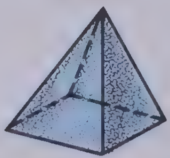
We **see** these physical objects.	We **think** about these space figures.	We **say** these names.
building 		rectangular prism
tank 		cylinder
Earth 		sphere

? How many other physical objects that remind you of these space figures can you name?

Discussing the Ideas

1. What simple name is often used for a
   A rectangular prism ? B sphere ?

2. What are some physical objects that remind you of these space figures?



rectangular pyramid

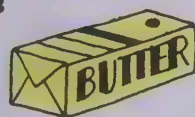
cone

3. Can you describe some space figures that are different from any of the ones shown on this page?

Using the Ideas

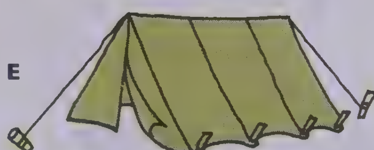
1. Which space figure below is suggested by each of these physical objects?

A

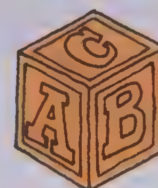

B


C

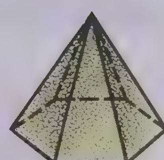

D

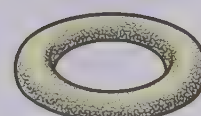

E


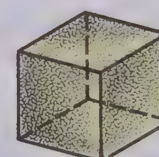
F


G


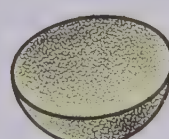

triangular prism

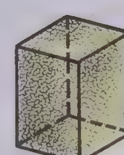

hexagonal pyramid


torus

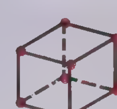

cube


truncated cone


hemisphere


square prism

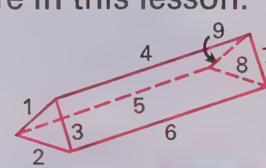
2. A cube has 8 **vertices.**



   A Which other figures have 8 vertices?

   B Give the number of vertices for each figure in this lesson.

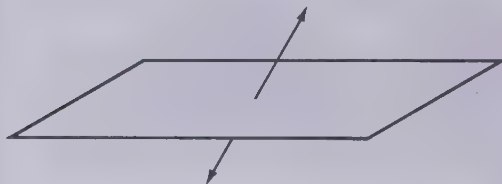
3. A triangular prism has 9 **edges.** Give the number of edges for each figure.



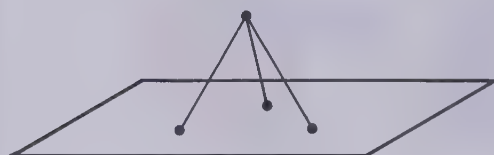
4. A cylinder has 2 flat **faces.** Give the number of flat faces each figure has.

Let's explore space relationships.

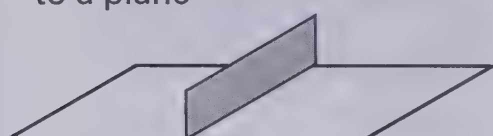
For each space figure on the left, give the letter of the physical object which best reminds you of the figure.

1. 

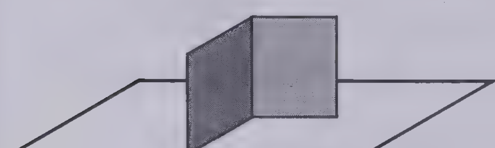
A line intersecting a plane

2. 

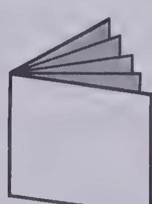
3 segments from a point to a plane

3. 

A plane through a line on another plane

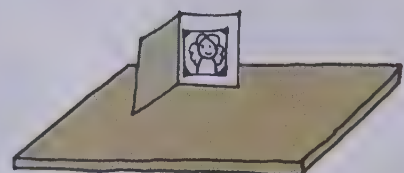
4. 

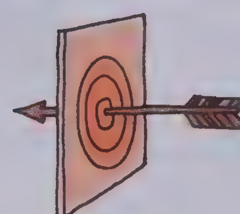
3 planes through a single point

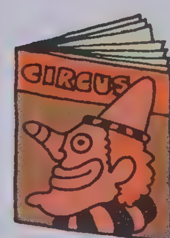
5. 

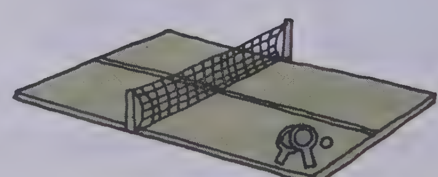
Several planes through one line

A 

B 

C 

D 

E 

Tell which picture best reminds you of the space figure that is described.

1. Two lines in space that do not meet and that are not in one plane

2. Four planes

3. Many lines in space, all going in the same direction

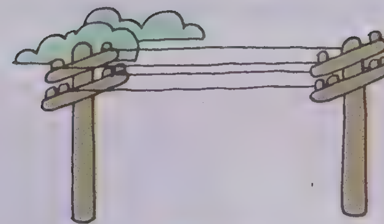
4. Two lines in one plane that do not cross

5. A circle touching a plane at one point

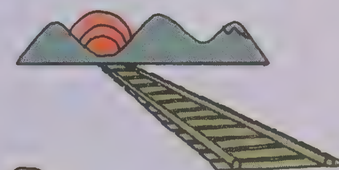
6. A plane, a circle, a sphere, and a truncated cone

7. A sphere on a plane

A

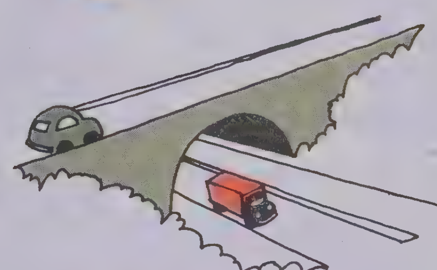


B

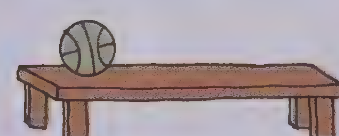


C

D



E



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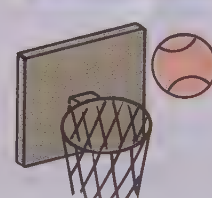
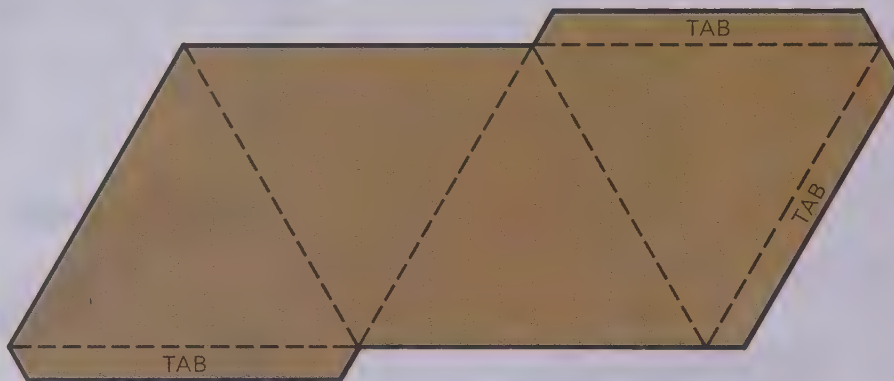


Investigating the Ideas



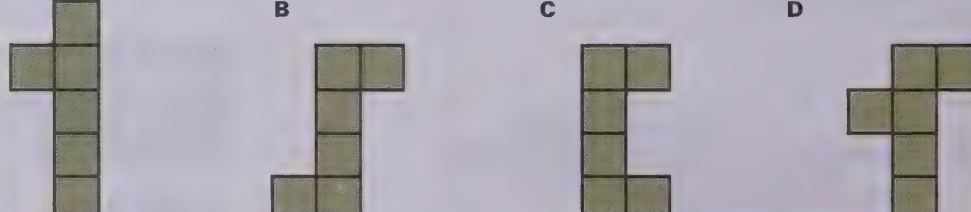
? Can you trace the pattern above, cut it along the solid lines, and fold on the dotted lines to form a space figure?

Discussing the Ideas

1. A What is the name of the space figure you made in the Investigation?
   B How many vertices does your figure have?
   C How many faces? How many edges?

2. Which of the patterns below do you think would form a **cube** if cut out and folded?

A B C D

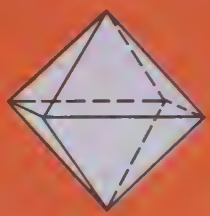


3. How many faces, edges, and vertices does this triangular prism have?

Trace the figures and make the space figures shown.

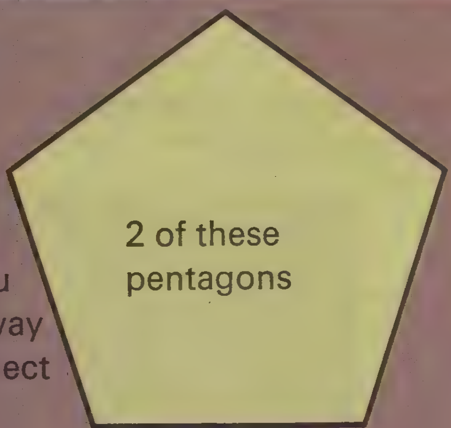
1

Can you connect 8 triangles like those in the Investigation to make this regular octahedron?

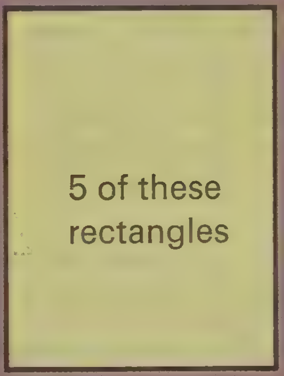


2

Can you find a way to connect



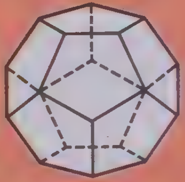
and



to make a pentagonal prism? →

★3

Can you connect 12 pentagons like those in exercise 2 to make this regular dodecahedron?



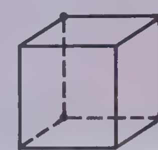
Investigating the Ideas

? Can you follow these directions and draw a larger picture of each of these space figures?

Cube

Draw a square for the "front" face.

Draw the corners of a square for the back face.

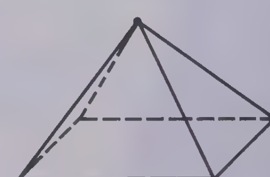


Make "hidden" edges dotted. Make other edges solid.

Rectangular Pyramid

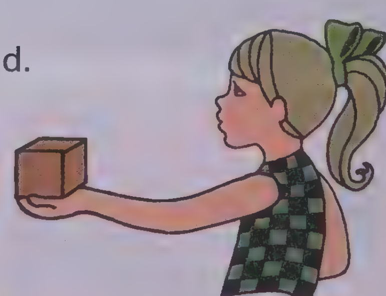
Draw a parallelogram, half solid, half dotted.

Mark the "top" vertex.

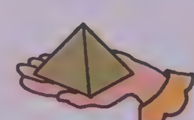


Make "hidden" edges dotted. Make other edges solid.

Discussing the Ideas

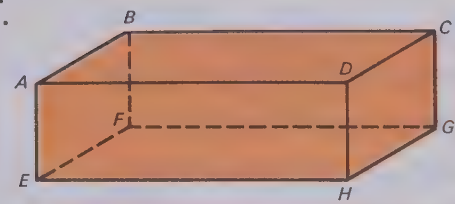


1. Suppose you hold a cube in your hand.
   A What is the greatest number of edges that could be hidden from your view?
   B What is the smallest number?



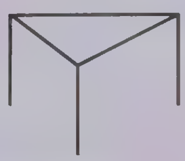
2. Could you hold a pyramid so that you can see all of its edges at once? Explain.

1. The front face of the rectangular prism shown here is named *ADHE*.
   A Name all the other faces.
   B Name the edges.
   C Which faces are hidden?
   D Which edges are hidden?

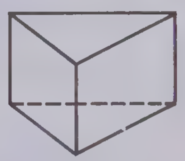


2. Follow the steps below and draw a **triangular prism.**

Draw a triangle.



Draw 3 parallel, congruent segments.



Make hidden edges dotted. Make other edges solid.

think

Draw and cut out a circle. Fold the paper so that the circle is divided into 12 equal parts. Label your circle like this:

If you start at 1 and draw to the dot 5 spaces away each time until you get back to 1, what design will you make? Can you make other designs like this?

3. Draw a triangular pyramid. Example 2 in the Investigation may help you.

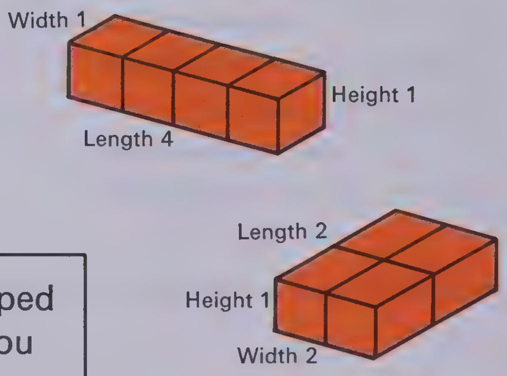
4. Draw a rectangular prism. Problem 1 above may help you.

5. Draw a cylinder.

★ 6. Pick out an object, such as a doghouse or a piano, and draw a picture of it. Be sure to include dotted lines to show hidden edges.

Investigating the Ideas

Here are the only two different-shaped boxes (rectangular prisms) that can be made from 4 cubes.



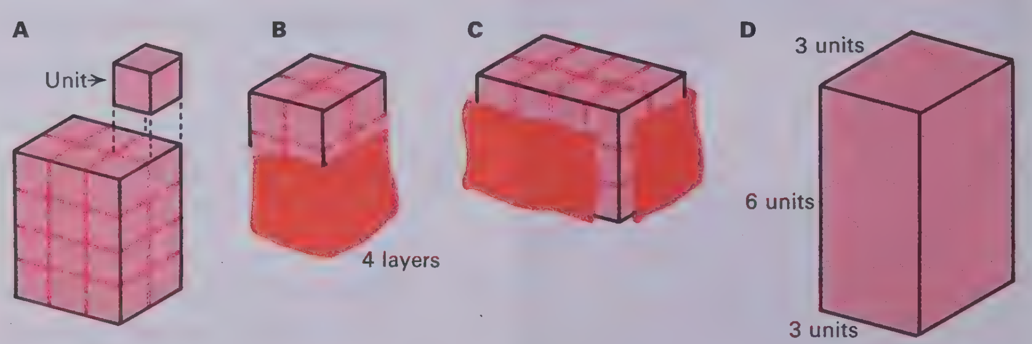
? How many different-shaped rectangular prisms can you make with 12 cubes?

Discussing the Ideas

1. The **dimensions** of a rectangular prism are its **length**, **width**, and **height**. What are the dimensions of each rectangular prism you found in the Investigation?

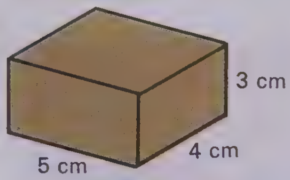
2. What would be the dimensions of a cube built from eight cubes?

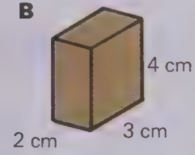
3. We count cubes (**cubic units**) to find the **volume** of space figures. Suppose you have made these figures with cubes. How would you find the volume of each figure?



4. What are the dimensions of each figure in exercise 3? Can you figure out a shortcut for finding the volume of a rectangular prism if you know the dimensions of the prism?

1. Use the dimensions shown on each figure to find the volume.

A


B


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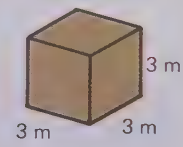

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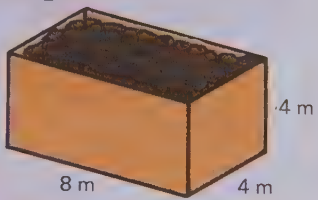
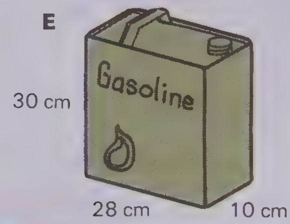


E


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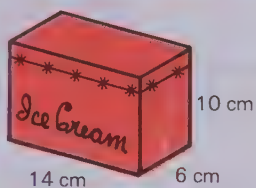

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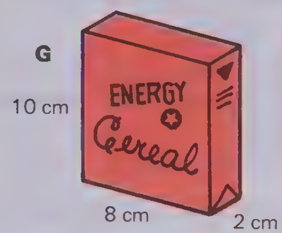
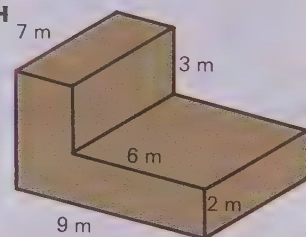


★ H


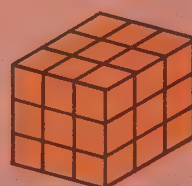
2. A Which holds more, the ice-cream carton or the box of cereal? What is the difference?

B A litre occupies about 1000 cubic centimetres. Does the gasoline can in exercise 1 E hold more or less than 9 litres?

C Does the wood box hold more or less than the L-shaped carton? What is the difference?

think

Suppose you used 27 blocks to form a cube. You painted its faces orange. Then you took it apart.

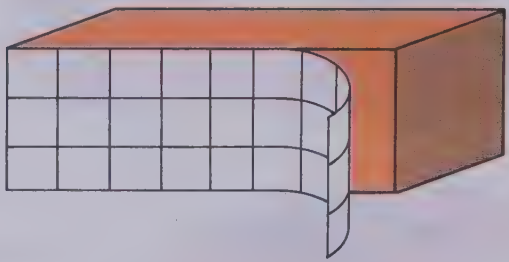


1. How many of the 27 blocks would have 4 or more orange faces? just 3 orange faces? just 2 orange faces? just 1 orange face? no orange faces?

2. Can you answer these questions for a 4-by-4-by-4 cube?

Investigating the Ideas

Use a small box. Cut out pieces of graph paper to cover the entire surface of the box.

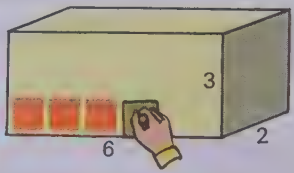


? Can you count how many square units it takes to cover the box?

Discussing the Ideas

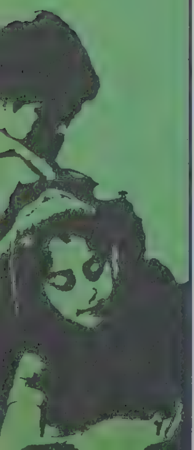
1. The number of square units it takes to "cover" the surface of a space figure is called the **surface area** of the figure. What is the surface area of a 1-unit cube? Why?

2. Suppose you use an inked stamper that prints a 1-unit square each time you press it against this box.
   A If the squares don't overlap, how many times will you have to stamp to cover the box completely?
   B What is the surface area of the box?



3. Can you figure out another way to find the surface area of the box in exercise 2?

4. Can you make up a problem which could be solved by finding the surface area of an object?

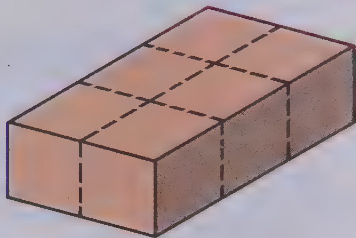


Using the Ideas

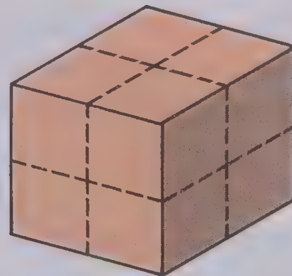
The square centimetre  is the unit for these exercises.

1. Find the surface area of each figure.

A



B



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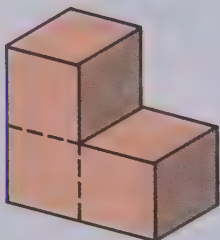
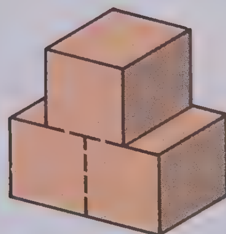
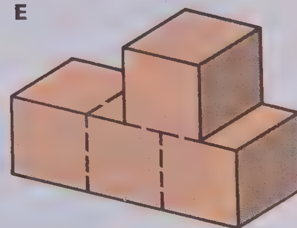


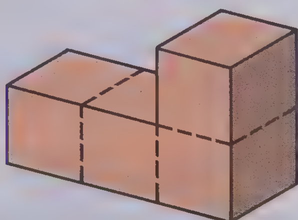
D



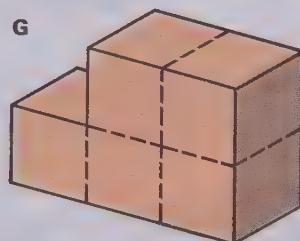
E



F

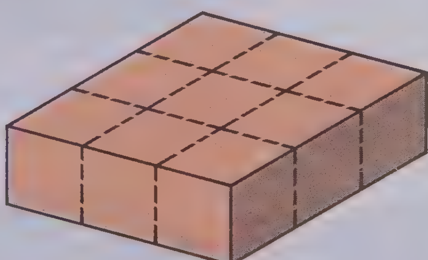


G

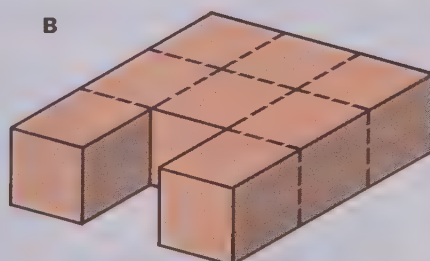


2. Give the volume and surface area of each figure.

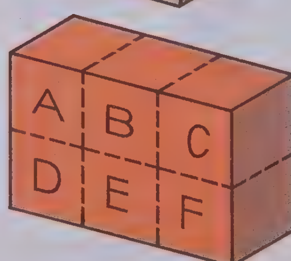
A

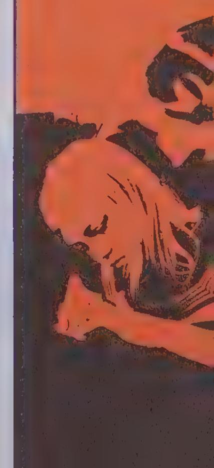


B



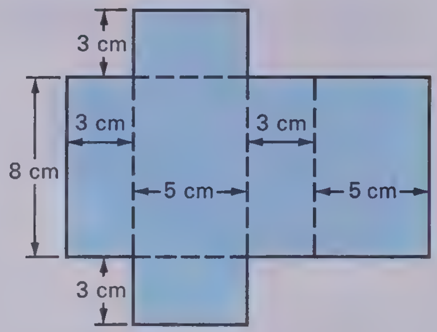
★ 3. The volume of this figure is 6. The surface area is 22. Give the letter of a cube that you could remove so the volume would be 5 and the surface area would still be 22.

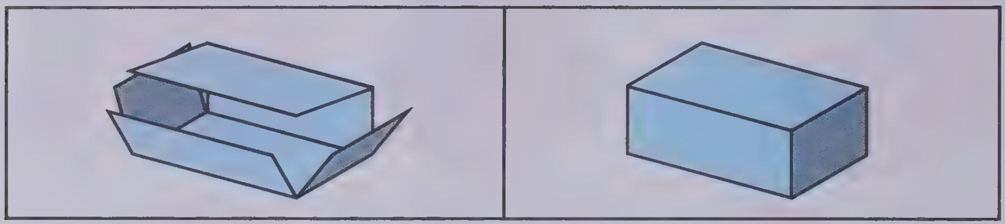




How do volume and surface area compare?

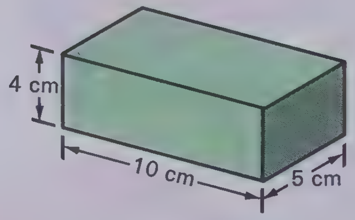
1. A Draw a figure like this. It is made of rectangles. The lengths and widths are shown on the drawing.
   B What is the area of the figure?
   C Cut out the figure and fold on the dotted lines as shown below. Use cellophane tape to fasten the sides together.





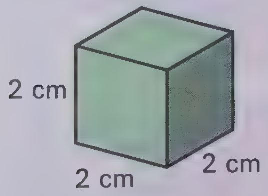
   D What is the **surface area** of the figure?
   E What is the **volume** of the figure?

2. Think about the rectangles that form the surface of a figure with the length, width, and height as shown. Find the surface area of this figure.

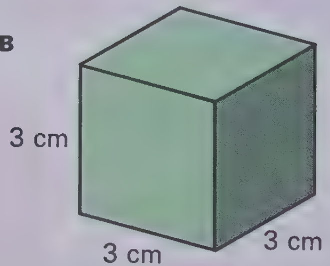


3. Find the volume and surface area of each figure.

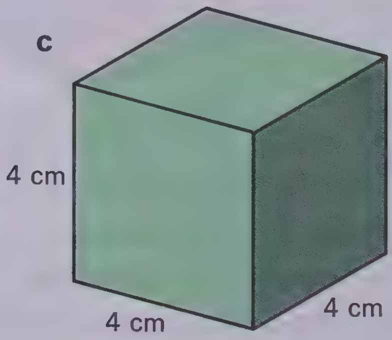
A



B

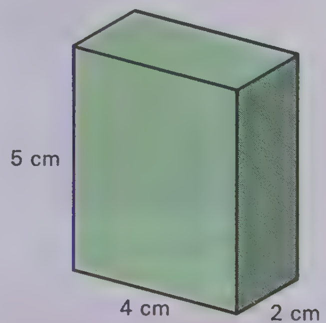


C

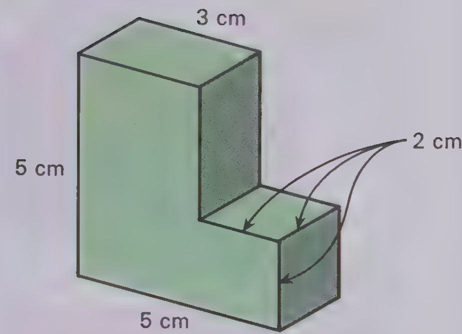


4. Find the **surface area** of each figure.

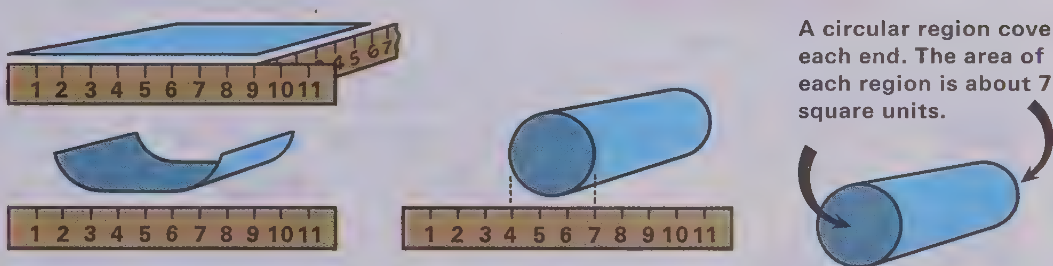
A



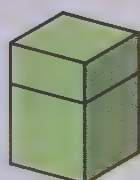
B



★ 5. Use the information given in the pictures to find the **surface area** of a cylinder that is constructed as shown.



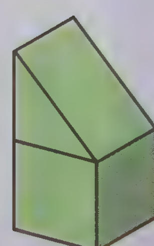
★ 6. The volume of this figure is $1\frac{1}{2}$. What is the surface area ?



★ 7. Give the surface area of the tallest possible tower made of 100 cubic centimetres.

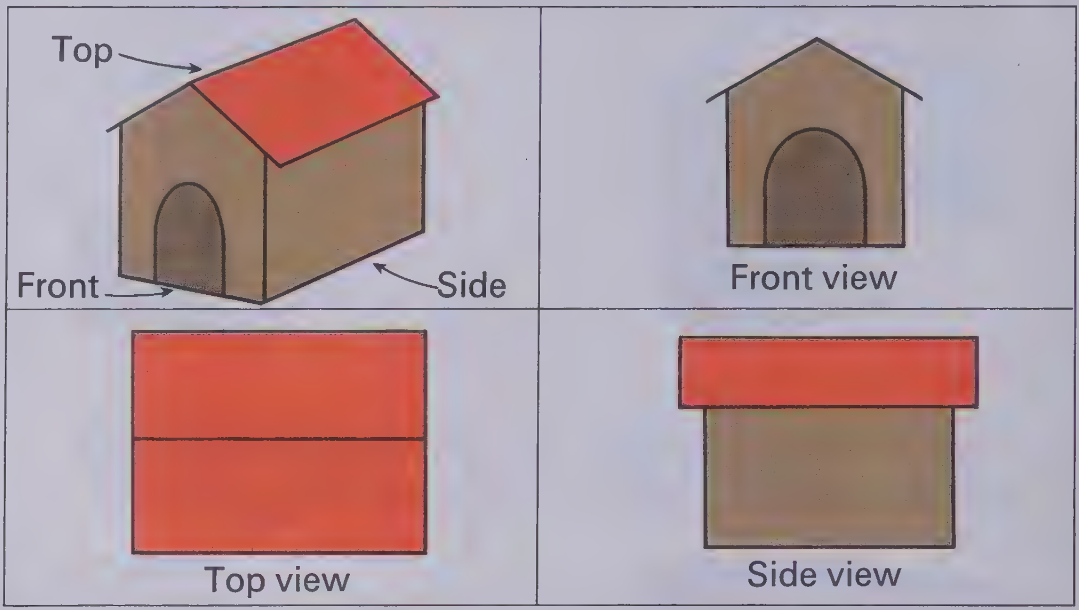
★ 8. The volume of this figure is $1\frac{1}{2}$. Is the surface area of the figure

A more than 8 ? B less than 8 ? C equal to 8 ?



Investigating the Ideas

Here are a **front view**, a **top view**, and a **side view** of a doghouse.



Can you draw the front, top, and side views of


a juice can?


a 2-holed brick?

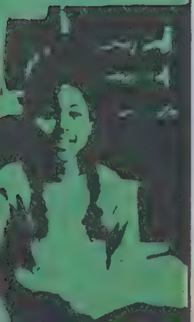

a table?

Discussing the Ideas

1. A Which objects above have front and side views that are the same?
   B Which objects have side and front views that are both rectangles?

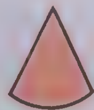
2. When do you think drawings of front, top, and side views might be used?

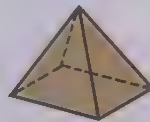
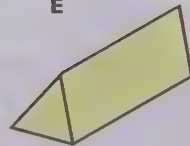
3. Can you describe objects with two views the same? all three views the same?



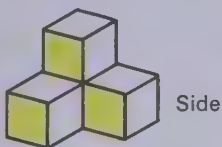
1. Draw the top view of each of these figures.

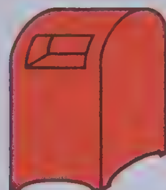
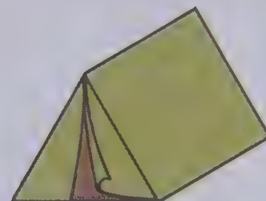
A

B

C

D

E


2. Draw the side view of each of these.

A


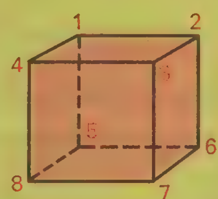
B

C


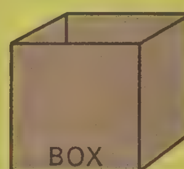
3. Draw the front views of the figures in exercises 1B, 1C, 1D, 2A, 2B.

4. A Describe a figure with front and side views the same.
   B Describe a figure (different from A) with top and front views the same.
   C Describe a figure (different from A and B) with all 3 views the same.

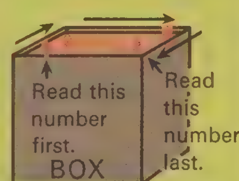
★ 5. Choose an object from your home or school (different from those mentioned in this lesson). Make accurate drawings of the front, side, and top views.

think





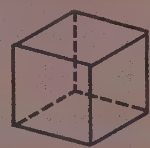
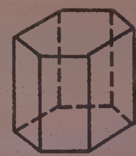
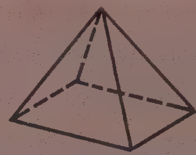
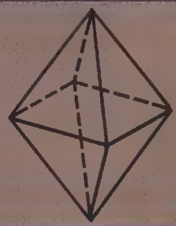
Suppose each vertex of this cube has a number. Here is one way the cube can be placed into this box. We call it **Position 1234**. How many other positions can you find?



Let's explore faces, vertices, and edges.

Investigating the Ideas

Study the figures in the table.

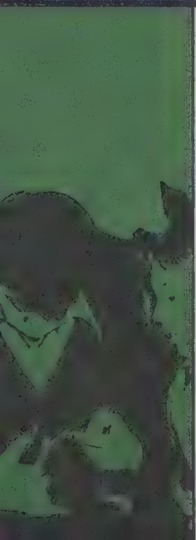
Figure				
Name	cube	hexagonal prism	rectangular pyramid	regular octahedron
Number of faces	?	8	?	?
Number of vertices	?	?	5	?
Number of edges	?	?	?	12

? Can you copy and complete the blue part of the table ?

Discussing the Ideas

1. Do you see any interesting patterns or relationships in your completed table ?

2. A regular icosahedron has 20 faces and 12 vertices. Without a model or picture, can you find out how many edges it has ? (Hint: For each figure in your table, add the number of faces to the number of vertices. Is this sum close to the number of edges ? How close ?)



1.

Triangular pyramid | Square pyramid | Pentagonal pyramid | Hexagonal pyramid | Octagonal pyramid

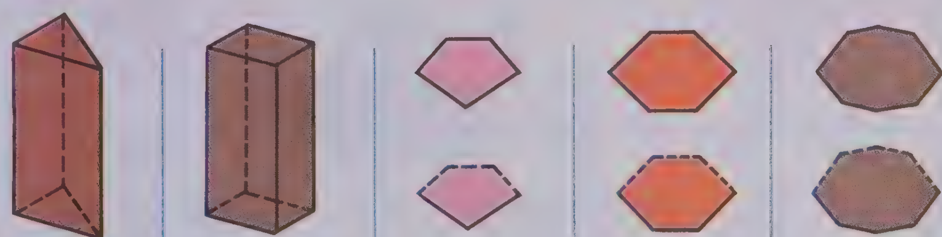
A How many vertices, faces, and edges does the triangular pyramid have?

B Trace and complete the drawing of the square pyramid. How many vertices, faces, and edges does the square pyramid have?

C Do you think each pyramid has the same number of faces as vertices? Trace and complete the other drawings to check your guess.

D Write down the number of vertices and faces for each pyramid. Without counting, give the number of edges for each pyramid.

2.

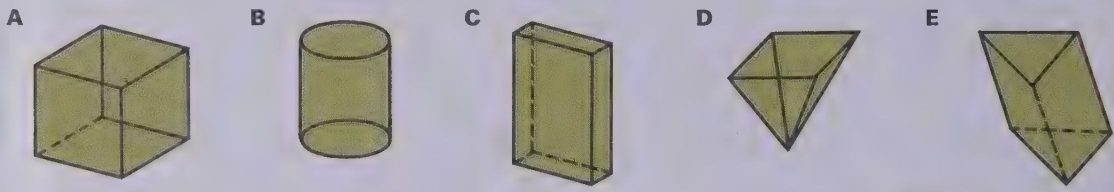


Triangular prism | Rectangular prism | Pentagonal prism | Hexagonal prism | Octagonal prism

A A triangle has 3 sides. A triangular prism has 9 edges. (Check it.) A rectangle has 4 sides. A rectangular prism has 12 edges. (Check it.) A pentagon has 5 sides. A pentagonal prism has __?__ edges.

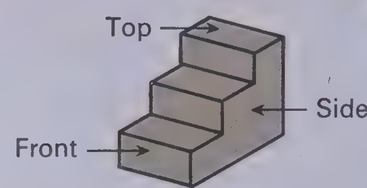
B How many faces, vertices, and edges do hexagonal and octagonal prisms have? Trace and complete the drawings to check your answers.

1. Which of these space figures is drawn correctly?

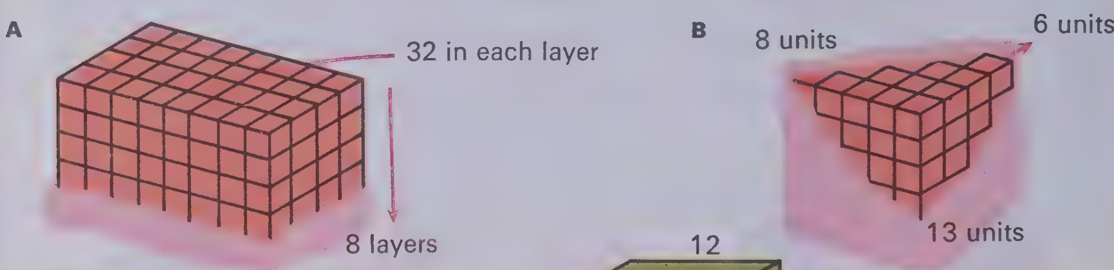


2. Draw a triangular pyramid.

3. Draw the **front view, top view,** and **side view** of these "stairs."

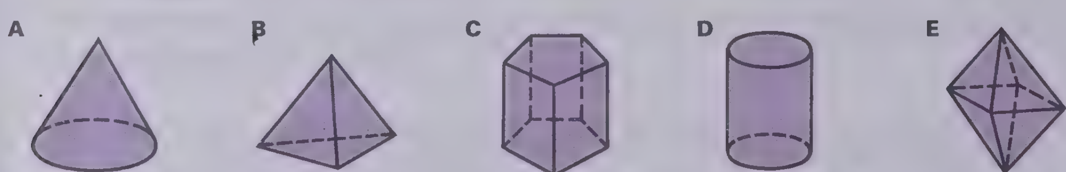


4. Find the volume of the figure indicated in each exercise.

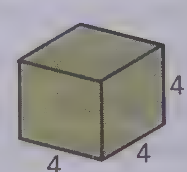


5. Find the surface area of the box.

6. Give the name of each space figure below.



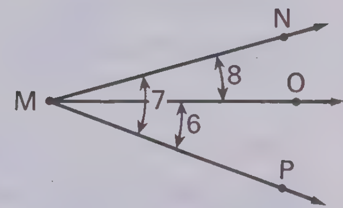
7. What are the volume and the surface area of a cube that measures 4 cm on each edge?



1. Find the sums and differences.

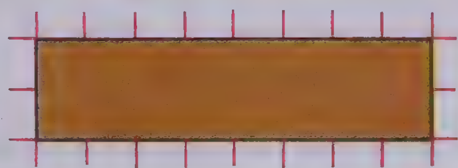
A $\begin{array}{r} 13\frac{7}{10} \\ +9\frac{3}{4} \\ \hline \end{array}$ B $\begin{array}{r} 15\frac{3}{4} \\ -8\frac{7}{10} \\ \hline \end{array}$ C $\begin{array}{r} 58\frac{7}{10} \\ +39\frac{1}{5} \\ \hline \end{array}$ D $\begin{array}{r} 6\frac{3}{4} \\ -2\frac{1}{5} \\ \hline \end{array}$ E $\begin{array}{r} 3\frac{3}{10} \\ +9\frac{4}{5} \\ \hline \end{array}$ F $\begin{array}{r} 16\frac{1}{2} \\ -2\frac{2}{5} \\ \hline \end{array}$

2. A Give the number name for $\angle NMO$.
   B What is the number name for $\angle NMP$?
   C What is the number name for $\angle OMP$?

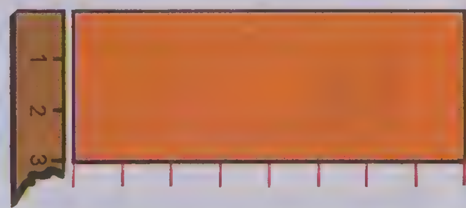


3. Use the unit shown to find the area of each rectangle.

A



B



4. Give the numbers for the gray screens.

Products		× ÷		Quotients
8	4	× ÷	2	2
27		× ÷		3
32		× ÷		2
6		× ÷		6
49		× ÷		1

think

Suppose ★ is an operation so that

$a \star b = (a \times b) + b$.

Example: $3 \star 4 = (3 \times 4) + 4 = 16$

1. Solve the equations:
   A $5 \star 0 = n$ C $5 \star 1 = n$
   B $0 \star 5 = n$ D $1 \star 5 = n$
2. Choose numbers a, b, and c to show that the grouping principle does not hold for ★.
   $(a \star b) \star c = a \star (b \star c)$

You are invited to explore

ACTIVITY CARD 14
Page 340

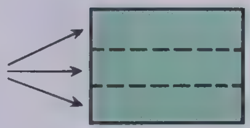
14 Multiplication and Division of Fractional Numbers

Let's explore regions and multiplication.

Investigating the Ideas

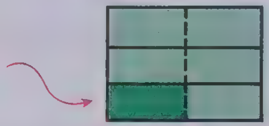
This paper was folded into thirds.

thirds



After one more folding, the paper shows half of one third.

$\frac{1}{2}$ of $\frac{1}{3}$

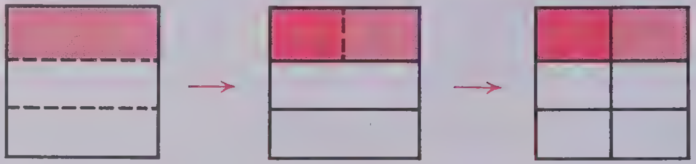


? Can you fold and color a paper to show each of these?
$\frac{1}{2}$ of $\frac{1}{2}$ $\frac{1}{2}$ of $\frac{1}{4}$ $\frac{1}{2}$ of $\frac{1}{8}$

Discussing the Ideas

Study example A. Then explain example B and solve the equation.

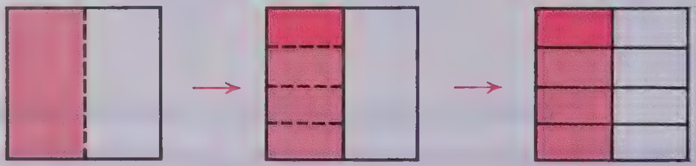
A



$\frac{1}{3}$ of the region is shaded pink. $\frac{1}{2}$ of $\frac{1}{3}$ is shaded red. $\frac{1}{6}$ of the region is shaded red.

We write:
$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

B

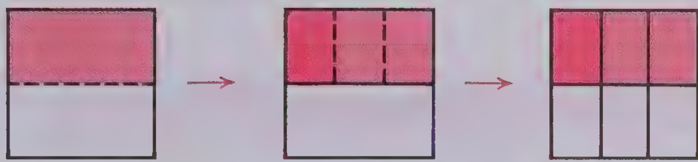


$\frac{1}{2}$ of the region is shaded pink. $\frac{1}{4}$ of $\frac{1}{2}$ is shaded red. $\frac{1}{8}$ of the region is shaded red.

We write:
$\frac{1}{4} \times \frac{1}{2} = n$

Using the Ideas

1. Give the fraction of the region that is shaded red. Then copy the equation and give the product n.

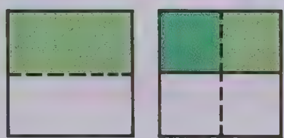


$\frac{1}{3} \times \frac{1}{2} = n$

$\frac{1}{2}$ of the region is shaded pink. $\frac{1}{3}$ of $\frac{1}{2}$ is shaded red. ▯ of the region is shaded red.

2. Study the figures. Then give the product.

A



$\frac{1}{2}$ $\frac{1}{2}$ of $\frac{1}{2}$

$\frac{1}{2} \times \frac{1}{2} = n$

B

$\frac{1}{2}$ $\frac{1}{5}$ of $\frac{1}{2}$

$\frac{1}{5} \times \frac{1}{2} = n$

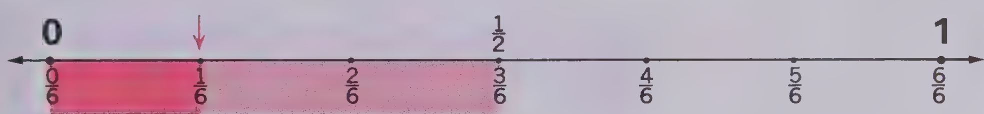
C

$\frac{1}{4}$ $\frac{1}{3}$ of $\frac{1}{4}$

$\frac{1}{3} \times \frac{1}{4} = n$

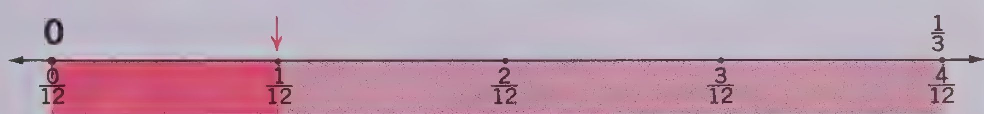
3. Study the number-line picture. Then copy the equation and give the product.

A



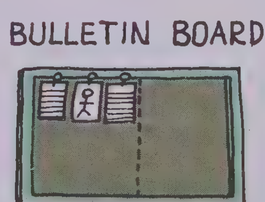
$\frac{1}{3}$ of $\frac{1}{2}$ We write: $\frac{1}{3} \times \frac{1}{2} = n$

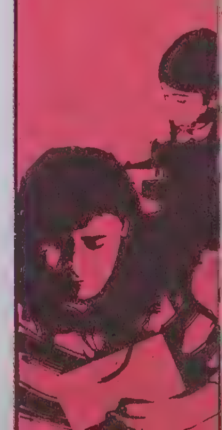
B



$\frac{1}{4}$ of $\frac{1}{3}$ We write: $\frac{1}{4} \times \frac{1}{3} = n$

4. The class used $\frac{1}{2}$ of the bulletin board for a science display. Sue's papers took $\frac{1}{3}$ of the space. What part of the whole board did Sue's papers cover? Write and solve an equation.

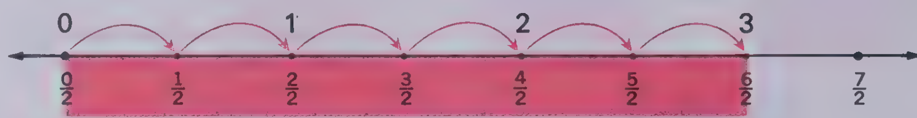




How can you find the product of a whole number and a fraction?

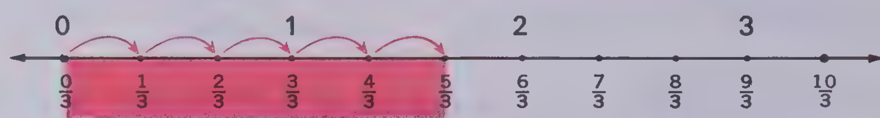
Investigating the Ideas

A



6 jumps of $\frac{1}{2}$ We write: $6 \times \frac{1}{2} = \frac{6}{2}$

B

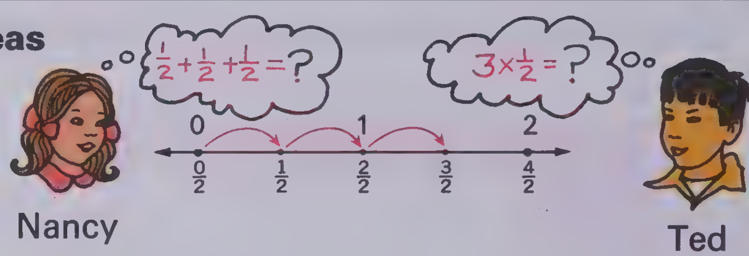


5 jumps of $\frac{1}{3}$ We write: $5 \times \frac{1}{3} = \frac{\square}{\square}$

What is the product for number line B?

> **?** Can you draw number lines and write the multiplication equations for these jumps?
> 7 jumps of $\frac{1}{4}$ 9 jumps of $\frac{1}{2}$

Discussing the Ideas



1. A Can you explain how Nancy is thinking about the number-line picture?
   B How is Ted thinking about it?
   C Do you think Nancy and Ted will get the same answer?

2. What multiplication equation does each number line suggest?

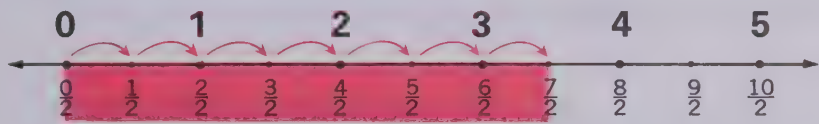
5 jumps of $\frac{1}{2}$

Jumping $\frac{1}{2}$ of 5

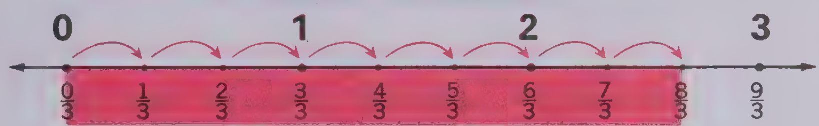
1. Study the number-line picture. Then copy the equation and give the product.

A 

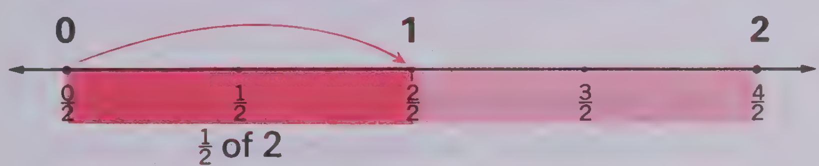
$3 \times \frac{1}{5} = n$

B 

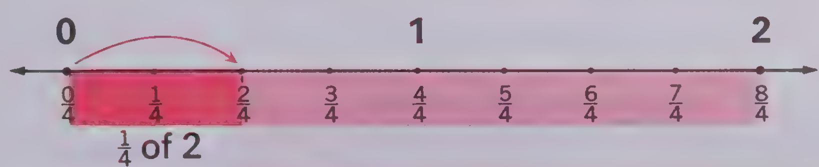
$7 \times \frac{1}{2} = n$

C 

$8 \times \frac{1}{3} = n$

D 

$\frac{1}{2} \times 2 = n$

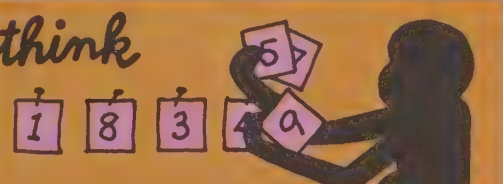
E 

$\frac{1}{4} \times 2 = n$

2. Solve the equations.

$5 \times \frac{1}{3} = \frac{5}{3}$	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
A $3 \times \frac{1}{2} = n$	G $\frac{1}{3} \times \frac{1}{4} = n$
B $7 \times \frac{1}{2} = n$	H $\frac{1}{2} \times \frac{1}{5} = n$
C $5 \times \frac{1}{6} = n$	I $\frac{1}{3} \times \frac{1}{3} = n$
D $9 \times \frac{1}{4} = n$	J $\frac{1}{6} \times \frac{1}{2} = n$
E $\frac{1}{2} \times 8 = n$	K $\frac{1}{4} \times \frac{1}{5} = n$
F $\frac{1}{7} \times 12 = n$	L $\frac{1}{7} \times \frac{1}{3} = n$

think



Find 3 different whole numbers, each less than 10, so that their product is equal to their sum.

Can the basic principles be used to find products?

Discussing the Ideas

1. Read and complete each of the **basic principles** for multiplication of fractional numbers.

   A THE 1 PRINCIPLE

   When you choose a fractional number and multiply by 1, the product is ______?______.

   B THE COMMUTATIVE PRINCIPLE (ORDER PRINCIPLE)

   When you multiply two fractional numbers, the order of the factors does not affect the ______?______.

   C THE ASSOCIATIVE PRINCIPLE (GROUPING PRINCIPLE)

   When you multiply fractional numbers, you can change the grouping and get the same ______?______.

2. Give the missing word.

   According to the order and grouping principles for multiplication of fractional numbers, you can rearrange the ______?______ in any way and get the same product.

3. Study the diagram and give the missing number.

   Find the product of these factors.

   $$\frac{2}{3} \times \frac{3}{4} = \left(2 \times \frac{1}{3}\right) \times \left(3 \times \frac{1}{4}\right) = 6 \times \frac{1}{12} = \boxed{}$$

   Find the product of these factors.

4. Can you give a rule for finding the product of two fractional numbers?

1. Give the products.

A $5 \times \frac{1}{3}$ D $5 \times \frac{1}{4}$ G $\frac{1}{4} \times \frac{1}{8}$ J $\frac{1}{5} \times \frac{1}{4}$

B $3 \times \frac{1}{2}$ E $\frac{1}{3} \times \frac{1}{4}$ H $\frac{1}{5} \times \frac{1}{6}$ K $\frac{1}{6} \times \frac{1}{3}$

C $4 \times \frac{1}{3}$ F $\frac{1}{5} \times \frac{1}{3}$ I $\frac{1}{8} \times \frac{1}{3}$ L $\frac{1}{5} \times \frac{1}{7}$

2. Give the products.

A $\frac{3}{4} \times \frac{2}{3} = (3 \times 2) \times (\frac{1}{4} \times \frac{1}{3}) = n$

B $\frac{4}{3} \times \frac{5}{2} = (4 \times 5) \times (\frac{1}{3} \times \frac{1}{2}) = n$

C $\frac{4}{5} \times \frac{2}{3} = (4 \times 2) \times (\frac{1}{5} \times \frac{1}{3}) = n$

D $\frac{3}{8} \times \frac{5}{4} = (3 \times 5) \times (\frac{1}{8} \times \frac{1}{4}) = n$

E $\frac{7}{2} \times \frac{2}{3} = (7 \times 2) \times (\frac{1}{2} \times \frac{1}{3}) = n$

F $\frac{5}{6} \times \frac{3}{2} = (5 \times 3) \times (\frac{1}{6} \times \frac{1}{2}) = n$

G $\frac{3}{4} \times \frac{7}{2} = (3 \times 7) \times (\frac{1}{4} \times \frac{1}{2}) = n$

H $\frac{5}{6} \times \frac{5}{4} = (5 \times 5) \times (\frac{1}{6} \times \frac{1}{4}) = n$

I $\frac{4}{3} \times \frac{3}{7} = (4 \times 3) \times (\frac{1}{3} \times \frac{1}{7}) = n$

J $\frac{5}{8} \times \frac{4}{3} = (5 \times 4) \times (\frac{1}{8} \times \frac{1}{3}) = n$

K $\frac{6}{4} \times \frac{3}{5} = (6 \times 3) \times (\frac{1}{4} \times \frac{1}{5}) = n$

L $\frac{4}{3} \times \frac{5}{3} = (4 \times 5) \times (\frac{1}{3} \times \frac{1}{3}) = n$

3. Give the products.

A $\frac{3}{4} \times \frac{2}{5}$ H $\frac{5}{6} \times \frac{3}{8}$

B $\frac{5}{3} \times \frac{3}{2}$ I $\frac{8}{3} \times \frac{3}{4}$

C $\frac{2}{7} \times \frac{5}{2}$ J $\frac{3}{4} \times \frac{5}{8}$

D $\frac{4}{5} \times \frac{7}{3}$ K $\frac{5}{6} \times \frac{7}{4}$

E $\frac{3}{8} \times \frac{2}{5}$ L $\frac{4}{7} \times \frac{7}{4}$

F $\frac{5}{4} \times \frac{3}{5}$ M $\frac{5}{5} \times \frac{3}{5}$

G $\frac{2}{3} \times \frac{2}{3}$ N $\frac{0}{7} \times \frac{3}{8}$

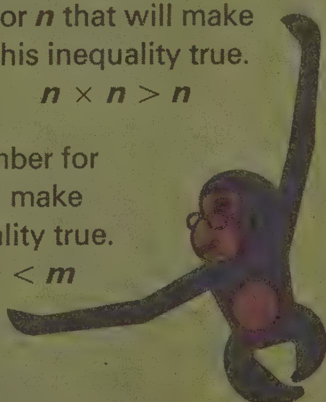
think

Give a number for n that will make this inequality true.

$n \times n > n$

Give a number for m that will make this inequality true.

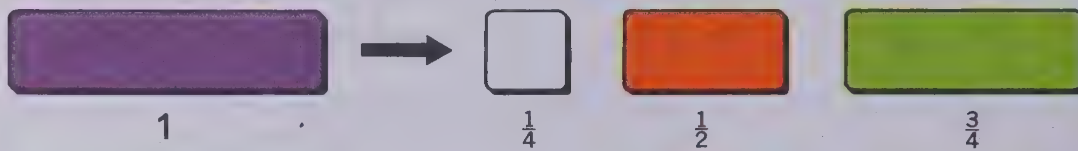
$m \times m < m$



More practice, page A-28, Set 53

Investigating the Ideas

Use the purple strip as the unit.



How many white strips will "match" the purple train?

Can you make a red or light green train that matches the purple train?

Discussing the Ideas

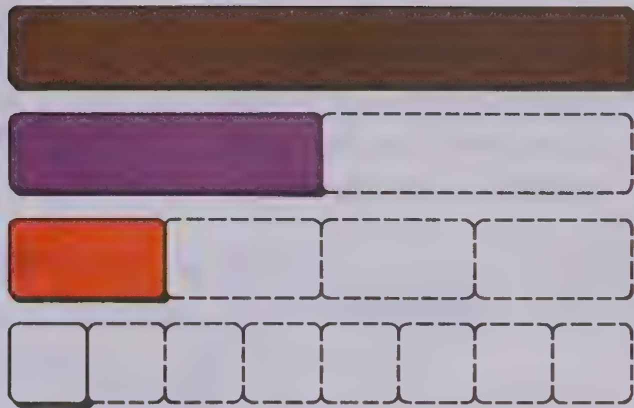
1. A How many white strips did it take to match the 3 purple strips?
   B How many fourths are in 3?
   C Solve: $3 \div \frac{1}{4} = n$

2. A How many halves are in 3?
   B Solve: $3 \div \frac{1}{2} = n$

3. A How many three-fourths are in 3?
   B Solve: $3 \div \frac{3}{4} = n$

4. Can you use your strips to find how many fourths are in three halves?

1. Give the missing fractions. When the brown strip is 1,



   A the purple strip is ▒.
   B the red strip is ▒.
   C the white strip is ▒.

2. A How many halves in 1 ?
   B Solve: $1 \div \frac{1}{2} = n$

3. A How many fourths in 1 ?
   B Solve: $1 \div \frac{1}{4} = n$

4. A How many eighths in 1 ?
   B Solve: $1 \div \frac{1}{8} = n$

5. A How many fourths in $\frac{1}{2}$?
   B Solve: $\frac{1}{2} \div \frac{1}{4} = n$

6. A How many eighths in $\frac{1}{2}$?
   B $\frac{1}{2} \div \frac{1}{8} = n$

7. A How many eighths in $\frac{1}{4}$?
   B $\frac{1}{4} \div \frac{1}{8} = n$

★ 8. Study the figure. Then solve the equations.

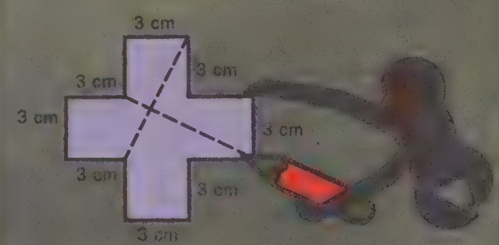
You find this quotient $n \times \frac{1}{2} = \frac{3}{8}$

$\frac{3}{8} \div \frac{1}{2} = n$ when you find this factor.

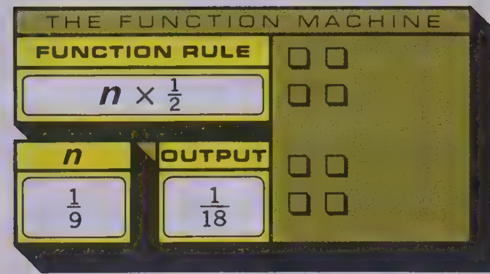
   A $\frac{1}{2} \times \frac{1}{4} = a \longrightarrow \frac{1}{8} \div \frac{1}{4} = b$
   B $\frac{1}{2} \times \frac{1}{4} = a \longrightarrow \frac{1}{8} \div \frac{1}{2} = b$
   C $\frac{3}{8} \times \frac{2}{3} = a \longrightarrow \frac{6}{24} \div \frac{2}{3} = b$
   D $\frac{1}{8} \times \frac{5}{6} = a \longrightarrow \frac{5}{48} \div \frac{1}{8} = b$
   E $\frac{7}{10} \times \frac{1}{2} = a \longrightarrow \frac{7}{20} \div \frac{1}{2} = b$

think

Draw a figure like the one shown below. Cut it out. Cut along the dotted lines. Now fit the four pieces together to make a square.



The Function Machine



We can use fractional numbers in the function machine. When the input number is $\frac{1}{9}$ and the function rule is $n \times \frac{1}{2}$, the output number is $\frac{1}{18}$.

Think about the function machine and give what you think should go in each gray space.

1. Function Rule: $n \times \frac{1}{3}$

	n	Output
A	5	
B	3	
C	$\frac{1}{6}$	
D	$\frac{2}{5}$	
E	0	
F	$\frac{4}{7}$	

2. Function Rule: $n \times \frac{1}{4}$

	n	Output
A	3	
B	$\frac{1}{6}$	
C	$\frac{3}{5}$	
D	4	
E		$\frac{1}{20}$
F		$\frac{4}{20}$

3. Function Rule: $\frac{2}{3} \times n$

	n	Output
A	5	
B	$\frac{1}{3}$	
C	$\frac{1}{4}$	
D	3	
E	$\frac{3}{5}$	
F		$\frac{8}{15}$

4. Function Rule: $\frac{3}{4} + n$

	n	Output
A	$\frac{1}{2}$	
B	$\frac{3}{4}$	
C	$\frac{1}{8}$	
D	$\frac{1}{3}$	
E	$\frac{2}{3}$	
F		$1\frac{1}{2}$

★5. A Function Rule:

	n	Output
	$\frac{1}{2}$	$\frac{1}{4}$
	3	$\frac{3}{2}$
	8	4
B	$\frac{5}{6}$	
C	$\frac{3}{7}$	
D		$\frac{1}{8}$

★6. A Function Rule:

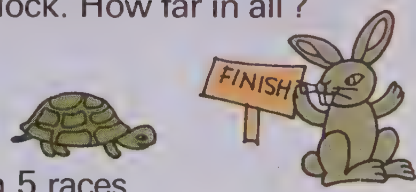
	n	Output
	$\frac{1}{2}$	1
	$\frac{3}{4}$	$\frac{5}{4}$
	1	$1\frac{1}{2}$
B	$\frac{2}{3}$	
C	$\frac{5}{4}$	
D	$1\frac{1}{5}$	

Solving Short Stories

1 $\frac{3}{5}$ km.
$\frac{1}{2}$ that far.
How far?

2 $\frac{3}{4}$ of a pie. Ate $\frac{1}{3}$ of that. How much?

3 Walked $1\frac{1}{4}$ blocks. Ran $\frac{2}{3}$ block. How far in all?



4 Used stamps:
$\frac{1}{10}$ cent. How much
are 50 stamps worth?

5 Ran 5 races.
Each was $\frac{3}{10}$ km. Ran how far?

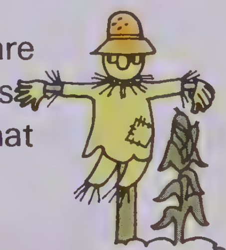
6 Square: Each side is $\frac{3}{10}$ metre. What is the perimeter?

7 $\frac{2}{10}$ litre of milk for 1 cake.
4 cakes. How much
milk is needed?

8 One day: $\frac{1}{7}$ of a week.
What part of a week
is $\frac{1}{2}$ of a day?

9 What part of an hour
is $\frac{1}{3}$ of $\frac{1}{4}$ of an hour?

10 Garden: $\frac{7}{10}$ of a square
kilometre. $\frac{2}{3}$ of that is
planted in corn. What
part of a square km
is planted in corn?



11 Loaf of bread: $\frac{9}{10}$ kg.
Used $\frac{1}{2}$ loaf for dinner.
How many kilograms left?

12 $\frac{5}{8}$ of the children were girls.
$\frac{1}{2}$ of the girls went to the library.
What part of the children
went to the library?

13 Drank $\frac{2}{3}$ glass of milk. The glass holds $\frac{1}{2}$ litre.
Drank what part of a litre?

14 8 puppies in a litter.

A What part of the litter is 1 pup?

B What part is 2 puppies?

C If 3 pups are male, what part is female? What part is male?

More practice, page A-29, Set 55

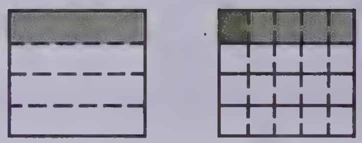
1. Study the figures. Then solve the equations.

A

$\frac{1}{5}$ $\quad$ $\frac{1}{2}$ of $\frac{1}{5}$

$\frac{1}{2} \times \frac{1}{5} = n$

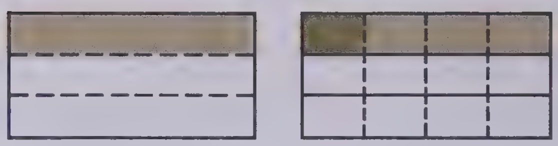
B



$\frac{1}{4}$ $\quad$ $\frac{1}{5}$ of $\frac{1}{4}$

$\frac{1}{5} \times \frac{1}{4} = n$

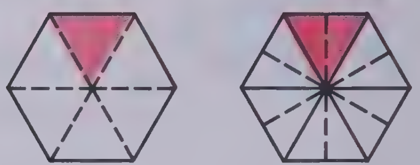
C



$\frac{1}{3}$ $\quad$ $\frac{1}{4}$ of $\frac{1}{3}$

$\frac{1}{4} \times \frac{1}{3} = n$

D

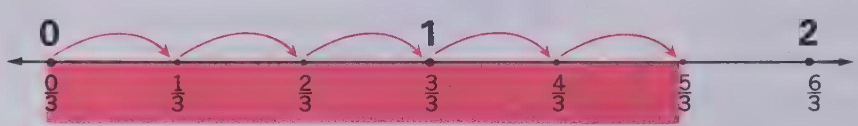


$\frac{1}{6}$ $\quad$ $\frac{1}{2}$ of $\frac{1}{6}$

$\frac{1}{2} \times \frac{1}{6} = n$

2. Study the number lines. Then give the products.

A

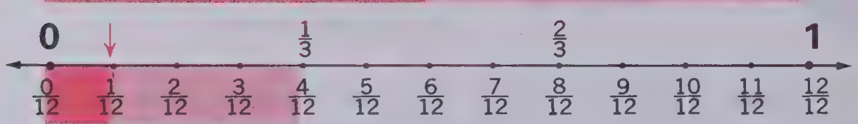


$5 \times \frac{1}{3} = n$

B

$\frac{1}{2} \times 7 = n$

C



$\frac{1}{4} \times \frac{1}{3} = n$

3. Give the number for ***a***. Then give the number for ***b***.

A Since $7 \times \frac{1}{3} = a$, we know that $\frac{1}{3} \times 7 = b$.

B Since $\frac{1}{3} \times \frac{1}{6} = a$, we know that $\frac{1}{6} \times \frac{1}{3} = b$.

C Since $5 \times \frac{1}{3} \times \frac{1}{2} = a$, we know that $\frac{5}{3} \times \frac{1}{2} = b$.

D Since $3 \times 4 \times \frac{1}{2} \times \frac{1}{5} = a$, we know that $\frac{3}{2} \times \frac{4}{5} = b$.

4. Find the products.

A $\frac{2}{3} \times \frac{3}{5} = (2 \times 3) \times (\frac{1}{3} \times \frac{1}{5}) = n$

B $\frac{2}{5} \times \frac{2}{5} = (2 \times 2) \times (\frac{1}{5} \times \frac{1}{5}) = n$

C $\frac{1}{8} \times \frac{3}{4} = (1 \times 3) \times (\frac{1}{8} \times \frac{1}{4}) = n$

D $\frac{5}{8} \times \frac{2}{3} = (5 \times 2) \times (\frac{1}{8} \times \frac{1}{3}) = n$

5. Solve each short story.

A Recess: $\frac{1}{4}$ hour.
Talked for $\frac{1}{2}$ of it.
Talked what part
of an hour?

B Track: $\frac{1}{4}$ km around.
Ran 2 km.
How many times around?

C Cake: $\frac{7}{10}$ kg sugar.
How much for 2 cakes?

D $\frac{1}{3}$ of a pie left.
Ate $\frac{1}{2}$ of the part left.
Ate what part of the pie?

6. Give the products.

A $\frac{3}{4} \times \frac{1}{6}$

B $\frac{5}{8} \times \frac{1}{3}$

C $\frac{1}{3} \times \frac{4}{5}$

D $\frac{1}{2} \times \frac{6}{7}$

E $\frac{3}{4} \times \frac{3}{10}$

F $\frac{2}{3} \times \frac{7}{4}$

G $\frac{4}{7} \times \frac{5}{3}$

H $\frac{3}{10} \times \frac{3}{4}$

I $\frac{1}{2} \times \frac{2}{5}$

J $\frac{1}{10} \times \frac{1}{10}$

K $\frac{2}{3} \times \frac{1}{2}$

L $\frac{7}{10} \times \frac{1}{4}$

7. Give the quotients.

A $1 \div \frac{1}{8} = n$

B $2 \div \frac{1}{2} = p$

C $2 \div \frac{1}{4} = r$

D $3 \div \frac{1}{2} = s$

think

Farmer Brown had some animals.
$\frac{1}{4}$ were horses. $\frac{1}{2}$ were cows.
The rest were pigs. He had 8 pigs.
How many horses and how many
cows did he have?

1. Compute the sums.

A $693 + 48 + 976 + 9283$

B $57.2 + 3.6 + 349.1 + 8.3$

C $68.34 + 2.26 + 389.45$

D $8\frac{1}{5} + 7\frac{2}{5} + 4\frac{3}{5} + 8\frac{4}{5}$

2. Give the sums, products, differences, and quotients.

A $\begin{array}{r} 964 \\ -485 \\ \hline \end{array}$ B $\begin{array}{r} 35 \\ \times 27 \\ \hline \end{array}$ C $\begin{array}{r} 983 \\ +47 \\ \hline \end{array}$ D $\begin{array}{r} 807 \\ -269 \\ \hline \end{array}$ E $\begin{array}{r} 6954 \\ +9765 \\ \hline \end{array}$ F $\begin{array}{r} 395 \\ \times 207 \\ \hline \end{array}$

G $7\overline{)5234}$ H $30\overline{)728}$ I $59\overline{)2130}$ J $43\overline{)3340}$

3. Solve the equations.

A $\frac{1}{3} + \frac{3}{4} = n$

B $\frac{5}{6} - \frac{1}{4} = n$

C $\frac{5}{3} + \frac{0}{5} = n$

D $\frac{4}{10} + \frac{3}{10} = n$

E $0.4 + 0.3 = n$

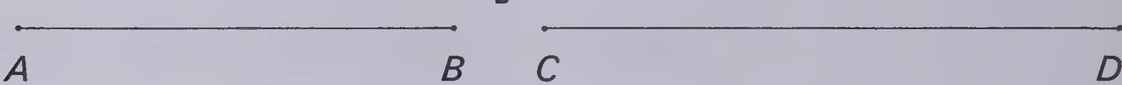
F $8\frac{4}{5} - 2\frac{1}{4} = n$

G $1.4 - 0.5 = n$

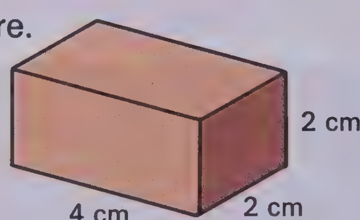
H $5\frac{3}{5} + 8\frac{2}{3} = n$

4. Give the length of each segment to the nearest centimetre.

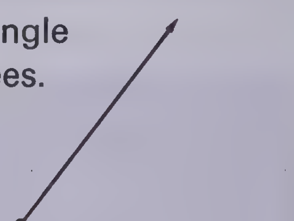
A B



5. Give the volume and surface area of this figure.



6. Give the measure of this angle in degrees.



7. Find the differences.

A $\begin{array}{r} 12 \text{ wk } 8 \text{ days} \\ 7 \text{ wk } 6 \text{ days} \\ \hline \end{array}$ B $\begin{array}{r} 17 \text{ h } 6 \text{ min} \\ 9 \text{ h } 8 \text{ min} \\ \hline \end{array}$

You are invited to explore

ACTIVITY CARD 15
Page 340

People

Oldest
113 years

Heaviest
485.5 kg

Shortest
58.1 cm

Strongest
Lifted 2724 kg

1. The height of the tallest man is how much greater
   A than the height of the tallest woman?
   B than the height of the shortest man?

Tallest

257.8 cm   270.5 cm

2. The oldest man died in 1814. In what year was he born?

3. Of course, the strongest man did not lift 2724 kg over his head, but he did lift it off the ground. How much greater is the weight he lifted than the weight of the heaviest man?

4. Some strong men can put as much as 170 kg over their heads. One of the best lifts by a small man occurred when a 57.7 kg man put slightly more than $2\frac{1}{2}$ times his own weight over his head. About how much was that?

5. Twelve times the weight of the tallest man is about as much as the strongest man can lift. About what does the tallest man weigh?

6. When the heaviest man was 10 years old, he weighed 172 kg. About how many times your weight is this?

*The records given, although subject to dispute, are reasonably accurate.

15 Graphing and Geometry

Can you graph points?

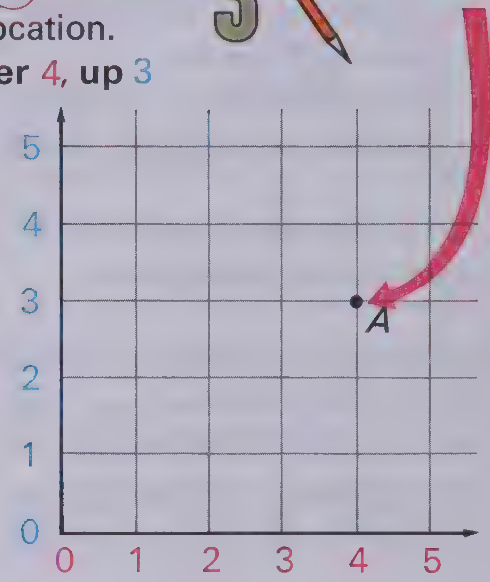
Investigating the Ideas

To graph point A with co-ordinates (4, 3)

1 LOOK at its **co-ordinates.** (4, 3)

2 THINK about its location. **Over 4, up 3**

3 MARK the dot.



Make a grid like this and graph these points.

A: (4,3)	*E:* (1,2)
B: (4,2)	*F:* (1,3)
C: (3,1)	*G:* (2,4)
D: (2,1)	*H:* (3,4)

? What geometric figure can you form by connecting the dots in order?

Discussing the Ideas

1. Explain how you would graph these points:

A (5,5)	D (0,0)
B (2,3)	E (4,0)
C (3,2)	F (0,4)

2. How can you show that the points for (5,3) and (3,5) are different points?

1. Give the missing number. Then give the co-ordinates.

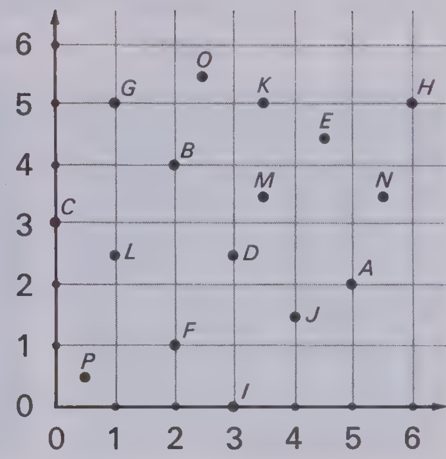
A Point *A* is 5 over and |||| up.
The co-ordinates of *A* are_ ?_.

B Point *B* is |||| over and 4 up.
The co-ordinates of *B* are_ ?_.

C Point *C* is 0 over and |||| up.
The co-ordinates of *C* are_ ?_.

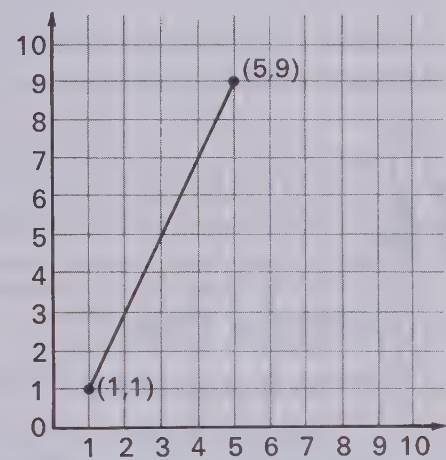
D Point *D* is |||| over and $2\frac{1}{2}$ up.
The co-ordinates of *D* are_ ?_.

E Point *E* is $4\frac{1}{2}$ over and |||| up.
The co-ordinates of *E* are_ ?_.



2. Give the co-ordinates of points *F* through *P*.

3. The first 2 points have been graphed and connected. Use your graph paper to copy and complete the picture by connecting these points in the order given: (1,1), (5,9), (9,1), (0,6), (10,6), (1,1).



4. Invent a picture and list the co-ordinates for the points of the picture. Then see if a classmate can draw the picture.

think

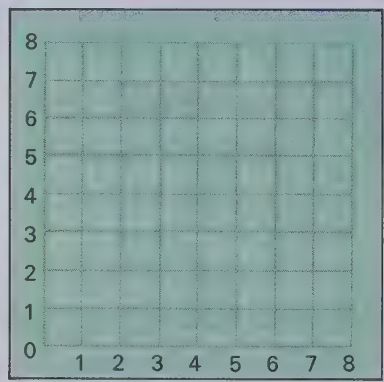
One morning a snail starts climbing up a board that is 309 cm long. Each day he climbs 61 cm. Each night he slips back 30 cm. On what day (8th, 9th, or 10th) does he reach the top?

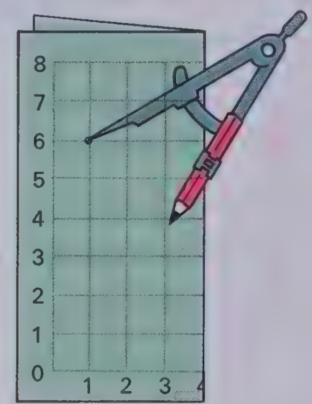
Investigating the Ideas

1 Cut out a piece of graph paper.

2 Fold it like this.

3 "Graph" each point with the compass tip. (Push through both parts.)





Use the steps above to graph these points:

(4,8), (0,8), (1,7), (1,6), (3,6), (3,7), (0,4), (0,3),
(1,2), (4,3), (2,3), (2,2), (4,1), (4,2), (0,0).

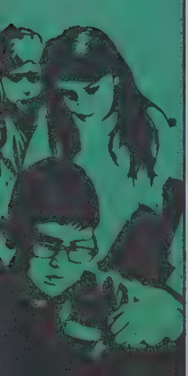
Connect them to form half of a figure. Now unfold the paper.

? Can you use the other holes to help you draw the other half of the figure?

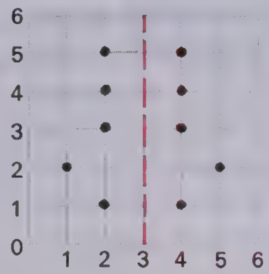
Discussing the Ideas

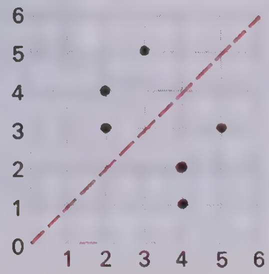
1. The line along the fold is called the **line of symmetry**.
   - A When you graphed the point (1,6) with your compass tip, what other point did you automatically graph on the other side of the line of symmetry?
   - B Answer the question in A for each of the points in the Investigation.

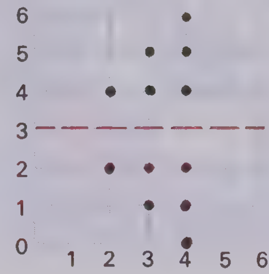
2. How many different triangles can you find in the left half of the completed figure above? Can you find each triangle in the right half of the figure that is congruent to a triangle in the left half?



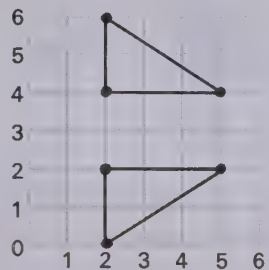
1. Suppose you folded each picture along the dashed line. In which pictures would the dots match each other?

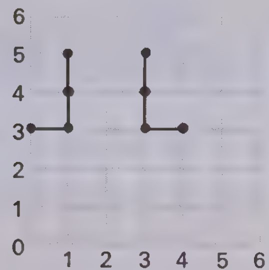
A


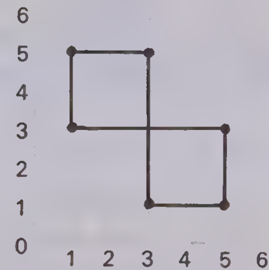
B


C


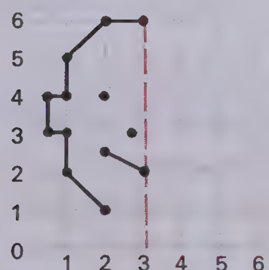
2. Where could you fold each picture so that the two parts will match?

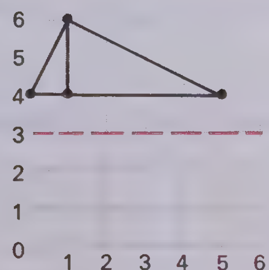
A


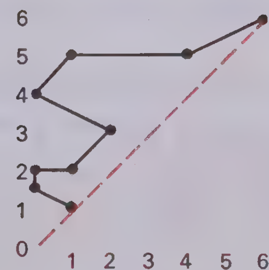
B


C


3. Copy each figure on graph paper. Then draw the "other half."

A


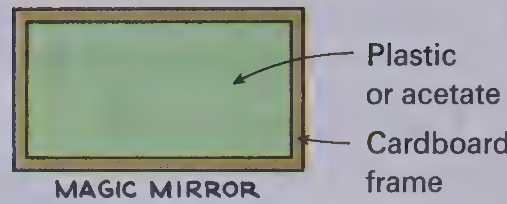
B


C


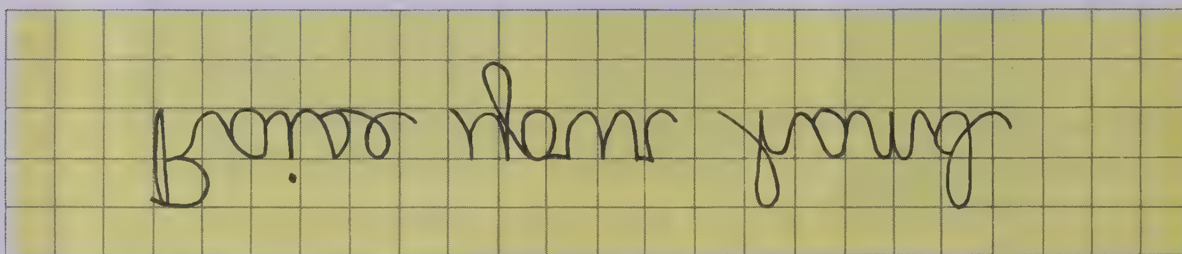
4. Draw half of a symmetric figure. Ask a classmate to draw the other half.

Let's explore line symmetry.

For these exercises you will need a "magic mirror." To make one, cut out a piece of acetate or plastic 8 by 13 centimetres. If necessary, make a cardboard frame to hold the plastic.

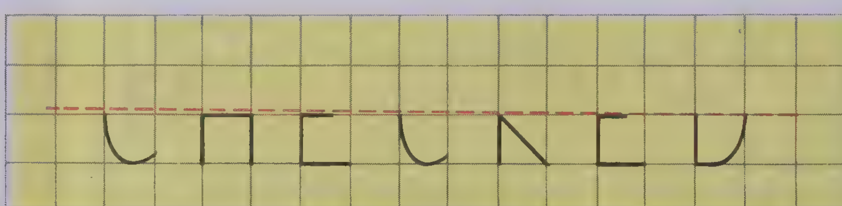


1. Here is a SECRET MESSAGE. Can you use your mirror to figure out what it says?



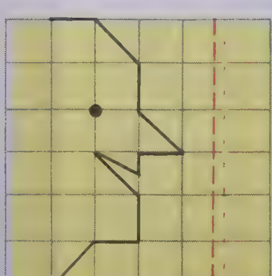
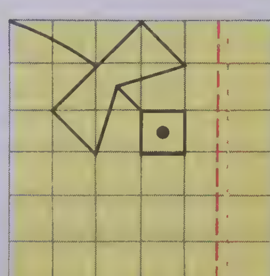
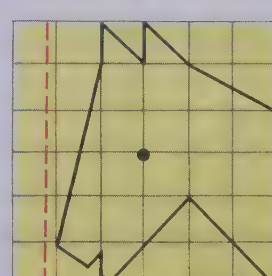
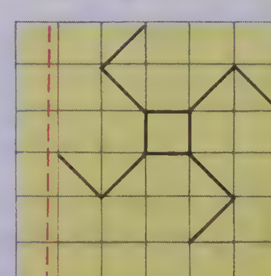
Can you make some messages for others to figure out?

2. Here are a "half word" and a "half numeral." Name them.

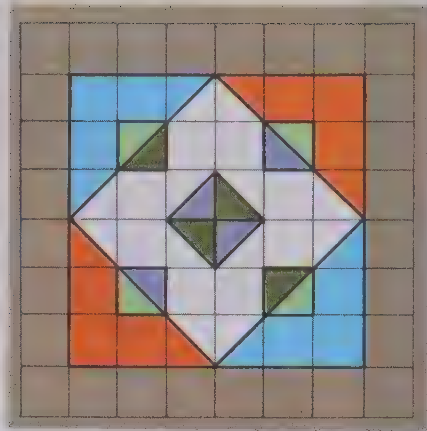


How many more "half words" and "half numerals" can you make?

3. Can you draw the reflection ("the other half") of each figure? Check your drawing with your mirror. Make some more like these.

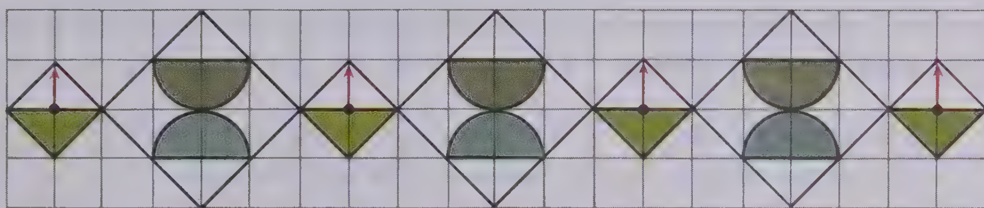





LINE-SYMMETRIC PATTERNS



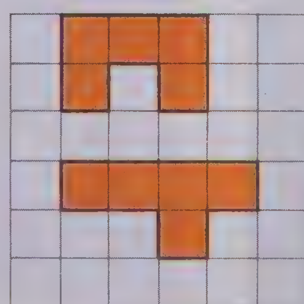
1. This graph-paper design is line-symmetric. (One half is a reflection of the other half.) Its two halves are exactly alike and will fit upon each other. How? Can you use your graph paper to make other line-symmetric designs?

2.

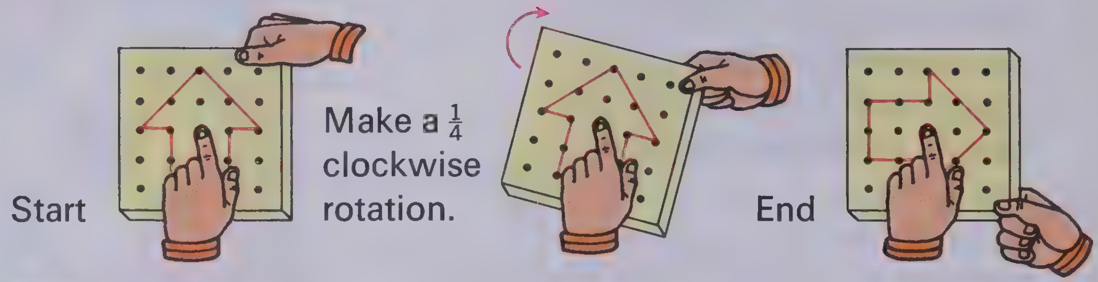


Strip designs like this can be used as borders in decorating. Where can you fold it so that one half is a reflection of the other half? Can you use your graph paper and make other border designs that are line-symmetric?

3. A **pentomino** is made up of 5 squares. Each of its squares can touch another one only along one complete common side. Here are two of the twelve possible pentominoes. One has a line of symmetry and the other does not. Which one? Can you find other pentominoes with lines of symmetry and draw them on graph paper?



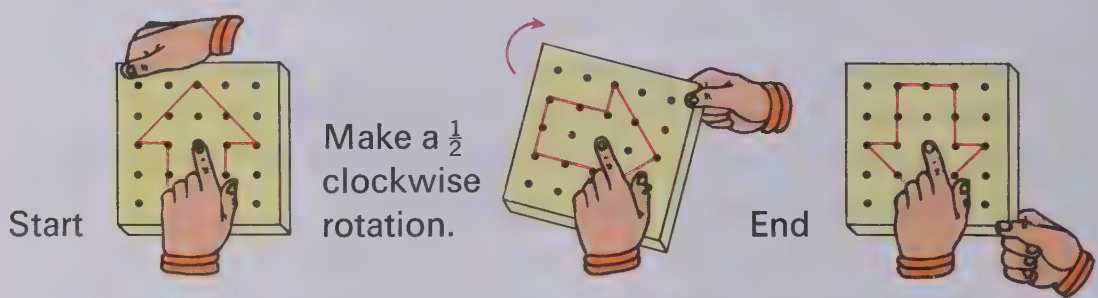
Investigating the Ideas



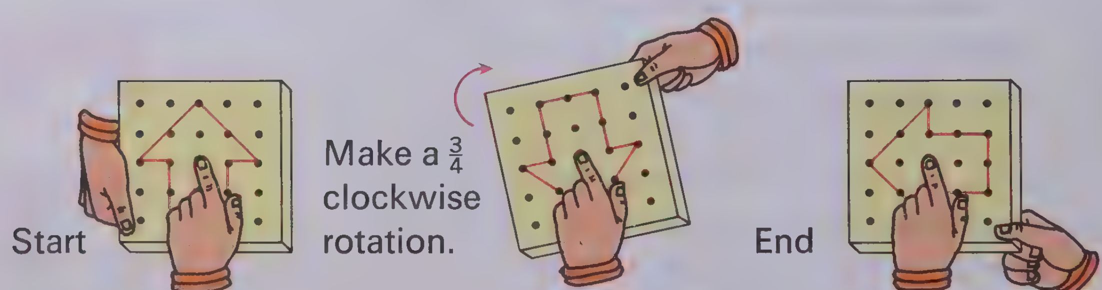
? Can you show on your geoboard and draw on dot paper some figures that will look exactly the same at the end of a $\frac{1}{4}$ rotation as at the beginning?

Discussing the Ideas

1. Here is a $\frac{1}{2}$ rotation. Which of your geoboard figures look the same after a $\frac{1}{2}$ rotation as at the beginning?

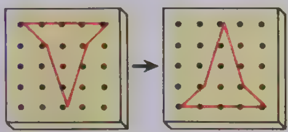


2. Here is a $\frac{3}{4}$ rotation. Can you show a geoboard figure that looks the same after a $\frac{3}{4}$ rotation as at the beginning?

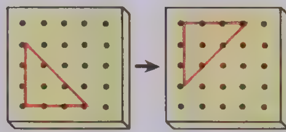


1. Each part shows a geoboard at the start and at the end of a rotation. Tell whether it was a $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ rotation.

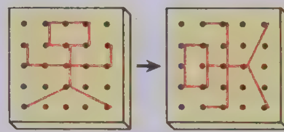
Start End

A 

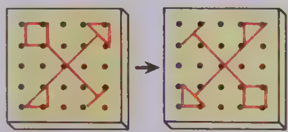
Start End

B 

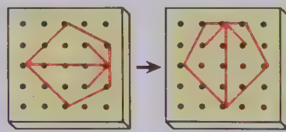
Start End

C 

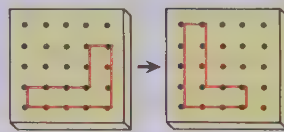
Start End

D 

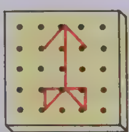
Start End

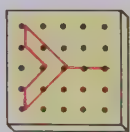
E 

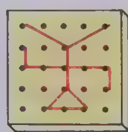
Start End

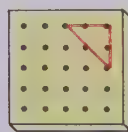
F 

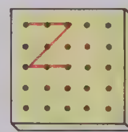
2. Draw a picture on dot or graph paper to show what each geoboard will look like after a $\frac{1}{2}$ rotation.

A 

B 

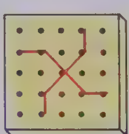
C 

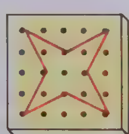
D 

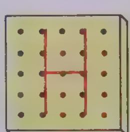
E 

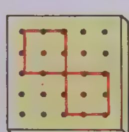
3. Complete exercise 2 for a $\frac{1}{4}$ rotation and for a $\frac{3}{4}$ rotation.

4. Which of these will look the same after a $\frac{1}{2}$ rotation? a $\frac{1}{4}$ rotation?

A 

B 

C 

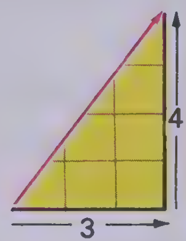
D 

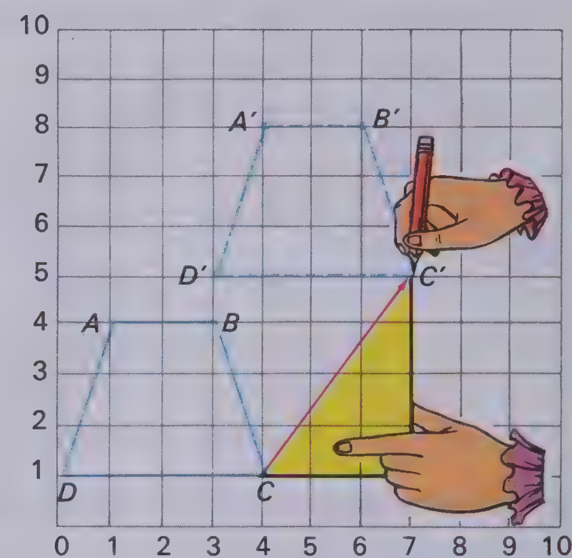
think

Suppose you put 2¢ in a bank on February 1, 4¢ on February 2, 8¢ on February 3, and so on, doubling the amount each day. After you made your last deposit on Valentine's Day, would you have enough money to buy a \$5 box of candy? How much would you have?

Investigating the Ideas

Anita made a "point slider" like this from graph paper. She called it a "right-3, up-4 point slider" and used it as shown on the graph to "slide" each point of the blue figure to a new position.



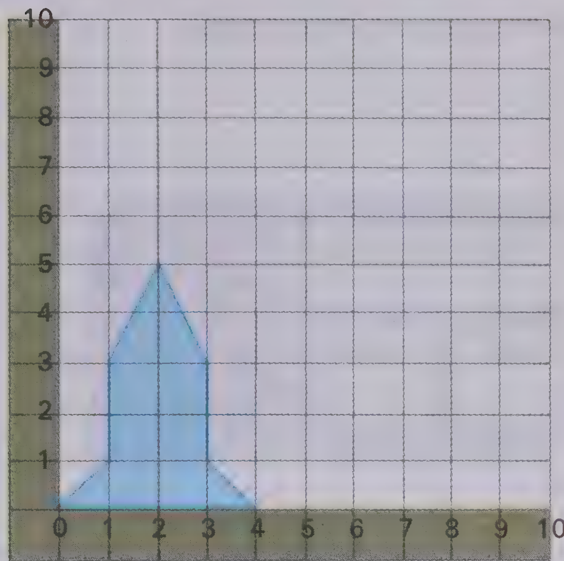


?	Can you make a "right-4, up-5 point slider" and use it to find the new position of the blue figure after sliding each point?	Show your results on graph paper.

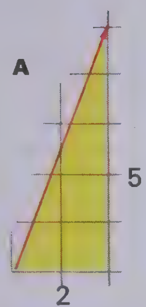
Discussing the Ideas

1. Explain why Anita called her "point slider" a "right-3, up-4 point slider."
2. When we "slide" a figure to a new position, we call this motion a **translation** of the figure. The red arrow on the "point slider" shows the direction and distance of the translation. In the picture above, how far was each point of the blue figure moved to obtain the new position? (Mark off the scale units along the edge of a piece of paper to make a ruler.)
3. Explain how you could use a "right-4, up-5 point slider" as a "left-4, down-5 point slider."

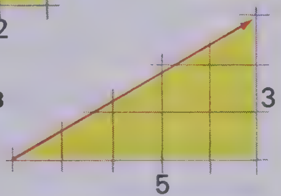
1. Copy this figure on your graph paper. Then draw on your graph paper the final position of the figure after using the translation indicated.



A



B

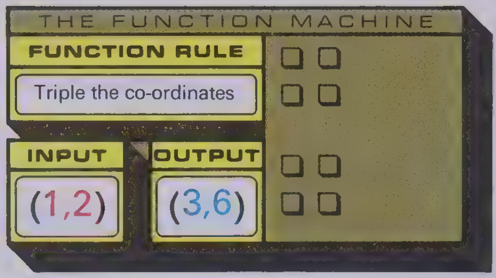


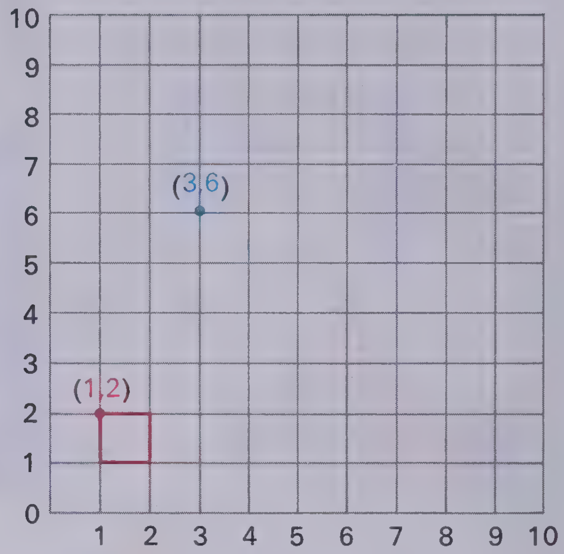
2. The beginning and final positions of the figures are given. Which "point sliders" were used?

★ 3. Can you draw a picture on your graph paper and show its new position after each of these translations?

A left 3, down 2 B right 3, down 4 C left 4, up 5

Investigating the Ideas





? Can you copy the red square on your paper and show the figure that would result if **all** the corner co-ordinates are tripled, graphed, and connected ?

Discussing the Ideas

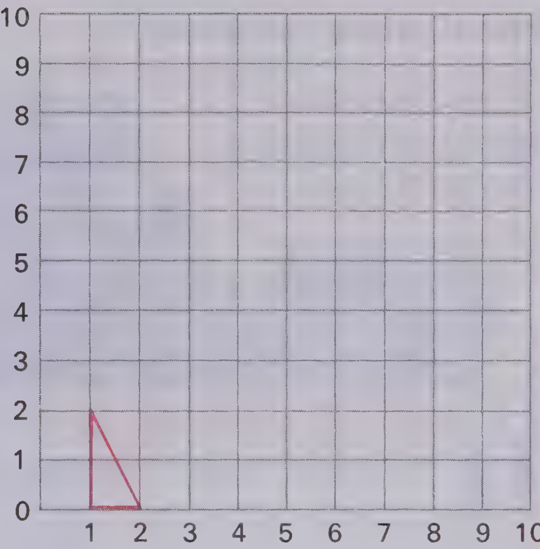
1. How would you describe the new figure formed by tripling the co-ordinates ?

2. A How many times as long is the side of the new figure as the side of the red square ?
   B How do the areas of the two figures compare ?

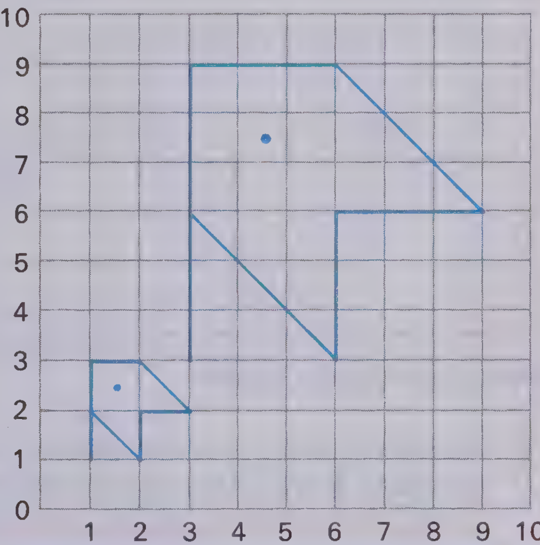
3. What figure do you think would result if the co-ordinates were doubled ? quadrupled ?

4. How does the distance from (0,0) to (3,6) compare with the distance from (0,0) to (1,2) ?

1. On your graph paper show the figure that would result if the co-ordinates of the triangle were
   - A doubled.
   - B tripled.
   - C quadrupled.
   - D multiplied by 5.



2. When the co-ordinates of one figure are multiplied to arrive at the co-ordinates of a second figure, the figures are **similar** to each other (the same shape). Here are two similar figures.
   - A The co-ordinates of the large figure are what multiple of the co-ordinates of the small figure?
   - B Show on your graph another figure similar to these figures.



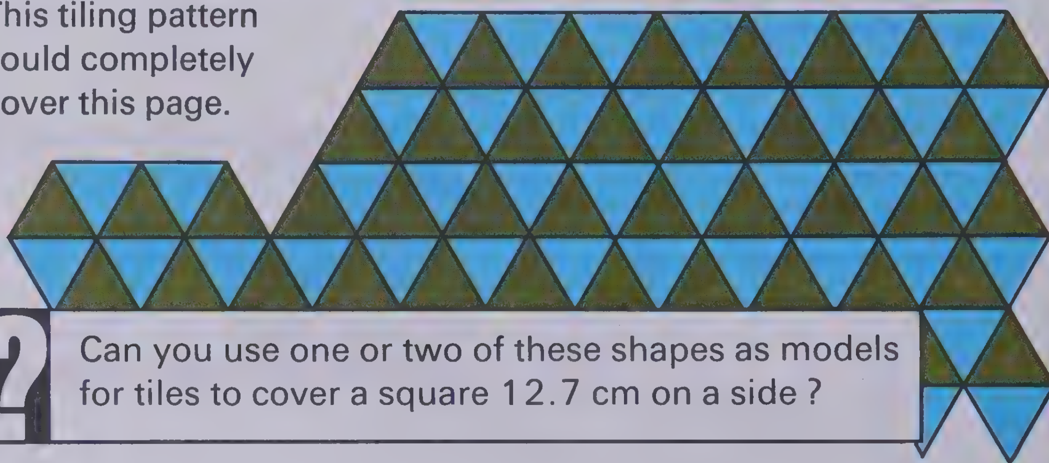
3. On your graph paper make
   - A two rectangles that are similar.
   - B two right triangles that are similar.
   - C two other interesting figures that are similar.

think

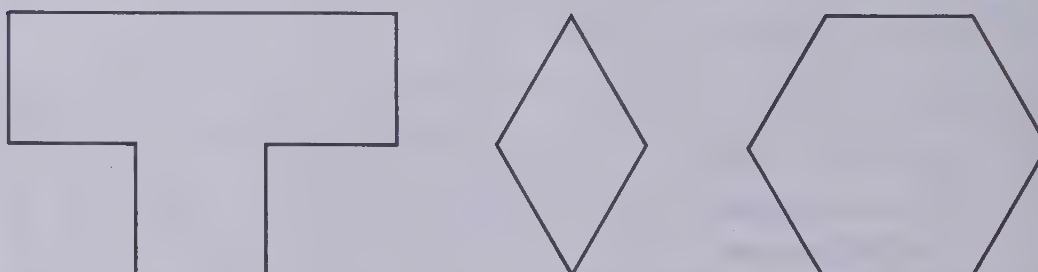
A metre stick makes a shadow 20 cm long. How tall is a flagpole that makes a 6-metre shadow?

Investigating the Ideas

This tiling pattern could completely cover this page.

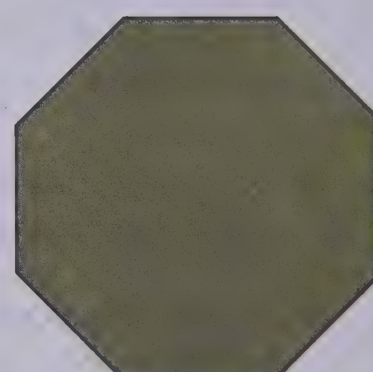


? Can you use one or two of these shapes as models for tiles to cover a square 12.7 cm on a side?



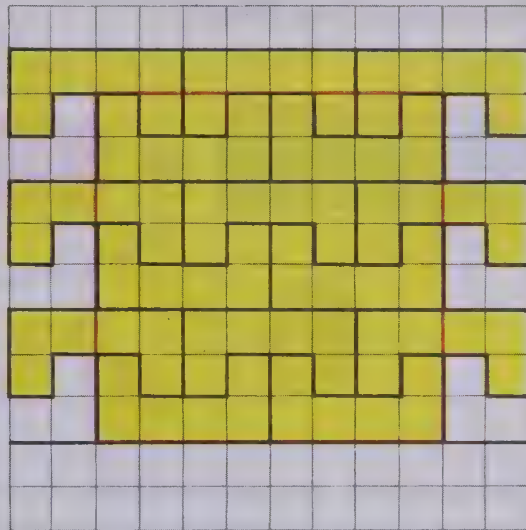
Discussing the Ideas

1. A pattern that covers a plane with tiling is called a **tessellation** of the plane.
   A Can you tessellate a plane with a regular octagon? Explain your answer.
   B Can you use another shape with the octagon to tessellate a plane?



2. Which regular polygons could be used to tessellate a plane?

1. This picture shows a covering of the red square with yellow pieces made up of 6 small squares. Can you find some different 6-square pieces that will completely cover the square? Show each tessellation.

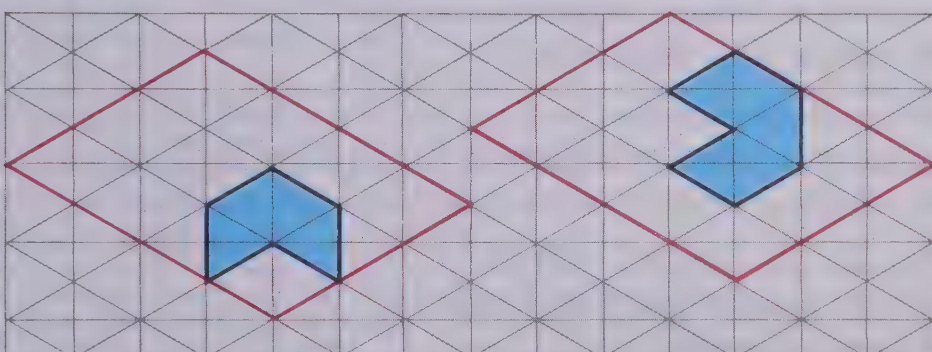


2. Can you invent a figure of your own that could be used to tessellate a plane? Show the tessellation on graph paper and color it so that an interesting pattern stands out.

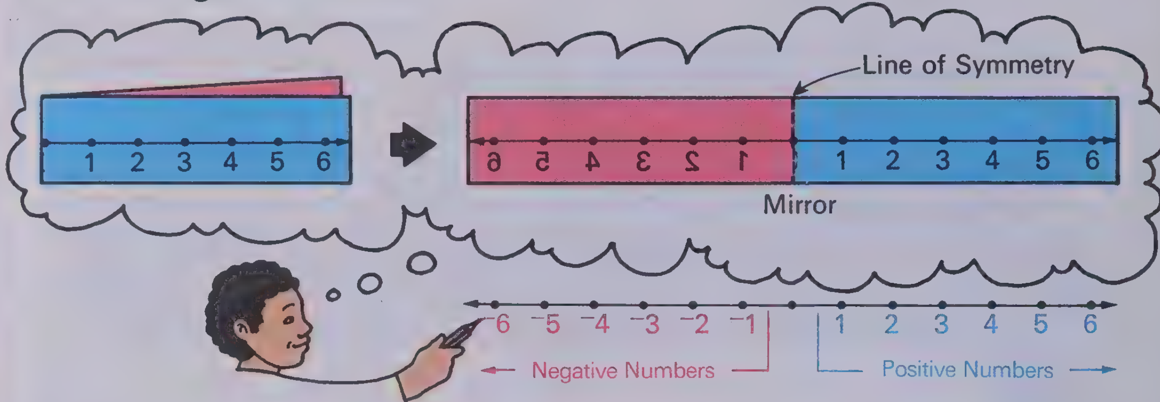
3. A Can this shape ⟶ be used to make a tessellation?

B Use four of the shapes to make a figure similar in shape but larger.

4. Can you trace this special graph paper grid and use the blue figure shown to draw a tessellation of each red figure?



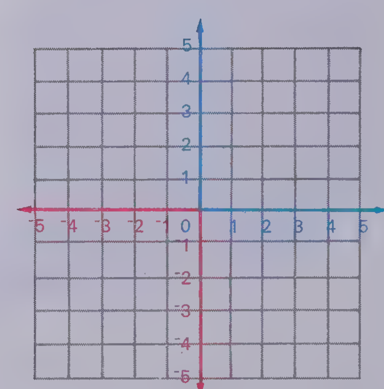
Discussing the Ideas



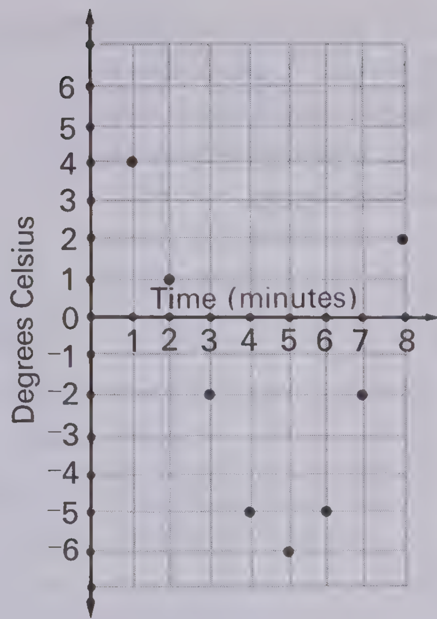
This whole number line shows numbers for the points **opposite** the **positive whole numbers.** Numerals for these numbers are in red. These "new" numbers we call **negative whole numbers.**

You can think of whole numbers in two ways. They tell you "how many." But they also tell you "what direction" (above or below zero). Positive and negative numbers are useful in describing many things in the world around you. They are also useful in graphing.

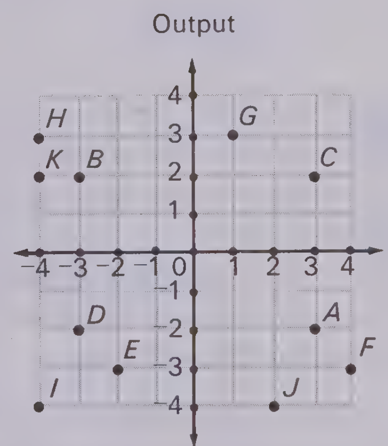
1. Explain how positive and negative numbers are used on a thermometer.
2. Explain how positive and negative numbers can be used for telling the amount of time before and after blast-off.
3. How can you use positive and negative numbers in a game when your score goes below zero ?
4. How can you use positive and negative numbers to talk about places above and below sea level ?
5. How can you use two crossed number lines to give co-ordinates for any point in a plane ?



1. A scientist was conducting an experiment with a certain type of motor oil. He was testing the oil at very low temperatures. The graph shows the temperature at various times (in minutes).
   - **A** What was the temperature of the oil at 1 min? What are the co-ordinates of that point?
   - **B** What was the temperature of the oil at 3 min? What are the co-ordinates of that point?
   - **C** What are the co-ordinates for the lowest recorded temperature?
   - **D** At what two periods of time was the temperature 5 degrees below zero?
   - **E** What would you guess about the temperature at $7\frac{1}{2}$ minutes?



2. In the graph on the right, the co-ordinates of point A are $(3, {}^{-}2)$ and the co-ordinates of B are $({}^{-}3, 2)$. Give the co-ordinates of points C through K.

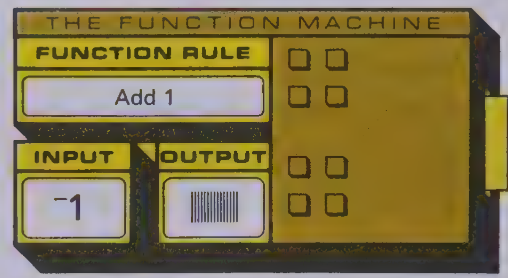


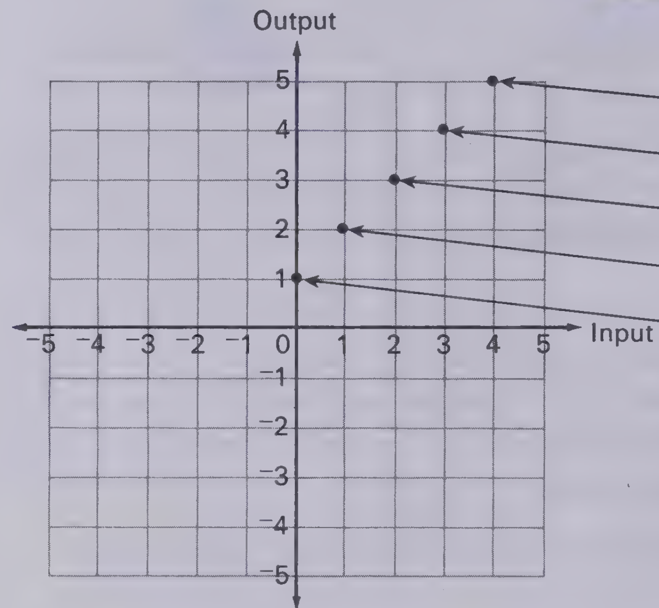
3. Draw a pair of axes like those for exercise 2. Graph the following co-ordinates. Label them A, B, C, and so on.

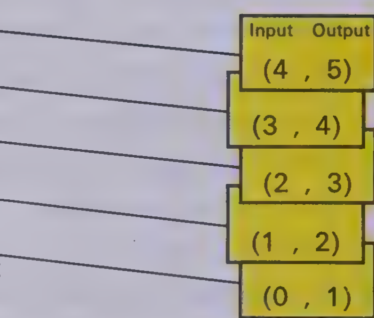
A $(2, 4)$	**C** $(2, {}^{-}4)$	**E** $({}^{-}3, {}^{-}4)$	**G** $(1, {}^{-}2)$
B $({}^{-}2, 4)$	**D** $({}^{-}2, {}^{-}4)$	**F** $({}^{-}4, 4)$	**H** $({}^{-}1, 2)$

Investigating the Ideas

Each time this function machine operates, it produces an input-output card. We can mark a point on the graph for each card.







Study the graph and the pattern of the input and output numbers on the cards.

?	Can you draw the input-output cards that the machine would produce for inputs ⁻1, ⁻2, ⁻3, and ⁻4 ?	Show their points on a graph.

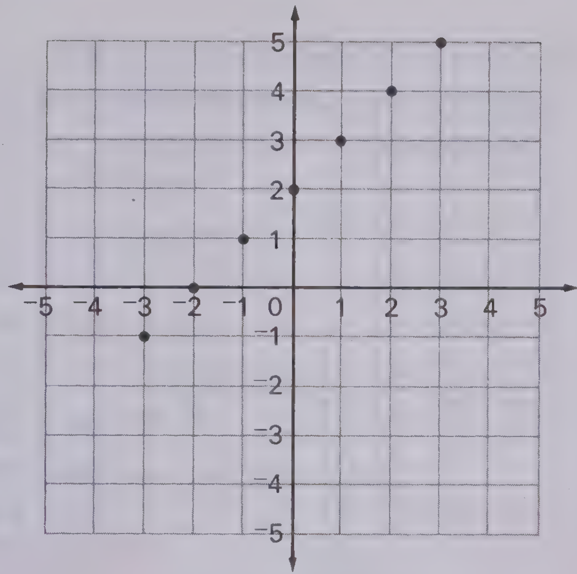
Discussing the Ideas

1. What do you notice about the set of points on the graph ? Describe other cards that will come from the machine.

2. How can the Investigation help you find these sums ?
   A $^{-}1 + 1$ B $^{-}2 + 1$ C $^{-}3 + 1$ D $^{-}4 + 1$

1. Complete the function table. The graph may help you.

Function Rule	
Add 2	
Input	Output
3	5
2	4
1	
0	
$^{-}1$	
$^{-}2$	
$^{-}3$	



Use the table or graph to help you solve these equations.

A $2 + {}^{-}2 = n$ B $2 + {}^{-}1 = n$ C $2 + {}^{-}3 = n$ D $2 + {}^{-}4 = n$

2. Give the missing numbers in the table. Then list and graph the input-output pairs.

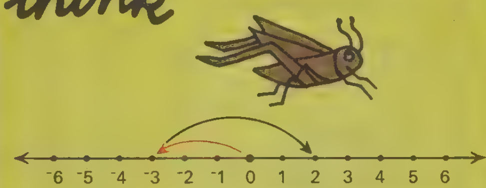
A

Function Rule	
Add 3	
Input	Output
3	6
2	5
1	
0	
$^{-}1$	
$^{-}2$	

★ **B**

Function Rule	
Subtract 1	
Input	Output
4	3
3	2
2	1
1	0
0	
$^{-}1$	

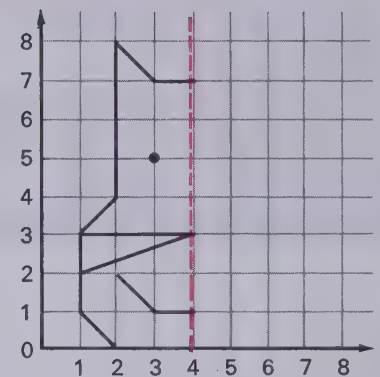
think

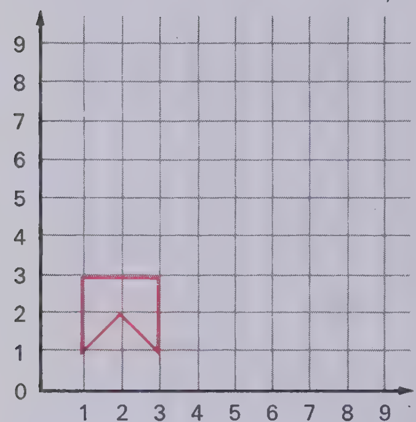


If a cricket jumps				he ends up at
3 left	then	5 right		
↓	↓	↓		↓
$^{-}3$	+	5	=	2
$^{-}4$	+	4	=	?
$^{-}2$	+	4	=	?
$^{-}6$	+	3	=	?

Reviewing the Ideas

1. Copy this figure on your graph paper. Then draw the other half to make a symmetric figure.



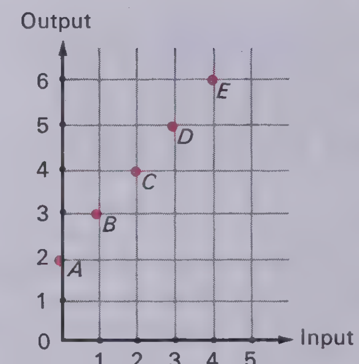


2. Draw the picture that results when you
   A move each point of this figure over 3 and up 2.
   B triple the co-ordinates of the points of this figure to get a new figure.

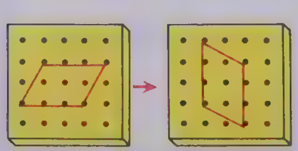
3. Copy the table on your paper. Complete your table by giving the number pairs for points *C*, *D*, and *E*. Give a function rule for this set of number pairs.

Function Rule

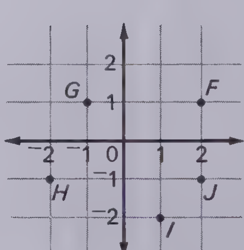
	Input	Output
A	0	2
B	1	3
C	2	
D		
E		



4. What rotation does the pair of geoboards show?



5. Give the co-ordinates of points F through J.



Keeping in Touch with Addition, Subtraction, Multiplication, Division, Fractional numbers

1. Solve the equations.

A $n \times 7 = 21$
B $5 \times n = 30$
C $28 = 4 \times n$
D $n = 5 \times 8$
E $6 \times n = 42$
F $n \times 3 = 24$
G $n = 9 \times 6$
H $49 = 7 \times n$

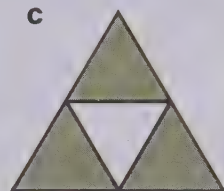
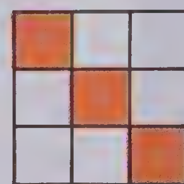
I $7 \times n = 63$
J $40 = n \times 8$
K $n = 5 \times 9$
L $6 \times n = 54$
M $56 = n \times 8$
N $7 \times 9 = n$
O $72 = 8 \times n$
P $n = 8 \times 8$

Q $(6 \times 4) + 6 = n$
R $(8 \times 6) + 3 = n$
S $56 = (9 \times 6) + n$
T $67 = (9 \times 7) + n$
U $48 = (n \times 5) + 3$
V $46 = (n \times 7) + 4$
W $81 - (n \times 9) = 0$
X $56 - (n \times 7) = 0$

2. Give the fractional part of the region that is shaded.

A

B

C

D


3. Find the sums.

A $\frac{1}{2} + \frac{3}{4}$ B $\frac{3}{8} + \frac{1}{4}$ C $\frac{2}{3} + \frac{1}{6}$ D $\frac{3}{10} + \frac{1}{2}$

4. Find the differences.

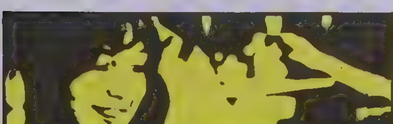
A $\frac{7}{8} - \frac{3}{4}$ B $\frac{9}{10} - \frac{2}{5}$ C $\frac{11}{20} - \frac{1}{5}$ D $\frac{7}{4} - \frac{3}{2}$

5. Find the sums and differences.

A $\begin{array}{r} 3\frac{1}{8} \\ +5\frac{3}{4} \\ \hline \end{array}$ B $\begin{array}{r} 8\frac{1}{2} \\ -1\frac{1}{4} \\ \hline \end{array}$ C $\begin{array}{r} 7\frac{1}{5} \\ +3\frac{9}{10} \\ \hline \end{array}$ D $\begin{array}{r} 6\frac{1}{4} \\ -3\frac{2}{3} \\ \hline \end{array}$

6. Give the products.

A $\frac{1}{2} \times 3$ B $\frac{1}{3} \times \frac{1}{4}$ C $\frac{2}{3} \times \frac{4}{5}$ D $\frac{3}{2} \times \frac{2}{3}$



You are invited to explore

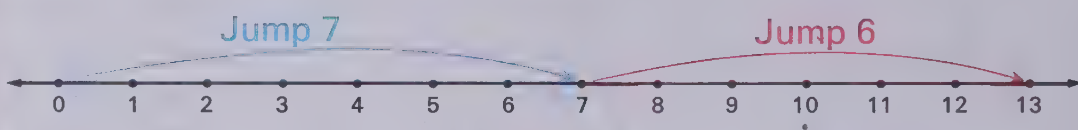
ACTIVITY CARD 16
Page 341

16 A New Mathematical System

Are there other kinds of arithmetic?

Investigating the Ideas

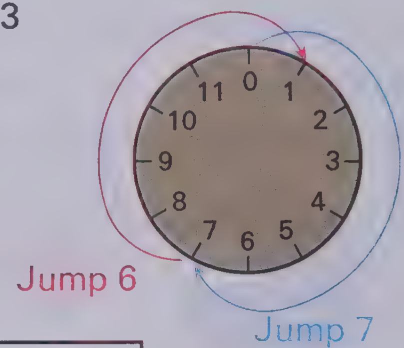
Here is an "ordinary" number line showing the sum of 7 and 6.



$7 + 6 = 13$

Here is a "number circle" showing the "sum" of 7 and 6.

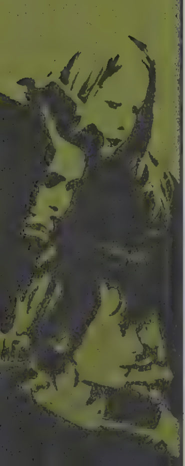
$7 + 6 = 1$



? Can you use this number circle to write some other strange-looking equations?

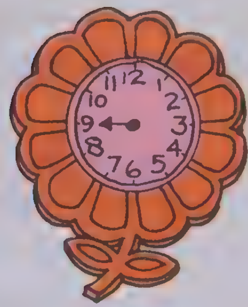
Discussing the Ideas

1. If the 0 is replaced by a 12, arithmetic on the number circle above is called "clock arithmetic." Can you explain why?
2. Explain why $7 + 6 = 1$ in clock arithmetic.
3. Bill wrote "$9 + 6 = 3$" on his paper. Jane said, "You made a mistake in your clock arithmetic. You must have meant to write $9 - 6 = 3$." Was Bill's equation correct in clock arithmetic?
4. Give an equation that is correct in both clock arithmetic and the arithmetic we usually study.
5. Explain why $9 + 12 = 9$ in clock arithmetic.



Using the Ideas

Jane does her clock problems by thinking about a clock with only one hand. Bill does his by counting to 12 and then starting over again:



1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, . . .

See if you can do these clock problems.

1. Tell what time it will be

- **A** 4 hours after 9 o'clock.
- **B** 2 hours after 11 o'clock.
- **C** 8 hours after 8 o'clock.
- **D** 3 hours after 4 o'clock.
- **E** 9 hours after 6 o'clock.
- **F** 6 hours after 9 o'clock.
- **G** 5 hours after 7 o'clock.
- **H** 7 hours after 5 o'clock.
- **I** 12 hours after 3 o'clock.
- **J** 2 hours after 12 o'clock.

2. Write clock equations for each part of exercise 1. For example, for exercise 1**A** the equation is $9 + 4 = 1$.

3. In "regular" arithmetic, the sum of a number and zero is that number. Which clock number acts like zero? Write two addition equations using this clock number.

4. Bill gave this problem to his father:

$$\square + \square = \square$$

He said, "You have to put the same clock number in each box." Can you find this clock number?

5. Solve these clock equations. Remember that you are working with clock numbers.

- **A** $6 + 7 = v$
- **B** $5 + 7 = s$
- **C** $9 + 9 = t$
- **D** $6 + 9 = m$
- **E** $8 + n = 2$
- **F** $6 + 10 = a$
- **G** $10 + 6 = f$
- **H** $9 + (4 + 6) = y$
- **I** $(9 + 4) + 6 = n$
- **J** $1 = 11 + b$
- **K** $8 = 6 + c$
- **L** $2 = 9 + q$
- **M** $8 + 12 = k$
- **N** $12 + 12 = g$
- **O** $6 + 6 = d$

● *Can other operations be performed in clock arithmetic?*

Investigating the Ideas

Study this clock-number function table and give the missing number.

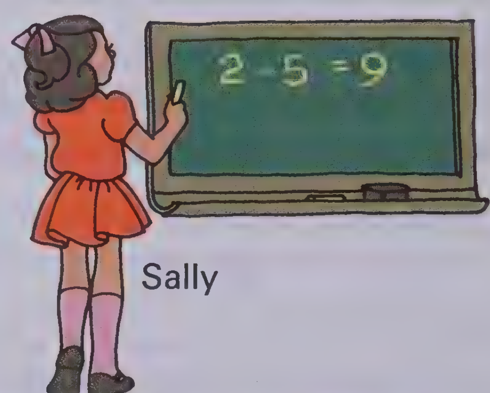
Function Rule	
Subtract ▯	
Input	Output
2	10
3	11
1	9
8	4
4	12

? Make up a function table for clock arithmetic and see if a classmate can guess your rule.

Discussing the Ideas

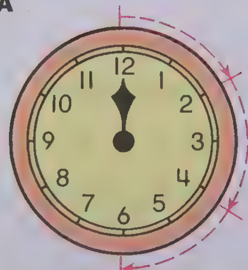
1. Use the clock to show that Sally's equation is correct.

2. A multiplication equation is given for picture A. Give multiplication equations for B and C.



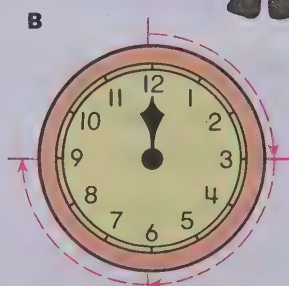
Sally

A

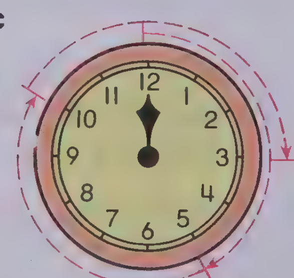


$3 \times 2 = 6$

B



C



Using the Ideas

1. Tell what time it will be

A 5 hours before 2 o'clock.
B 6 hours before 2 o'clock.
C 3 hours before 12 o'clock.
D 12 hours before 3 o'clock.
E 7 hours before 4 o'clock.
F 4 hours before 7 o'clock.
G 6 hours before 3 o'clock.
H 3 hours before 6 o'clock.
I 12 hours before 5 o'clock.
J 3 hours before 3 o'clock.

2. Write clock equations for each part of exercise 1.
(Example: A $2 - 5 = 9$)

3. Remember that you are working with clock numbers as you solve these clock equations. Check each equation by addition.

A $2 - 6 = s$
B $8 + 5 = t$
C $1 - 5 = m$
D $9 + 4 = n$
E $9 + 9 = b$
F $9 - 9 = y$
G $12 + 12 = n$
H $12 - 12 = f$
I $3 - 12 = c$
J $3 - 5 = k$
K $1 - g = 11$
L $5 - d = 6$

4. Solve the clock equations. Use the clock face or repeated clock-arithmetic addition.

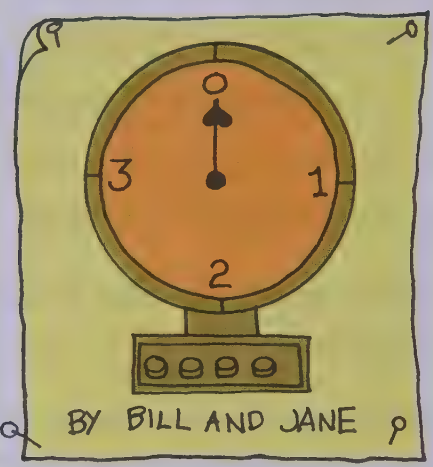
A $4 \times 2 = t$
B $2 \times 5 = r$
C $6 \times 2 = s$
D $4 \times 5 = n$
E $5 \times 12 = m$
F $8 \times 12 = a$
G $12 \times 2 = f$
H $12 \times 4 = y$
I $3 \times 7 = b$
J $7 \times 3 = c$
K $2 \times 10 = q$
L $10 \times 2 = d$

★ **5.** Find the missing factors. More than one answer is possible in parts E, F, G, and H.

A $n \times 5 = 10$
B $a \times 7 = 9$
C $c \times 1 = 12$
D $r \times 5 = 3$
E $b \times 3 = 12$
F $t \times 6 = 12$
G $f \times 2 = 8$
H $y \times 4 = 12$
I $d \times 5 = 1$

Investigating the Ideas

Bill and Jane made a clock like the one in the picture. Since every answer would be 0, 1, 2, or 3, they decided to use only those numbers in their equations.



Can you use this clock to write some clock equations?

Discussing the Ideas

1. Suppose Jane and Bill's clock had a "4" on it. Solve and explain each of these equations.

   A $3 + 4 = n$
   B $2 + 4 = r$
   C $1 - 4 = s$
   D $4 - 4 = a$
   E $2 \times 4 = c$
   F $3 \times 4 = q$

2. Can you explain why Jane and Bill decided to put a "0" in place of a "4" on their clock?

3. Make a special clock of your own and write some equations for your clock. Check your work with a classmate.

Using the Ideas

Remember, this clock has only four numbers: 0, 1, 2, and 3.

1. Tell what time it will be

- A 2 hours after 0 o'clock.
- B 3 hours after 0 o'clock.
- C 1 hour after 2 o'clock.
- D 1 hour after 3 o'clock.
- E 2 hours after 2 o'clock.
- F 2 hours after 3 o'clock.
- G 3 hours after 2 o'clock.
- H 3 hours after 3 o'clock.
- I 2 hours after 1 o'clock.
- J 3 hours after 1 o'clock.

2. Write clock equations for each part of exercise 1.

3. Tell what time it will be

- A 2 hours before 0 o'clock.
- B 1 hour before 0 o'clock.
- C 2 hours before 3 o'clock.
- D 2 hours before 2 o'clock.
- E 1 hour before 3 o'clock.
- F 2 hours before 1 o'clock.
- G 3 hours before 1 o'clock.
- H 3 hours before 2 o'clock.

4. Write clock equations for each part of exercise 3.

5. Solve these clock equations.

- A $2 + 1 = c$
- B $2 + 2 = v$
- C $0 - 2 = d$
- D $3 + 2 = m$
- E $3 \times 3 = f$
- F $2 - 3 = s$
- G $1 - 3 = n$
- H $3 + 1 = t$
- I $0 - 1 = b$
- J $1 + n = 0$
- K $3 + e = 0$
- L $2 \times 2 = w$

★ **6.** Copy and complete the multiplication and addition tables.

+	0	1	2	3
0				
1				
2				
3				

×	0	1	2	3
0				
1				
2				
3				

Reviewing the Ideas

1. Copy and complete each addition table for "twelve-clock" numbers.

A

+	6	8
6	12	
8		

B

+	7	9
5		
10		

2. Solve these equations for "twelve-clock" numbers.

A $7 + 6 = a$ C $1 + 12 = c$ E $8 + 6 = e$
B $9 + 9 = b$ D $10 + 4 = d$ F $12 + 11 = f$

3. Find the differences of these "twelve-clock" numbers.

A $8 - 7 = a$ C $8 - 9 = c$ E $8 - 11 = r$
B $8 - 8 = b$ D $8 - 10 = d$ F $8 - 12 = f$

4. Find the products of these "twelve-clock" numbers.

A $4 \times 4 = \mathrm{a}$ C $5 \times 5 = c$ E $3 \times 4 = e$
B $2 \times 8 = b$ D $11 \times 2 = d$ F $6 \times 3 = f$

5. Find two different "twelve-clock" numbers for n in this equation.

$$n \times 6 = 12$$

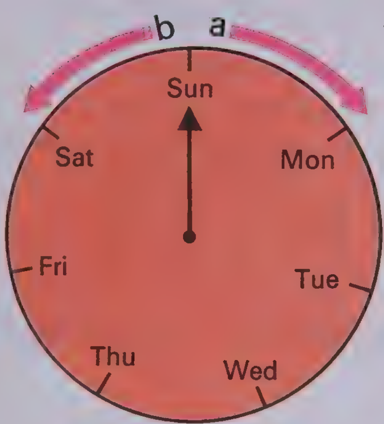
6. Find the sums and products of these "four-clock" numbers.

A $3 + 2 = m$
B $2 + 3 = n$
C $2 \times 1 = r$
D $2 \times 2 = s$
E $2 + 2 = t$
F $1 + 3 = u$
G $2 \times 3 = a$

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

"Four-clock" numbers

7. Study this special "clock" and the function tables. Try to find how the function rules work. Then give the missing symbols.



A Function Rule $\xrightarrow{3a}$

Input	Output
Mon	Thu
Wed	Sat
Sun	Wed
Fri	___
Sat	___

B Function Rule $\xrightarrow{5b}$

Input	Output
Mon	Wed
Wed	Fri
Sun	___
Fri	___
Sat	___

C Function Rule ___

Input	Output
Sun	Tue
Fri	Sun
Wed	Fri
Sat	Mon
Thu	___

8. Study the example. Then give the missing day.

Example: Mon $\xrightarrow{3a}$ Thu

A Thu $\xrightarrow{2a}$ ___ B Fri $\xrightarrow{2b}$ ___ C Sat $\xrightarrow{5a}$ ___

think

Here is a very special function table. The rule is not easy to guess but your work in clock arithmetic may help. Copy the table and give the missing rule and numbers.

Function Rule ___

n	Output
7	1
2	2
5	2
6	0
1	1
0	0
9	___
14	___

Keeping in Touch with

Computing
Measurement
Geometry
Place value
Functions
Principles
Fractional numbers
Number theory

1. Give the correct sign ($<$, $>$, $=$).

A $375 \; \blacksquare \; 300+60+5$

B $4793 \; \blacksquare \; 4000+800+90+3$

C $762 \; \blacksquare \; (7\times100)+(6\times10)+5$

D $9685 \; \blacksquare \; (97\times100)+(8\times10)+5$

2. Solve the equations.

A $8\times0=n$

B $54\times1=a$

C $96\div1=t$

D $(9\times6)\times5=9\times(n\times5)$

E $57+n=63+57$

F $0\div42=r$

G $8\times9=(n\times9)+(4\times9)$

H $(t\times8)\div8=60$

I $56-(n\times7)=0$

3. Give the number for *a*.
Then give the number for *b*.

A $(54+39)+78=a \longrightarrow 54+(39+78)=b$

B $50\times4\times10=a \longrightarrow 50\times40=b$

C $(3\times30)+(3\times2)=a \longrightarrow 3\times32=b$

D $24\times9=a \longrightarrow b\div24=9$

E $50\times283=a \longrightarrow 49\times283=b-283$

4. Give the missing numbers.

Function Rule

$(2\times n)+9$

	n	Output
A	6	$\blacksquare$
B	$\blacksquare$	29
C	37	$\blacksquare$

5. A jug holds 57.75 cubic centimetres. **Estimate** the number of cubic centimetres in 8 jugs.

6. Compute the following.

A $\begin{array}{r} 65 \\ +37 \\ \hline \end{array}$ B $\begin{array}{r} 96 \\ \times8 \\ \hline \end{array}$ C $\begin{array}{r} 83 \\ -48 \\ \hline \end{array}$ D $\begin{array}{r} 47 \\ \times24 \\ \hline \end{array}$ E $\begin{array}{r} 307 \\ -38 \\ \hline \end{array}$ F $\begin{array}{r} 72 \\ \times72 \\ \hline \end{array}$

G $\begin{array}{r} 875 \\ +367 \\ \hline \end{array}$ H $\begin{array}{r} 4987 \\ +9634 \\ \hline \end{array}$ I $\begin{array}{r} 3004 \\ -1865 \\ \hline \end{array}$ J $\begin{array}{r} 637 \\ \times456 \\ \hline \end{array}$ K $\begin{array}{r} 5083 \\ -2987 \\ \hline \end{array}$

L $7\overline{)168}$ M $60\overline{)540}$ N $9\overline{)4653}$ O $61\overline{)3275}$ P $45\overline{)3569}$

7. A What is the greatest common factor of 21 and 28?
   B Give in lowest terms: $\frac{21}{28}$
   C Construct a set of 10 equivalent fractions that includes $\frac{21}{28}$ and the lowest-terms fraction for $\frac{21}{28}$.
   D Draw a number line and locate the point for $\frac{21}{28}$ on the number line.
   E Which fraction in the set of fractions equivalent to $\frac{21}{28}$ would you use if you wanted to add $\frac{21}{28}$ to $\frac{5}{12}$?

8. Find the sums and differences.

   A $\frac{3}{8} + \frac{2}{8}$ B $4\frac{1}{5} + 6\frac{3}{5}$ C $\frac{17}{50} - \frac{1}{5}$ D $2\frac{6}{10} + 3\frac{1}{5}$ E $8\frac{1}{2} - 4\frac{2}{3}$ F $37\frac{3}{5} + 48\frac{9}{15}$

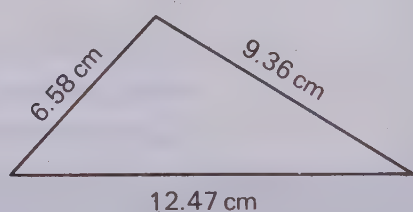
   G $\frac{3}{10} + \frac{41}{100}$ H $\frac{2}{3} + \frac{3}{4}$ I $5\frac{3}{5} + 6\frac{1}{4}$ J $8\frac{3}{5} - 6\frac{1}{10}$

9. Find the sums and differences.

   A $0.8 + 0.4$ B $0.28 + 0.56$ C $5.46 + 2.97$ D $8.43 - 2.76$ E $93.675 + 4.869$

   F $67.07 - 0.34$ G $9.687 + 8.34$ H $0.56 + 5.3$ I $4.684 - 0.89$

10. Solve the problems.
   A Spelling scores: 84, 95, 88, 97. Average score?
   B 75 kilometres per hour. Drive 13 hours. How many kilometres?
   C Height on birthday: 88.5 cm. Height next birthday: 91.25 cm. How much taller?
   D Travel 513 kilometres in 9 hours. How many kilometres per hour?
   E Find the perimeter.



Add, Subtract, Multiply, or Divide?

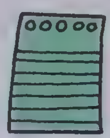
No numbers are given in these short story problems. Use **A**, **S**, **M**, or **D** to tell which operation (**A**ddition, **S**ubtraction, **M**ultiplication, or **D**ivision) you would use to solve each problem.

1. Bicycling: Rode ▯ km. Pushed ▯ km. How far in all?

2. Hiking: Walked and ran ▯ km.
Ran only ▯ km. Walked how far?

3. Bought ▯ candy bars for ▯ cents.
How much for one candy bar?

4. ▯ packages of notebook paper.
▯ sheets in each package.
How many sheets of paper in all?



5. John had ▯ marbles, Bill had ▯ marbles, and Mike had ▯ marbles. How many marbles all together?

6. ▯ rocks in each box.
▯ rocks in all.
How many boxes of rocks?

7. Dorothy bought ▯ stamps.
Paid ▯ cents for each stamp.
How much money did Dorothy spend?

8. Bought a bicycle for ▯.
Sold it for ▯.
Sold it for how much less?

9. Drove ▯ km.
Used ▯ litres of gas.
Drove how many km on 1 litre of gas?

★ 10. Given four different test grades.
What is the average test score?
(Hint: May use more than one operation.)

You are invited to explore

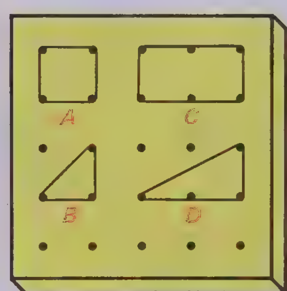
ACTIVITY CARD 17
Page 341

Mathematical Activities

How to Use the Activity Cards

Do you like to explore things for yourself? These Activity Cards will give you some exciting experiences with mathematics. Each card presents a different idea for you to explore. Often you will find that a card will give you ideas for additional activities on your own.

ACTIVITY CARD 1



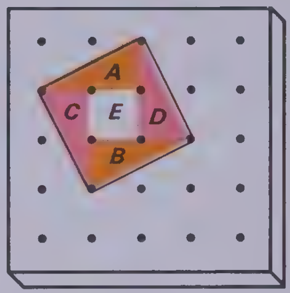
Every right triangle has an area that is half of some square or rectangle.

The area of square *A* is 1, so the area of triangle *B* is __?__.

The area of rectangle *C* is 2, so the area of triangle *D* is __?__.

How many **right triangles** of different sizes can you find on the geoboard and draw on dot paper? What is the area of each?

ACTIVITY CARD 2



What is the area of A? B? C? D? E?

What is the area of the large square?

Can you find on the geoboard and draw on dot paper a square with area 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 16? (All but three of these are possible.)

ACTIVITY CARD 3

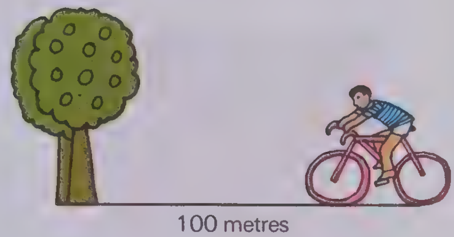
It might surprise you to find out how much water is wasted by a dripping faucet.

Turn on a faucet just enough so that it drips. Can you find how much water (and money) would be wasted by the dripping faucet in a year?

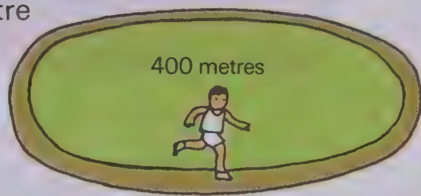
ACTIVITY CARD 4

How fast do you

- walk ?
- run ?
- ride a bicycle ?



Try figuring out at least one of these. (It will help if you measure off a 100-metre track or perhaps use a school track, which may be about 400 metres.)



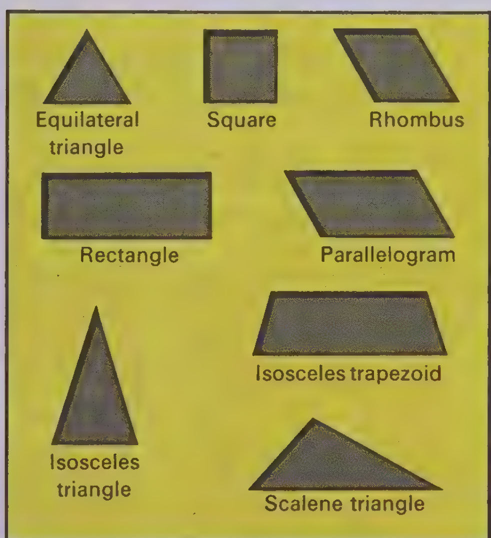
ACTIVITY CARD 5

Cut out figures from a sheet of cardboard so the holes look like this.

Mark an **F** on the front and a **B** on the back of each cutout figure.

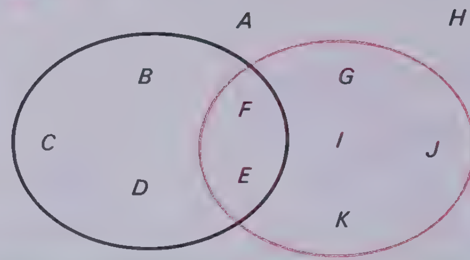
In how many different ways can you put each figure back into its hole ?

(Draw a picture to show the different ways for each.)



ACTIVITY CARD 6

Make a chart like the one below and see how many of the letters you can fill in.



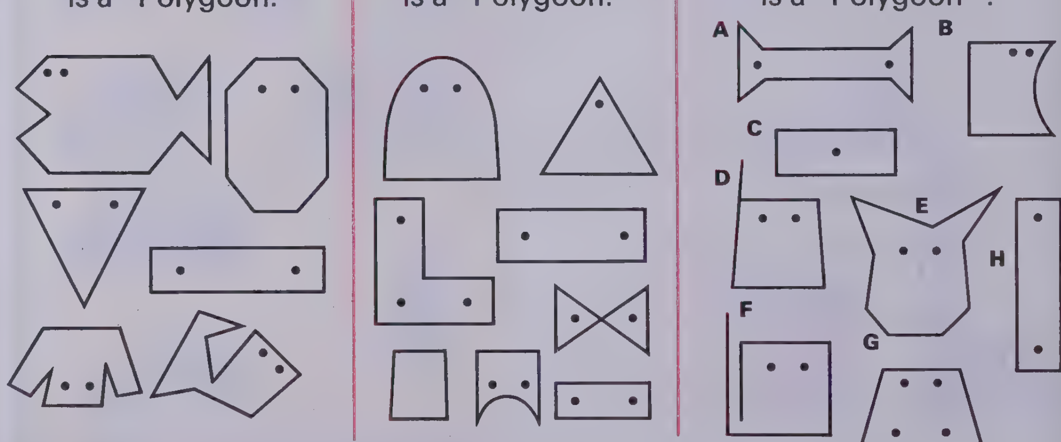
A list of the letters that are					
in but not ◯	in neither nor ◯	not in	in both and ◯	in or ◯	in ◯ but not

ACTIVITY CARD 7

Each of these is a "Polygoon."

None of these is a "Polygoon."

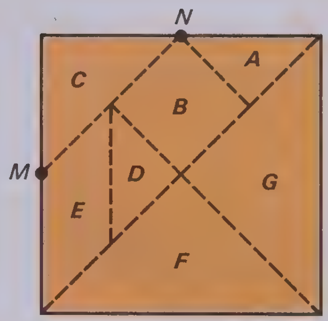
Which of these is a "Polygoon"?



Can you draw some "Polygoons" of your own?

ACTIVITY CARD 8

Draw a 8-cm square and cut it into 7 tangram pieces as shown. (*M* and *N* are the middle points of the sides.)



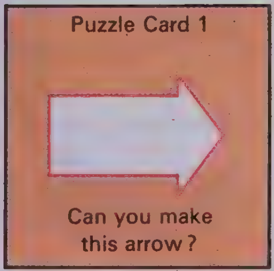
Can you place **all 7** pieces together to form a triangle? a rectangle? a parallelogram? a trapezoid? another interesting figure?

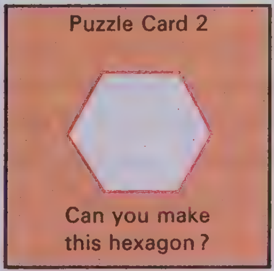
(Draw a picture to show how you made each figure.)

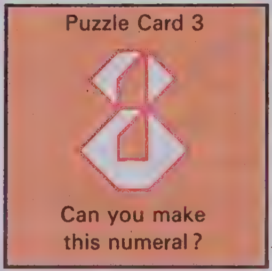
ACTIVITY CARD 9

Can you make a set of at least ten puzzle cards to use with the tangram pieces?

Here are three samples. Each card contains an exact outline of a figure that can be made by using **all seven** tangram pieces.

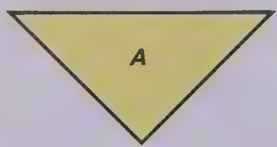


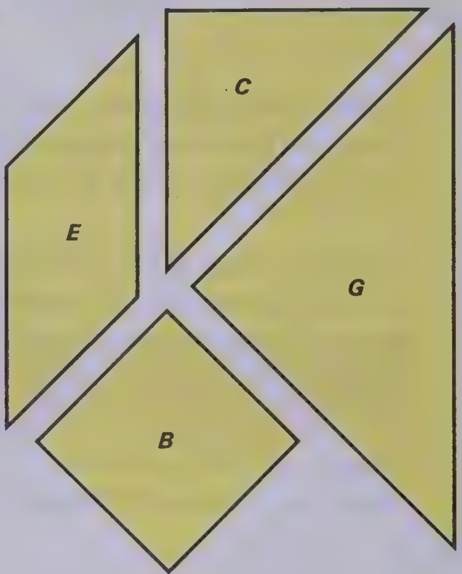




ACTIVITY CARD 10

The area of
tangram piece *A*
is 1 square unit.





Can you find the area of
these other tangram pieces?
(Use the tangram pieces
you made in **Activity 8.**)

ACTIVITY CARD 11

In how many different ways
can you measure yourself?

Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

Pulse	Length of step
Height	Number of calories used
Weight	Area of bottom of foot
Arm span	Distance you can jump
Grip strength	

ACTIVITY CARD 12

Write the letters R, E, and A on cards and put them in a hat. Draw them out of the hat.

Can you list all the ways you could draw them out? There are 6 ways.

	1st	2nd	3rd
One way is	R	E	A .

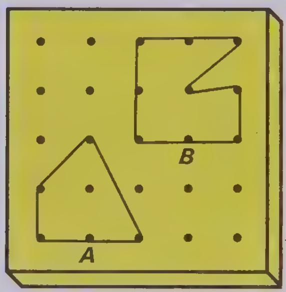
How many of these will form words?

Is it very likely that you will get a word on any one drawing?

ACTIVITY CARD 13

Figure *A* is a 4-sided polygon (quadrilateral) without "dents."

Figure *B* is a 6-sided polygon (hexagon) with one "dent."

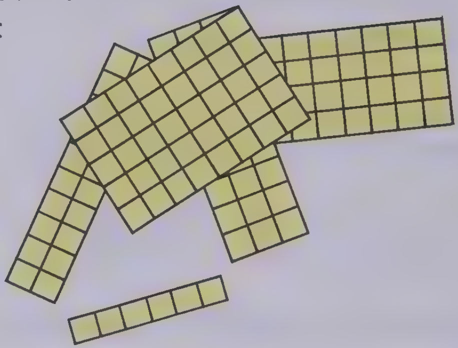


Can you find on the geoboard and draw on dot paper a polygon (without "dents") that has 5 sides? 6 sides? 7 sides? 8 sides? more than 8 sides?

(If you can, give the name of each polygon you find.)

ACTIVITY CARD 14

From 1-centimetre graph paper cut five rectangles with these dimensions:



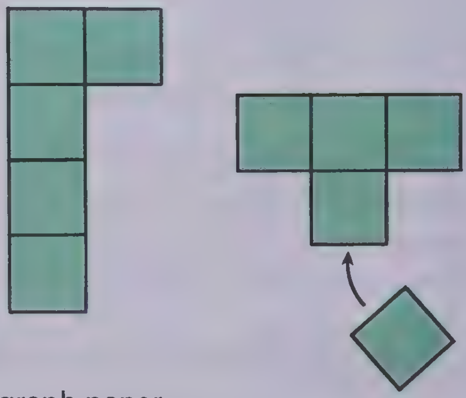
A 3 by 9 centimetres
B 5 by 8 centimetres
C 4 by 7 centimetres
D 10 by 2 centimetres
E 1 by 6 centimetres

Can you arrange the rectangles so that they form a square?

ACTIVITY CARD 15

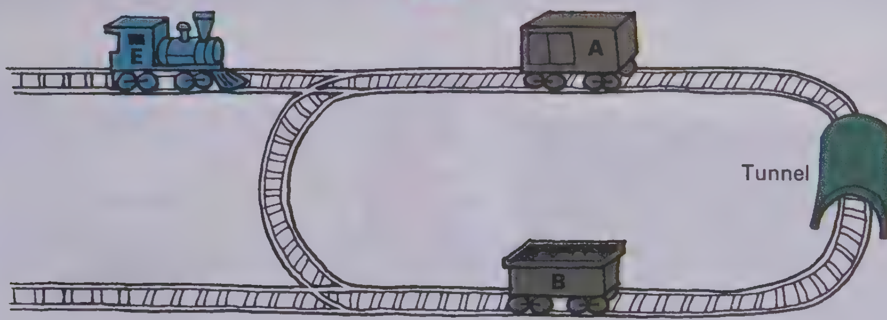
Cut out 5 square regions from cardboard.

How many different shapes can you form by placing the sides of the squares together, like this?



Show your different shapes on graph paper.

ACTIVITY CARD 16

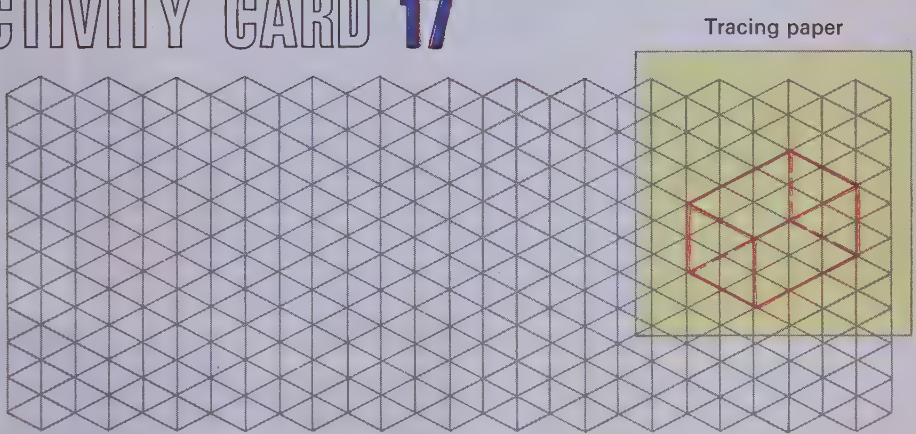


Here is a toy train problem.

Engine E can push or pull cars A and B.
The engine can go through the tunnel but the cars cannot.

Can you reverse the positions of A and B and put engine E back in its starting position?

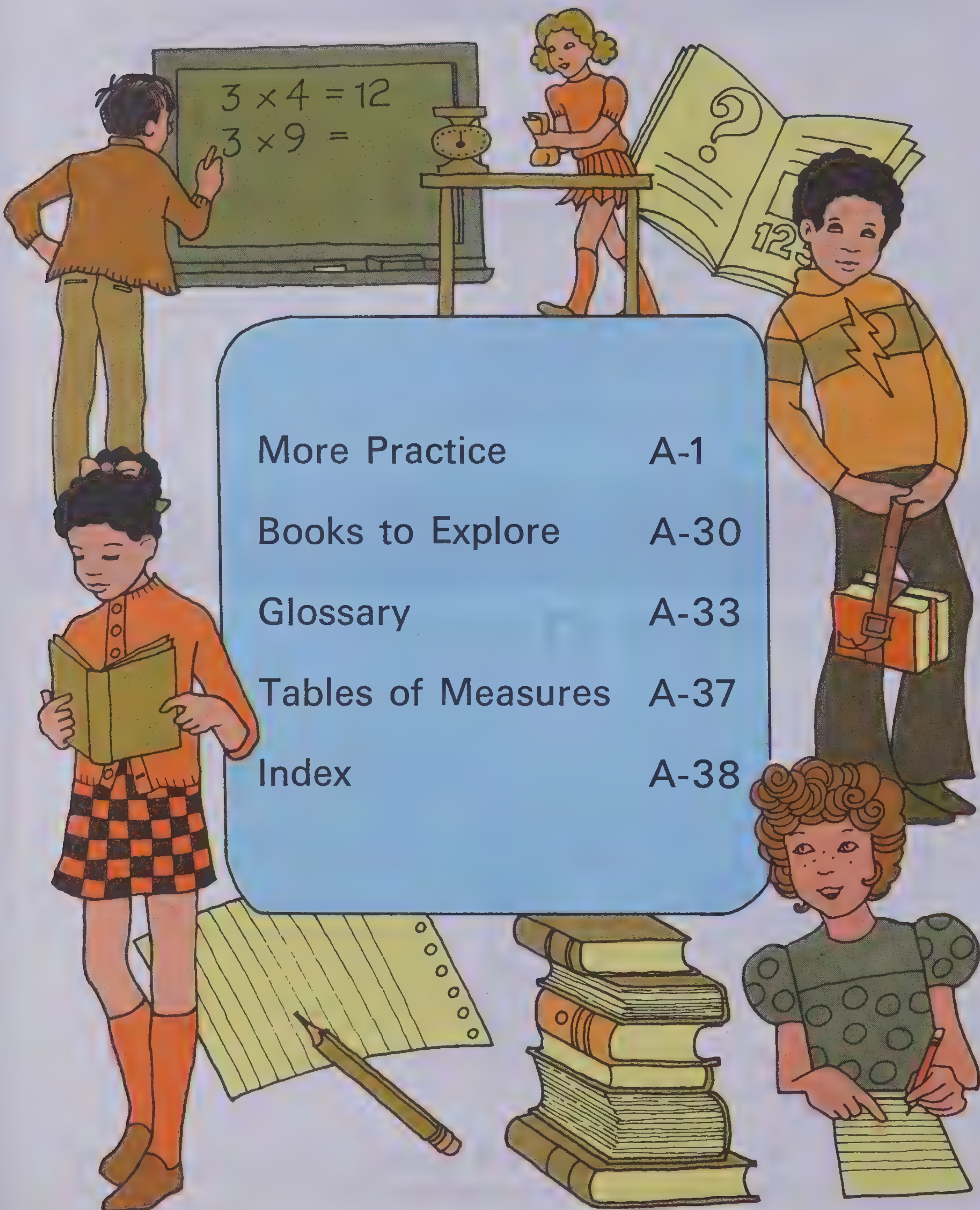
ACTIVITY CARD 17



Someone placed tracing paper over this grid and drew along some of the lines to make a box.

Can you use tracing paper and the grid to draw a box? a dog house? stairsteps? other interesting figures?

Appendix



Set 1 *For use with page 23*

Solve the equations.

1. $6385 = 6000 + n + 80 + 5$
2. $7082 = m + 80 + 2$
3. $3490 = 3000 + 400 + y$
4. $7233 = 7000 + v + 30 + 3$
5. $4106 = 4000 + 100 + b$
6. $1087 = t + 80 + 7$
7. $2194 = 2000 + p + 90 + 4$
8. $8183 = 8000 + 100 + s + 3$
9. $9007 = 9000 + n$
10. $6911 = j + 900 + 10 + 1$
11. $2613 = 2000 + 600 + m + 3$
12. $7508 = 7000 + 500 + b$
13. $9089 = 9000 + n + 9$
14. $8650 = 8000 + 600 + s$
15. $2930 = f + 900 + 30$
16. $3057 = 3000 + r + 7$
17. $2873 = 2000 + n + 70 + 3$
18. $6704 = 6000 + y + 4$

Reflected answers, Set 1: 1. 300, 2. 7000, 3. 90, 10. 6000, 11. 10, 12. 8

Set 2 *For use with page 25*

For the numeral 948 306 125, tell what digit is in each of these places.

1. hundreds'
2. millions'
3. ones'
4. hundred millions'
5. ten thousands'
6. ten millions'
7. hundred thousands'
8. tens'
9. thousands'

Use the numeral 607 854 932 to answer each exercise below.

10. 8 is in the __?__ place.
11. 3 is in the __?__ place.
12. 0 is in the __?__ place.
13. 7 is in the __?__ place.
14. 9 is in the __?__ place.
15. 6 is in the __?__ place.
16. 4 is in the __?__ place.
17. 5 is in the __?__ place.

Reflected answers, Set 2: 1. 1, 4. 9, 7. 3, 10. hundred thousands', 14. hundreds'

Set 3	*For use with page 27*

For each number, write an equation as in the example.
(Example: $7896 = 7000 + 800 + 90 + 6$)

1. 2358	**4.** 22 793	**7.** 772 853	**10.** 8 700 108
2. 5501	**5.** 52 438	**8.** 396 923	**11.** 4 502 824
3. 7483	**6.** 72 230	**9.** 790 374	**12.** 43 227 038

For each number, write an equation as in the example.
[Example: $3241 = (3 \times 1000) + (2 \times 100) + (4 \times 10) + 1$]

13. 6984	**15.** 5083	**17.** 40 261	**19.** 175 011
14. 7362	**16.** 17 626	**18.** 341 163	**20.** 2 419 421

Solve.

21. $900+70+4=x$	**24.** $(2\times1000)+(6\times100)+(1\times10)+9=r$
22. $1000+200+90+4=n$	**25.** $(3\times1000)+(2\times10)+6=t$
23. $(8\times100)+(3\times10)+7=q$	**26.** $(8\times1000)+(3\times100)+(5\times10)+2=z$

Reflected answers, Set 3: 1. 2000 + 300 + 50 + 8. 4. 20,000 + 2000 + 700 + 90 + 3. 7. 700,000 + 70,000 + 2000 + 800 + 50 + 3. 10. 8,000,000 + 700,000 + 100 + 8. 21. 974. 24. 2619

Set 4	*For use with page 29*

Give the correct sign (< or >) for each ●.

1. 73 ● 67	**7.** 8384 ● 7384	**13.** 795 668 ● 759 668
2. 206 ● 260	**8.** 6287 ● 6587	**14.** 437 202 ● 437 220
3. 437 ● 427	**9.** 4391 ● 4319	**15.** 661 369 ● 661 269
4. 734 ● 743	**10.** 26 844 ● 26 518	**16.** 9 264 577 ● 9 354 577
5. 896 ● 968	**11.** 39 263 ● 39 463	**17.** 8 242 357 ● 8 224 357
6. 519 ● 549	**12.** 59 393 ● 59 339	**18.** 8 320 811 ● 8 328 011

Reflected answers, Set 4: 1. >, 7. >, 13. >

Set 5 *For use with page 43*

Tell which operation or operations (+, −, ×, ÷) you would use to find the answer if numbers were given.

1. |||| insects.
   |||| legs each.
   How many legs in all?
2. Book |||| pages long.
   |||| pages in each chapter.
   How many chapters?
3. Had |||| dollars.
   Spent ||||.
   How much left?
4. |||| boys. |||| girls.
   |||| people on each team.
   How many teams?
5. Had |||| records.
   Bought |||| more.
   Gave away ||||.
   How many records now?
6. Drove |||| kilometres on Monday.
   |||| kilometres on Tuesday.
   Used |||| litres of gas.
   How many kilometres to one litre of gas?

Reflected answers, Set 5: 1. ×, 4. +, ÷

Set 6 *For use with page 45*

Rewrite each equation. Use an operation different from the one given.

1. $8 - 5 = 3$	**11.** $72 \div 9 = 8$	**21.** $4 \times 5 = 20$
2. $6 \times 4 = 24$	**12.** $9 + 8 = 17$	**22.** $32 \div 4 = 8$
3. $35 \div 5 = 7$	**13.** $11 - 6 = 5$	**23.** $18 + 7 = 25$
4. $16 + 5 = 21$	**14.** $63 \div 7 = 9$	**24.** $25 - 18 = 7$
5. $7 \times 7 = 49$	**15.** $9 \times 3 = 27$	**25.** $6 \times 6 = 36$
6. $36 \div 9 = 4$	**16.** $8 + 7 = 15$	**26.** $27 \div 3 = 9$
7. $7 + 6 = 13$	**17.** $15 \div 3 = 5$	**27.** $35 + 6 = 41$
8. $2 \times 9 = 18$	**18.** $3 \times 4 = 12$	**28.** $8 \times 7 = 56$
9. $15 - 9 = 6$	**19.** $12 - 8 = 4$	**29.** $63 - 54 = 9$
10. $4 \times 7 = 28$	**20.** $81 \div 9 = 9$	**30.** $5 \times 9 = 45$

Reflected answers, Set 6: 1. $3 + 5 = 8$, 2. $24 \div 6 = 4$ or $24 \div 4 = 6$
11. $9 \times 8 = 72$, 12. $17 - 9 = 8$ or $17 - 8 = 9$,
21. $20 \div 5 = 4$ or $20 \div 4 = 5$, 22. $4 \times 8 = 32$

Set 7 — *For use with page 51*

Find the missing addend in the addition equation.
Then write the subtraction equation with the correct difference.

1. $n + 9 = 16$ $16 - 9 = n$	**3.** $r + 4 = 13$ $13 - 4 = r$	**5.** $9 + q = 14$ $14 - 9 = q$	**7.** $s + 28 = 35$ $35 - 28 = s$
2. $8 + w = 13$ $13 - 8 = w$	**4.** $t + 6 = 11$ $11 - 6 = t$	**6.** $y + 74 = 82$ $82 - 74 = y$	**8.** $14 + v = 23$ $23 - 14 = v$

Find the missing factor in the multiplication equation.
Then write the division equation with the correct quotient.

9. $x \times 3 = 12$ $12 \div 3 = x$	**11.** $b \times 8 = 56$ $56 \div 8 = b$	**13.** $4 \times m = 36$ $36 \div 4 = m$	**15.** $c \times 8 = 64$ $64 \div 8 = c$
10. $7 \times w = 42$ $42 \div 7 = w$	**12.** $d \times 9 = 54$ $54 \div 9 = d$	**14.** $8 \times h = 40$ $40 \div 8 = h$	**16.** $k \times 6 = 48$ $48 \div 6 = k$

Reflected answers, Set 7: 1. $n = 7$, 3. $r = 9$, 5. $q = 5$, 7. $s = 7$

Set 8 — *For use with page 53*

Find as many of these as you can in 1 minute.

1. 2×4	**4.** 3×7	**7.** 7×8	**10.** 0×0	**13.** 9×2
2. 7×5	**5.** 5×9	**8.** 4×5	**11.** 8×4	**14.** 6×4
3. 0×8	**6.** 4×7	**9.** 3×9	**12.** 6×8	**15.** 5×5

Find as many of these as you can in $1\frac{1}{2}$ minutes.

16. $16 \div 4$	**19.** $25 \div 5$	**22.** $36 \div 9$	**25.** $81 \div 9$	**28.** $63 \div 7$
17. $16 \div 8$	**20.** $32 \div 4$	**23.** $24 \div 6$	**26.** $35 \div 7$	**29.** $0 \div 9$
18. $27 \div 3$	**21.** $40 \div 8$	**24.** $45 \div 9$	**27.** $16 \div 2$	**30.** $6 \div 3$

Reflected answers, Set 8: 1. 8, 4. 21, 7. 56, 10. 0, 13. 18, 16. 4, 19. 5, 22. 4, 25. 9, 28. 9

Set 9 — *For use with page 57*

Solve the equations.

1. $(2 \times 9) + 6 = m$	**11.** $(4 \times 5) + 9 = b$	**21.** $(7 \times n) + 4 = 25$
2. $(4 \times 7) + 5 = k$	**12.** $(7 \times 7) + 7 = h$	**22.** $(w \times 4) + 3 = 19$
3. $(6 \times 3) + 4 = j$	**13.** $(6 \times 9) + 3 = z$	**23.** $(4 \times p) + 5 = 37$
4. $(8 \times 3) + 2 = c$	**14.** $(5 \times 5) + 4 = a$	**24.** $(3 \times s) + 6 = 15$
5. $(4 \times 9) + 7 = s$	**15.** $(7 \times 8) + 2 = v$	**25.** $(c \times 1) + 7 = 14$
6. $(7 \times 5) + 2 = d$	**16.** $(6 \times 7) + 3 = r$	**26.** $(6 \times m) + 3 = 39$
7. $(8 \times 0) + 3 = q$	**17.** $(8 \times 2) + 9 = u$	**27.** $(j \times 5) + 6 = 16$
8. $(2 \times 6) + 6 = t$	**18.** $(9 \times 9) + 2 = x$	**28.** $(7 \times x) + 2 = 37$
9. $(8 \times 5) + 1 = w$	**19.** $(6 \times 5) + 0 = c$	**29.** $(i \times 6) + 4 = 28$
10. $(9 \times 7) + 8 = f$	**20.** $(4 \times 3) + 6 = t$	**30.** $(t \times 8) + 1 = 65$

Reflected answers, Set 9: 1. 24, 2. 33, 11. 29, 12. 56, 21. 3, 22. 4

Set 10 — *For use with page 65*

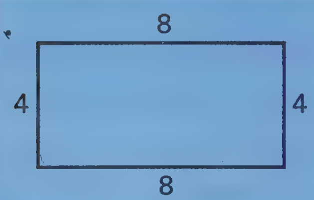
Solve the equations.

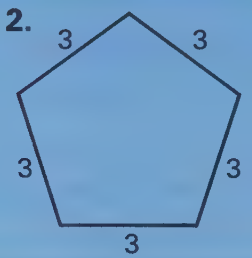
1. $9 \times 9 = (6 \times 9) + (s \times 9)$	**13.** $5 \times 43 = (5 \times m) + (5 \times 3)$
2. $7 \times 9 = (5 \times 9) + (r \times 9)$	**14.** $2 \times 24 = (2 \times 20) + (n \times 4)$
3. $8 \times 8 = (6 \times 8) + (b \times 8)$	**15.** $6 \times 14 = (c \times 10) + (6 \times 4)$
4. $5 \times 12 = (5 \times 10) + (5 \times q)$	**16.** $5 \times 21 = (5 \times 20) + (5 \times t)$
5. $4 \times 11 = (m \times 10) + (4 \times 1)$	**17.** $8 \times 32 = (8 \times j) + (8 \times 2)$
6. $6 \times 16 = (6 \times 10) + (t \times 6)$	**18.** $9 \times 19 = (9 \times 10) + (d \times 9)$
7. $5 \times 13 = (5 \times k) + (5 \times 3)$	**19.** $7 \times 23 = (k \times 20) + (7 \times 3)$
8. $7 \times 14 = (7 \times 10) + (7 \times n)$	**20.** $4 \times 27 = (4 \times p) + (4 \times 7)$
9. $9 \times 15 = (9 \times 10) + (z \times 5)$	**21.** $6 \times 48 = (6 \times 40) + (6 \times c)$
10. $4 \times 18 = (d \times 10) + (4 \times 8)$	**22.** $3 \times 62 = (3 \times 60) + (r \times 2)$
11. $3 \times 26 = (3 \times j) + (3 \times 6)$	**23.** $5 \times 36 = (h \times 30) + (5 \times 6)$
12. $7 \times 31 = (7 \times 30) + (7 \times a)$	**24.** $8 \times 29 = (8 \times x) + (8 \times 9)$

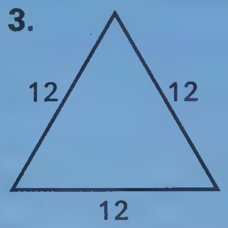
Reflected answers, Set 10: 1. 3, 2. 2, 3. 2, 13. 40, 14. 2, 15. 6

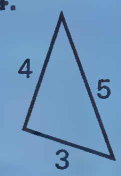
Set 11 *For use with page 89*

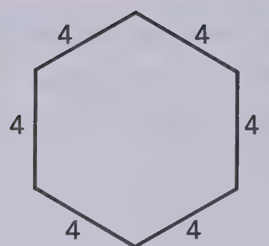
Find the perimeter of each figure.

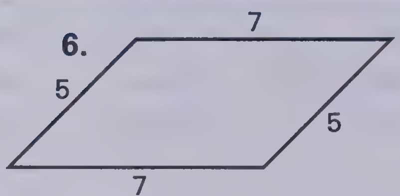
1.


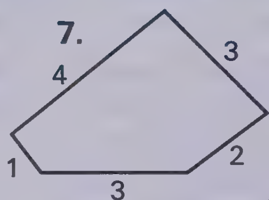
2.


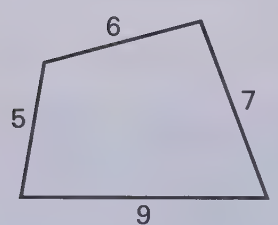
3.


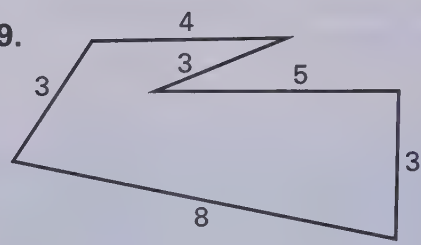
4.


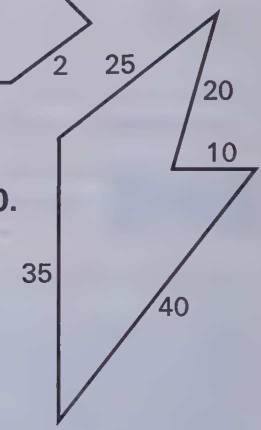
5.


6.


7.


8.


9.


10.


Reflected answers, Set 11: 1. 24, 2. 15, 3. 36, 4. 12

Set 12 *For use with page 93*

Find the area of the following triangles. The base and height are given.

1. $b = 6$, $h = 4$	**2.** $b = 7$, $h = 8$	**3.** $b = 5$, $h = 12$	**4.** $b = 13$, $h = 18$	**5.** $b = 9$, $h = 16$
6. $b = 30$, $h = 10$	**7.** $b = 20$, $h = 17$	**8.** $b = 22$, $h = 7$	**9.** $b = 14$, $h = 11$	**10.** $b = 24$, $h = 10$

Reflected answers, Set 12: 1. 12, 2. 28, 3. 30, 4. 117, 5. 72

Set 13 *For use with page 99*

Give the multiple of 10 that you think should go in each |||||.

1. To estimate 33 + 58, we can find the sum 30 + |||||.
2. To estimate 87 − 49, we can find the difference ||||| − 50.
3. To estimate 62 × 76, we can find the product 60 × |||||.
4. To estimate 348 + 213, we can find the sum 350 + |||||.
5. To estimate 477 ÷ 63, we can find the quotient ||||| ÷ 60.

Give the multiple of 100 that you think should go in each |||||.

6. To estimate 701 + 882, we can find the sum 700 + |||||.
7. To estimate 261 − 172, we can find the difference ||||| − 200.
8. To estimate 861 × 244, we can find the product 900 × |||||.
9. To estimate 778 × 419, we can find the product ||||| × 400.
10. To estimate 623 ÷ 196, we can find the quotient 600 ÷ |||||.

Reflected answers, Set 13: 1. 60, 6. 900

Set 14 *For use with page 101*

Find the products.

1. 9 × 30	**11.** 70 × 30	**21.** 300 × 40	**31.** 8 × 7000
2. 8 × 40	**12.** 40 × 70	**22.** 600 × 70	**32.** 90 × 9000
3. 40 × 7	**13.** 80 × 90	**23.** 40 × 200	**33.** 6000 × 40
4. 50 × 3	**14.** 70 × 60	**24.** 90 × 300	**34.** 20 × 3000
5. 6 × 50	**15.** 30 × 90	**25.** 800 × 40	**35.** 7000 × 50
6. 7 × 90	**16.** 50 × 40	**26.** 500 × 60	**36.** 500 × 500
7. 10 × 30	**17.** 20 × 90	**27.** 70 × 900	**37.** 700 × 800
8. 40 × 10	**18.** 70 × 50	**28.** 400 × 40	**38.** 200 × 400
9. 60 × 80	**19.** 60 × 30	**29.** 200 × 10	**39.** 600 × 100
10. 30 × 50	**20.** 40 × 90	**30.** 30 × 800	**40.** 900 × 300

Reflected answers, Set 14: 1. 270, 2. 320, 3. 280, 11. 2100, 12. 2800, 13. 7200, 21. 12,000, 22. 42,000, 23. 8000, 31. 56,000, 32. 810,000, 33. 240,000

Set 15 | *For use with page 103*

Solve.

1. 240 ÷ 6 = ***t***	**10.** 180 ÷ 60 = ***c***	**19.** 320 ÷ 40 = ***b***	**28.** 420 ÷ 60 = ***x***
2. 350 ÷ 7 = ***h***	**11.** 120 ÷ 30 = ***q***	**20.** 360 ÷ 40 = ***s***	**29.** 560 ÷ 70 = ***p***
3. 320 ÷ 4 = ***w***	**12.** 250 ÷ 50 = ***x***	**21.** 350 ÷ 50 = ***f***	**30.** 490 ÷ 70 = ***n***
4. 150 ÷ 3 = ***p***	**13.** 160 ÷ 40 = ***a***	**22.** 420 ÷ 70 = ***r***	**31.** 810 ÷ 90 = ***j***
5. 560 ÷ 7 = ***b***	**14.** 240 ÷ 30 = ***k***	**23.** 560 ÷ 80 = ***m***	**32.** 480 ÷ 60 = ***b***
6. 240 ÷ 8 = ***n***	**15.** 280 ÷ 70 = ***z***	**24.** 640 ÷ 80 = ***j***	**33.** 540 ÷ 90 = ***s***
7. 280 ÷ 7 = ***d***	**16.** 140 ÷ 20 = ***d***	**25.** 540 ÷ 60 = ***y***	**34.** 150 ÷ 30 = ***h***
8. 210 ÷ 3 = ***s***	**17.** 270 ÷ 30 = ***u***	**26.** 630 ÷ 70 = ***i***	**35.** 360 ÷ 60 = ***t***
9. 480 ÷ 6 = ***m***	**18.** 270 ÷ 90 = ***g***	**27.** 720 ÷ 90 = ***d***	**36.** 120 ÷ 60 = ***r***

Reflected answers, Set 15: 1. 40, 2. 50, 10. [illegible], 11. [illegible], 19. 8, 20. [illegible], 28. 7, 29. 8

Set 16 | *For use with page 107*

Choose the best estimate for each problem.

1. There are 52 weeks in one year. Choose the best estimate for the number of weeks in 7 years.
   A 420 weeks **B** 350 weeks **C** 280 weeks

2. A theatre has 47 seats in each row. Choose the best estimate for the number of seats in the theatre if there are 32 rows.
   A 1500 **B** 1200 **C** 150

3. 1 hour is 60 minutes. Choose the best estimate for the number of hours in 5390 minutes.
   A 100 **B** 90 **C** 80

Reflected answers, Set 16: 1. B

Set 17 *For use with page 115*

Find the sums.

1. 34 +45	**2.** 56 +87	**3.** 42 +57	**4.** 136 +157	**5.** 427 +382
6. 487 45 +262	**7.** 28 856 +28	**8.** 382 994 +481	**9.** 1481 464 +168	**10.** 562 2363 +3905
11. 283 636 34 +440	**12.** 1934 759 50 +621	**13.** 3281 5182 56 +354	**14.** 9533 65 536 +2292	**15.** 4116 8476 5022 +4273

Reflected answers, Set 17: 1. 79, 2. 143, 3. 99, 4. 293, 5. 809

Set 18 *For use with page 117*

Find the differences.

1. 39 −24	**2.** 51 −35	**3.** 416 −39	**4.** 165 −97	**5.** 367 −39
6. 356 −163	**7.** 873 −436	**8.** 721 −563	**9.** 500 −187	**10.** 802 −496
11. 1500 −59	**12.** 1608 −567	**13.** 5308 −795	**14.** 2834 −242	**15.** 7438 −842
16. 6900 −3486	**17.** 8043 −5964	**18.** 7004 −5437	**19.** 5702 −3986	**20.** 6700 −3985

Reflected answers, Set 18: 1. 15, 2. 16, 3. 377, 4. 68, 5. 328

Set 19 *For use with page 123*

Find the products.

1. 12×3	2. 34×5	3. 17×5	4. 56×4	5. 48×7	6. 76×4
7. 324×7	8. 572×9	9. 856×4	10. 457×3	11. 863×2	12. 294×7
13. 566×8	14. 354×9	15. 768×6	16. 9534×6	17. 7604×8	18. 6875×2
19. 2442×8	20. 3905×6	21. 5027×9	22. 9701×4	23. 6762×3	24. 1836×7

Reflected answers, Set 19: 1. 36, 2. 170, 3. 85, 4. 224, 5. 336, 6. 304

Set 20 *For use with page 125*

Find the products.

1. 24×15	2. 57×26	3. 33×54	4. 64×25	5. 65×34	6. 67×27
7. 79×18	8. 80×89	9. 253×23	10. 147×45	11. 568×63	12. 476×72
13. 935×87	14. 467×94	15. 851×56	16. 304×69	17. 436×275	18. 514×308
19. 175×302	20. 643×449	21. 863×705	22. 972×667	23. 781×708	24. 825×191

Reflected answers, Set 20: 1. 360, 2. 1482, 3. 1782, 4. 1600, 5. 2210, 6. 1809

Set 21 *For use with page 139*

Find the average of the numbers in each set.

1. {5, 8, 2}	**6.** {21, 34, 20}	**11.** {189, 373, 284}
2. {6, 7, 5}	**7.** {74, 65, 29}	**12.** {9, 12, 15, 16}
3. {9, 6, 2, 3}	**8.** {41, 24, 31}	**13.** {107, 56, 88, 97}
4. {8, 9, 9, 2}	**9.** {64, 80, 123}	**14.** {81, 76, 121, 54}
5. {37, 38, 54}	**10.** {137, 76, 75}	**15.** {465, 392, 436}

Reflected answers, Set 21: 1. 5, 6. 25, 11. 282

Set 22 *For use with page 141*

Find the quotients and remainders.

1. 2)15	**11.** 4)234	**21.** 9)198	**31.** 9)5264	**41.** 9)42 345
2. 4)30	**12.** 7)537	**22.** 4)297	**32.** 5)3957	**42.** 6)27 386
3. 3)19	**13.** 6)498	**23.** 5)596	**33.** 3)1832	**43.** 5)24 730
4. 5)34	**14.** 8)637	**24.** 8)166	**34.** 7)3254	**44.** 8)61 723
5. 7)52	**15.** 3)224	**25.** 3)856	**35.** 4)2376	**45.** 5)32 176
6. 9)65	**16.** 6)465	**26.** 8)340	**36.** 7)4552	**46.** 4)20 593
7. 2)13	**17.** 9)710	**27.** 8)701	**37.** 8)5839	**47.** 6)43 741
8. 3)23	**18.** 2)157	**28.** 6)164	**38.** 9)5555	**48.** 9)60 592
9. 6)29	**19.** 3)408	**29.** 3)575	**39.** 8)7356	**49.** 7)78 700
10. 5)38	**20.** 7)163	**30.** 4)437	**40.** 4)4859	**50.** 5)44 394

Reflected answers, Set 22: 1. 7R1, 2. 7R2, 11. 58R2, 12. 76R5,
21. 22, 22. 74R1, 31. 584R8, 32. 791R2, 41. 4705, 42. 4564R2

Set 23 *For use with page 145*

Find the quotients and remainders.

1. 40)332	**9.** 40)2560	**17.** 60)1964	**25.** 50)2150
2. 70)701	**10.** 30)2460	**18.** 20)1735	**26.** 80)4530
3. 20)129	**11.** 80)2290	**19.** 90)4983	**27.** 70)5540
4. 60)916	**12.** 70)3842	**20.** 60)4560	**28.** 90)7020
5. 50)487	**13.** 20)6621	**21.** 70)5833	**29.** 60)8365
6. 80)552	**14.** 50)2398	**22.** 80)2169	**30.** 20)4224
7. 30)264	**15.** 60)1176	**23.** 40)8825	**31.** 30)8183
8. 90)675	**16.** 40)2871	**24.** 50)9704	**32.** 40)3005

Reflected answers, Set 23: 1. 8R12, 2. 10R1, 9. 64, 10. 82, 17. 32R44, 18. 86R15, 25. 43, 26. 56R50

Set 24 *For use with page 157*

Find the quotients and remainders.

1. 29)946	**8.** 51)1020	**15.** 75)4782	**22.** 61)3729
2. 39)948	**9.** 65)5385	**16.** 31)9362	**23.** 36)2976
3. 58)875	**10.** 41)2876	**17.** 59)4200	**24.** 93)5760
4. 42)846	**11.** 72)5167	**18.** 78)6054	**25.** 28)1378
5. 38)856	**12.** 83)6572	**19.** 84)6720	**26.** 59)38 562
6. 39)978	**13.** 35)1050	**20.** 42)8735	**27.** 72)53 676
7. 52)679	**14.** 62)4965	**21.** 76)6450	**28.** 79)48 320

Reflected answers, Set 24: 1. 32R18, 2. 24R12, 8. 20, 9. 82R55, 15. 63R57, 16. 302, 22. 61R8, 23. 82R24

Set 25 *For use with page 161*

Find the quotients and remainders. Use the shortcut.

1. 32)256	**9.** 61)3176	**17.** 46)3875	**25.** 63)5463
2. 43)306	**10.** 35)2329	**18.** 29)1672	**26.** 65)5476
3. 35)216	**11.** 64)5462	**19.** 37)9786	**27.** 58)4683
4. 20)147	**12.** 41)3457	**20.** 19)1456	**28.** 29)2465
5. 58)558	**13.** 45)1574	**21.** 72)3972	**29.** 26)9541
6. 20)185	**14.** 35)2234	**22.** 68)4763	**30.** 68)5432
7. 59)236	**15.** 39)2654	**23.** 65)2976	**31.** 54)7490
8. 37)357	**16.** 57)3528	**24.** 52)2934	**32.** 49)4538

Reflected answers, Set 25: 1. 8, 2. 7R5, 9. 52R4, 10. 66R19, 17. 84R11, 18. 57R19, 25. 86R45, 26. 84R16

Set 26 *For use with page 163*

Find the quotients and remainders. Check your work.

1. 61)18 625	**4.** 38)58 364	**7.** 78)23 580	**10.** 74)29 656
2. 89)65 871	**5.** 85)58 403	**8.** 87)10 055	**11.** 22)16 387
3. 26)16 382	**6.** 92)74 614	**9.** 94)65 386	**12.** 18)13 428

Solve each story problem.

13. If 1152 students want yearbooks, how many dozen yearbooks must be ordered?

14. 14 168 people wish to go to a football game. If each bus can hold 28 people, how many buses are needed?

Reflected answers, Set 26: 1. 305R20, 4. 1535R34, 7. 302R24, 10. 400R56

Set 27 *For use with page 165*

Find the answers to these money problems.

1. \$ 3.42 × 5	**2.** \$ 0.86 × 6	**3.** \$10.67 × 8	**4.** \$ 3.50 × 34	**5.** \$ 7.35 × 67
6. \$ 4.31 × 54	**7.** \$ 5.17 × 12	**8.** \$ 8.06 × 34	**9.** \$56.30 × 25	**10.** \$47.85 × 56
11. \$20.57 × 98	**12.** \$32.70 × 86	**13.** \$20.42 × 9	**14.** \$53.02 × 27	**15.** \$26.57 × 74
16. 5)\$0.55	**17.** 7)\$6.30	**18.** 4)\$2.88	**19.** 3)\$12.15	**20.** 4)\$16.40
21. 29)\$1.45	**22.** 51)\$4.08	**23.** 75)\$4.50	**24.** 65)\$4.55	**25.** 31)\$2.17
26. 31)\$18.60	**27.** 49)\$98.98	**28.** 32)\$20.48	**29.** 76)\$72.20	**30.** 23)\$10.81

Solve the problems.

31. Dave earns \$4.95 a week delivering papers. How much will he earn in 6 weeks?

32. A 12-gram package costs \$0.84. How much is this per gram?

33. Glenn spends \$1.65 each week for lunches. How much does he spend in one month (4 weeks) for lunches?

34. Mr. Wall paid \$2.48 for gas for his car. If he got 14 litres of gas, how much did he pay per litre?

35. Nick saves \$20.50 each month. How much will he save in one year?

36. Bill wants to save enough for a \$66.40 bicycle in 16 weeks. How much must he save each week?

Reflected answers, Set 27: 1. \$17.10, 2. \$5.16, 3. \$85.36, 4. \$119.00, 5. \$492.45, 26. \$0.60, 27. \$2.02, 28. \$0.64, 29. \$0.95, 30. \$0.47

Set 28 *For use with page 173*

List all the factors of each number.

1. 18	**3.** 40	**5.** 56	**7.** 64	**9.** 72
2. 24	**4.** 54	**6.** 63	**8.** 69	**10.** 75

Give the missing numbers.

11. If 10 is a factor of a number, then |||| and |||| are factors of that number.

12. If 14 is a factor of a number, then |||| and |||| are factors of that number.

13. If 2 and 11 are factors of a number, then |||| is a factor of the number.

14. If 3 and 5 are factors of a number, then |||| is a factor of the number.

Reflected answers, Set 28: 1. {1, 2, 3, 6, 9, 18}, 3. {1, 2, 4, 5, 8, 10, 20, 40}, 5. {1, 2, 4, 7, 8, 14, 28, 56}, 7. {1, 2, 4, 8, 16, 32, 64}, 9. {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}

Set 29 *For use with page 177*

For each exercise, give the union and the intersection of the two sets.

1. S = {4, 7, 10}
T = {2, 4}

2. A = {0, 1, 2, 3, 4, 5}
B = {2, 4, 6, 8, 10}

3. M = {8, 9, 10}
N = {9, 10, 12, 13}

4. C = {0, 1, 2, 3, 4, 5, 6, 7, 8}
D = {8, 9, 10}

5. Q = {0, 1, 2}
R = {4, 5, 6}

6. E = {4, 5, 6, 7, 8, 9, 10}
F = {1, 2, 3, 4}

Reflected answers, Set 29: 1. S ∪ T = {2,4,7,10}, S ∩ T = {4}, 2. A ∪ B = {0,1,2,3,4,5,6,8,10}, A ∩ B = {2,4}

Set 30 *For use with page 179*

1. List the factors of 10.
2. List the factors of 15.
3. List the common factors of 10 and 15.
4. What is the greatest common factor of 10 and 15?
5. List the factors of 12.
6. List the factors of 27.
7. List the common factors of 12 and 27.
8. What is the greatest common factor of 12 and 27?

Give the greatest common factor of each pair of numbers.

9. 8, 20	**12.** 28, 35	**15.** 36, 24	**18.** 10, 16
10. 9, 24	**13.** 26, 6	**16.** 12, 21	**19.** 15, 20
11. 4, 17	**14.** 14, 28	**17.** 2, 13	**20.** 18, 30

Reflected answers, Set 30: 1. {1,2,5,10}, 2. {1,3,5,15}, 3. {1,5}, 4. 5

Set 31 *For use with page 181*

1. List the multiples (to 36) of 6.
2. List the multiples (to 40) of 4.
3. List the common multiples of 4 and 6.
4. Give the least common multiple of 4 and 6.

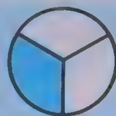
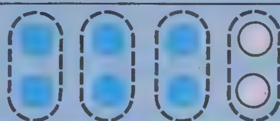
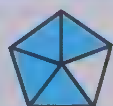
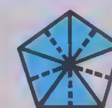
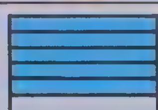
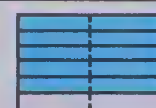
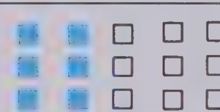
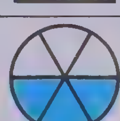
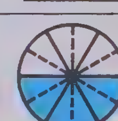
Give the least common multiple for each pair of numbers.

5. 2 and 7	**9.** 13 and 3	**13.** 10 and 3	**17.** 12 and 8
6. 8 and 3	**10.** 5 and 8	**14.** 4 and 12	**18.** 2 and 9
7. 9 and 5	**11.** 2 and 6	**15.** 9 and 6	**19.** 12 and 2
8. 7 and 4	**12.** 6 and 7	**16.** 10 and 2	**20.** 3 and 9

4. 12, 5. 14, 9. 39, 13. 30, 17. 24
2. {0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40}, 3. {0, 12, 24, 36},
Reflected answers, Set 31: 1. {0, 6, 12, 18, 24, 30, 36},

Set 32 | *For use with page 191*

Find the missing fractions.

1.		____ is equivalent to $\frac{2}{6}$.	
2.		$\frac{2}{5}$ is equivalent to ____.	
3.		____ is equivalent to ____.	
4.		____ is equivalent to ____.	
5.		____ is equivalent to ____.	
6.		____ is equivalent to ____.	
7.		____ is equivalent to ____.	
8.		____ is equivalent to ____.	
9.		____ is equivalent to ____.	
10.		____ is equivalent to ____.	

Reflected answers, Set 32: 1. $\frac{1}{3}$ 2. $\frac{4}{10}$ 3. $\frac{3}{4}$, $\frac{6}{8}$

Set 33 *For use with page 193*

Give the next three fractions in each set.

1. $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \ldots\}$
2. $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}, \ldots\}$
3. $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \ldots\}$
4. $\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \frac{5}{40}, \ldots\}$
5. $\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \frac{12}{28}, \frac{15}{35}, \ldots\}$
6. $\{\frac{5}{9}, \frac{10}{18}, \frac{15}{27}, \frac{20}{36}, \frac{25}{45}, \ldots\}$
7. $\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \frac{5}{50}, \ldots\}$
8. $\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \frac{35}{50}, \ldots\}$
9. $\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \ldots\}$
10. $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \ldots\}$

Reflected answers, Set 33: 1. $\{\frac{6}{18}, \frac{7}{21}, \frac{8}{24}\}$ 6. $\{\frac{30}{54}, \frac{35}{63}, \frac{40}{72}\}$

Set 34 *For use with page 197*

Which pairs of fractions are equivalent?

1. $\frac{3}{4}, \frac{7}{9}$
2. $\frac{1}{5}, \frac{20}{100}$
3. $\frac{5}{3}, \frac{25}{15}$
4. $\frac{2}{3}, \frac{12}{15}$
5. $\frac{4}{7}, \frac{7}{10}$
6. $\frac{4}{5}, \frac{17}{20}$
7. $\frac{3}{10}, \frac{30}{100}$
8. $\frac{1}{2}, \frac{3}{5}$
9. $\frac{8}{3}, \frac{40}{16}$
10. $\frac{3}{2}, \frac{6}{4}$
11. $\frac{3}{10}, \frac{15}{50}$
12. $\frac{9}{5}, \frac{7}{4}$
13. $\frac{1}{3}, \frac{3}{8}$
14. $\frac{8}{5}, \frac{17}{10}$
15. $\frac{2}{5}, \frac{5}{10}$
16. $\frac{6}{15}, \frac{4}{10}$
17. $\frac{7}{10}, \frac{13}{20}$
18. $\frac{8}{6}, \frac{4}{3}$
19. $\frac{9}{3}, \frac{12}{4}$
20. $\frac{15}{20}, \frac{3}{4}$
21. $\frac{24}{20}, \frac{18}{15}$
22. $\frac{12}{5}, \frac{14}{6}$
23. $\frac{18}{4}, \frac{15}{3}$
24. $\frac{75}{100}, \frac{15}{20}$
25. $\frac{24}{15}, \frac{16}{10}$
26. $\frac{8}{3}, \frac{14}{5}$
27. $\frac{17}{9}, \frac{13}{6}$
28. $\frac{5}{2}, \frac{25}{20}$
29. $\frac{7}{10}, \frac{35}{50}$
30. $\frac{20}{6}, \frac{50}{15}$

Reflected answers, Set 34: 1. not equivalent, 2. equivalent, 11. equivalent, 12. not equivalent, 21. equivalent, 22. not equivalent

Set 35 *For use with page 201*

Give the lowest-terms fraction for each of the following:

1. $\frac{5}{25}$	7. $\frac{8}{20}$	13. $\frac{9}{24}$	19. $\frac{12}{20}$	25. $\frac{25}{100}$	31. $\frac{30}{100}$	37. $\frac{5}{10}$
2. $\frac{15}{20}$	8. $\frac{4}{30}$	14. $\frac{9}{15}$	20. $\frac{8}{30}$	26. $\frac{8}{18}$	32. $\frac{12}{30}$	38. $\frac{40}{60}$
3. $\frac{6}{10}$	9. $\frac{8}{24}$	15. $\frac{12}{20}$	21. $\frac{4}{50}$	27. $\frac{4}{20}$	33. $\frac{35}{100}$	39. $\frac{60}{90}$
4. $\frac{3}{30}$	10. $\frac{6}{15}$	16. $\frac{6}{30}$	22. $\frac{8}{10}$	28. $\frac{10}{30}$	34. $\frac{10}{40}$	40. $\frac{15}{30}$
5. $\frac{2}{4}$	11. $\frac{7}{35}$	17. $\frac{25}{100}$	23. $\frac{2}{3}$	29. $\frac{25}{50}$	35. $\frac{5}{20}$	41. $\frac{25}{40}$
6. $\frac{60}{100}$	12. $\frac{5}{30}$	18. $\frac{1}{10}$	24. $\frac{9}{30}$	30. $\frac{70}{100}$	36. $\frac{14}{30}$	42. $\frac{14}{21}$

Reflected answers, Set 35: 1. $\frac{1}{5}$, 7. $\frac{2}{5}$, 13. $\frac{3}{8}$, 19. $\frac{3}{5}$, 25. $\frac{1}{4}$, 31. $\frac{3}{10}$, 37. $\frac{1}{2}$

Set 36 *For use with page 211*

Give the correct point for the fractional number that is indicated by the set of equivalent fractions.

1.


$\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$

2.


$\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\}$

3.


$\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$

4.


$\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \ldots\}$

5.


$\{\frac{7}{8}, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \ldots\}$

6.


$\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \ldots\}$

Reflected answers, Set 36: 1. B, 2. B

Set 37 *For use with page 213*

Give the missing numerator or denominator.

1. $\frac{1}{2} = \frac{\square}{4}$
2. $\frac{1}{10} = \frac{5}{\square}$
3. $\frac{1}{7} = \frac{\square}{14}$
4. $\frac{1}{3} = \frac{5}{\square}$
5. $\frac{2}{5} = \frac{\square}{15}$
6. $\frac{3}{8} = \frac{6}{\square}$
7. $\frac{2}{3} = \frac{\square}{9}$
8. $\frac{3}{5} = \frac{6}{\square}$
9. $\frac{3}{4} = \frac{\square}{100}$
10. $\frac{1}{5} = \frac{4}{\square}$
11. $\frac{1}{4} = \frac{\square}{20}$
12. $\frac{7}{10} = \frac{14}{\square}$
13. $\frac{1}{6} = \frac{\square}{30}$
14. $\frac{5}{10} = \frac{10}{\square}$
15. $\frac{4}{5} = \frac{\square}{25}$
16. $\frac{5}{7} = \frac{10}{\square}$
17. $\frac{3}{10} = \frac{\square}{20}$
18. $\frac{7}{10} = \frac{35}{\square}$
19. $\frac{1}{8} = \frac{\square}{16}$
20. $\frac{1}{9} = \frac{\square}{27}$
21. $\frac{1}{10} = \frac{\square}{10}$
22. $\frac{4}{5} = \frac{12}{\square}$
23. $\frac{1}{4} = \frac{\square}{4}$
24. $\frac{7}{10} = \frac{21}{\square}$
25. $\frac{2}{5} = \frac{\square}{20}$

Reflected answers, Set 37: 1. 2, 6. 16, 11. 5, 16. 14, 21. 1

Set 38 *For use with page 215*

Give the correct sign ($>$, $=$, or $<$) for each $\square$.

1. $\frac{1}{4} \square \frac{1}{3}$
2. $\frac{1}{2} \square \frac{2}{5}$
3. $\frac{1}{10} \square \frac{1}{4}$
4. $\frac{1}{4} \square \frac{2}{10}$
5. $\frac{7}{8} \square \frac{3}{4}$
6. $\frac{1}{3} \square \frac{4}{6}$
7. $\frac{4}{9} \square \frac{5}{8}$
8. $\frac{1}{3} \square \frac{2}{5}$
9. $\frac{3}{4} \square \frac{15}{20}$
10. $\frac{3}{5} \square \frac{9}{15}$
11. $\frac{3}{4} \square \frac{9}{10}$
12. $\frac{3}{5} \square \frac{2}{3}$
13. $\frac{1}{2} \square \frac{50}{100}$
14. $\frac{1}{3} \square \frac{5}{20}$
15. $\frac{7}{8} \square \frac{3}{4}$
16. $\frac{10}{20} \square \frac{4}{8}$
17. $\frac{5}{8} \square \frac{3}{8}$
18. $\frac{3}{10} \square \frac{1}{3}$
19. $\frac{5}{6} \square \frac{4}{5}$
20. $\frac{4}{7} \square \frac{3}{4}$
21. $\frac{2}{3} \square \frac{3}{4}$
22. $\frac{3}{5} \square \frac{7}{10}$
23. $\frac{25}{100} \square \frac{1}{4}$
24. $\frac{1}{2} \square \frac{5}{8}$
25. $\frac{1}{3} \square \frac{2}{6}$
26. $\frac{2}{3} \square \frac{5}{6}$
27. $\frac{9}{10} \square \frac{4}{5}$
28. $\frac{3}{4} \square \frac{15}{20}$
29. $\frac{4}{5} \square \frac{11}{15}$
30. $\frac{3}{8} \square \frac{2}{5}$
31. $\frac{28}{50} \square \frac{1}{2}$
32. $\frac{5}{30} \square \frac{1}{6}$
33. $\frac{1}{2} \square \frac{1}{3}$
34. $\frac{1}{4} \square \frac{2}{8}$
35. $\frac{10}{30} \square \frac{3}{9}$
36. $\frac{3}{4} \square \frac{3}{8}$
37. $\frac{2}{3} \square \frac{3}{5}$
38. $\frac{7}{10} \square \frac{3}{4}$
39. $\frac{1}{3} \square \frac{2}{5}$
40. $\frac{2}{3} \square \frac{5}{7}$

Reflected answers, Set 38: 1. <, 2. >, 11. <, 12. <, 21. <, 22. <, 31. >, 32.

Set 39 — *For use with page 217*

Solve each short story problem.

1. Gail's hand: 9 centimetres long.
   Mark's hand: 1 decimetre.
   Who has the longer hand?

2. Henry ate 4 eggs.
   Angie ate $\frac{1}{4}$ dozen eggs.
   Who ate more eggs?

3. Bob ate dinner in $\frac{1}{4}$ hour.
   It took Alex 20 minutes to eat dinner. Who ate faster?

4. Ann bought $\frac{1}{2}$ metre of material.
   Joan bought $\frac{3}{5}$ metre.
   Who bought more material?

5. Frank worked $\frac{3}{4}$ hour.
   Pete worked 50 minutes.
   Who worked longer?

6. Sam read $\frac{3}{8}$ of his book.
   Tony read $\frac{1}{2}$ of the same book.
   Who read more?

Reflected answers, Set 39: 1. Mark, 2. Henry

Set 40 — *For use with page 219*

Solve each story problem.

1. The ratio of boys to girls is 2 to 3. There are 15 girls. How many boys are there?

2. The ratio of tires to cars is 4 to 1. There are 24 tires. How many cars are there?

3. The ratio of tables to chairs is 1 to 6. There are 5 tables. How many chairs are there?

4. The ratio of swings to students is 4 to 15. There are 30 students. How many swings are there?

5. The ratio of eggs to cartons is 12 to 1. There are 5 cartons. How many eggs are there?

6. The ratio of dogs to bones is 3 to 8. There are 9 dogs. How many bones are there?

Reflected answers, Set 40: 1. 10, 2. 6

Set 41 *For use with page 227*

Find the sums and differences.

1. $\frac{1}{5} + \frac{3}{5}$	**6.** $\frac{6}{10} - \frac{1}{10}$	**11.** $\frac{3}{9} - \frac{2}{9}$	**16.** $\frac{2}{15} + \frac{12}{15}$	**21.** $\frac{6}{5} - \frac{2}{5}$
2. $\frac{7}{8} + \frac{7}{8}$	**7.** $\frac{7}{8} + \frac{4}{8}$	**12.** $\frac{7}{25} - \frac{4}{25}$	**17.** $\frac{7}{10} - \frac{6}{10}$	**22.** $\frac{9}{20} - \frac{4}{20}$
3. $\frac{6}{4} - \frac{5}{4}$	**8.** $\frac{8}{30} + \frac{2}{30}$	**13.** $\frac{6}{7} - \frac{1}{7}$	**18.** $\frac{2}{10} + \frac{7}{10}$	**23.** $\frac{25}{100} - \frac{20}{100}$
4. $\frac{4}{25} + \frac{7}{25}$	**9.** $\frac{3}{8} + \frac{2}{8}$	**14.** $\frac{7}{15} - \frac{5}{15}$	**19.** $\frac{3}{5} + \frac{3}{5}$	**24.** $\frac{3}{10} + \frac{3}{10}$
5. $\frac{4}{7} + \frac{5}{7}$	**10.** $\frac{21}{100} + \frac{19}{100}$	**15.** $\frac{7}{10} + \frac{9}{10}$	**20.** $\frac{4}{5} - \frac{1}{5}$	**25.** $\frac{9}{20} + \frac{7}{20}$

Solve each short story problem.

26. Drove $\frac{3}{10}$ kilometre,
then drove $\frac{6}{10}$ kilometre farther.
Drove how far?

27. Read $\frac{3}{8}$ of a book,
then read $\frac{1}{8}$ more. Read
how much of the book?

Reflected answers, Set 41: 1. $\frac{4}{5}$, 6. $\frac{5}{10}$ or $\frac{1}{2}$, 11. $\frac{1}{9}$, 16. $\frac{14}{15}$, 21. $\frac{4}{5}$

Set 42 *For use with page 231*

Make lists of equivalent fractions until you find 2 with the same denominator.
Then find the sum or difference.

1. $\frac{1}{3} + \frac{1}{6}$	**4.** $\frac{1}{2} - \frac{1}{4}$	**7.** $\frac{3}{4} + \frac{1}{8}$	**10.** $\frac{3}{8} - \frac{1}{4}$	**13.** $\frac{2}{3} - \frac{2}{9}$
2. $\frac{1}{3} - \frac{1}{6}$	**5.** $\frac{3}{10} + \frac{1}{4}$	**8.** $\frac{3}{4} - \frac{1}{8}$	**11.** $\frac{2}{3} - \frac{1}{5}$	**14.** $\frac{1}{2} + \frac{1}{10}$
3. $\frac{1}{2} + \frac{1}{4}$	**6.** $\frac{3}{10} - \frac{1}{4}$	**9.** $\frac{1}{4} + \frac{3}{8}$	**12.** $\frac{2}{3} + \frac{1}{5}$	**15.** $\frac{1}{2} - \frac{1}{10}$

16. $\begin{array}{r} \frac{4}{5} \\ +\frac{1}{10} \\ \hline \end{array}$	**17.** $\begin{array}{r} \frac{4}{5} \\ -\frac{1}{10} \\ \hline \end{array}$	**18.** $\begin{array}{r} \frac{2}{3} \\ -\frac{1}{2} \\ \hline \end{array}$	**19.** $\begin{array}{r} \frac{2}{3} \\ +\frac{1}{2} \\ \hline \end{array}$	**20.** $\begin{array}{r} \frac{3}{5} \\ +\frac{1}{10} \\ \hline \end{array}$	**21.** $\begin{array}{r} \frac{3}{5} \\ -\frac{1}{10} \\ \hline \end{array}$

Reflected answers, Set 42: 1. $\frac{3}{6}$ or $\frac{1}{2}$, 4. $\frac{1}{4}$, 7. $\frac{7}{8}$, 10. $\frac{1}{8}$, 13. $\frac{4}{9}$

Set 43 *For use with page 235*

Give the missing numerators.

1. $\frac{2}{5} = \frac{\square}{10}$ 3. $\frac{2}{3} = \frac{\square}{15}$ 5. $\frac{3}{10} = \frac{\square}{50}$ 7. $\frac{1}{2} = \frac{\square}{6}$ 9. $\frac{1}{5} = \frac{\square}{100}$

2. $\frac{3}{5} = \frac{\square}{15}$ 4. $\frac{1}{2} = \frac{\square}{20}$ 6. $\frac{3}{5} = \frac{\square}{15}$ 8. $\frac{4}{10} = \frac{\square}{30}$ 10. $\frac{2}{5} = \frac{\square}{50}$

Find the sums and differences.

11. $\frac{1}{4} + \frac{1}{10}$ 12. $\frac{1}{4} - \frac{1}{10}$ 13. $\frac{1}{4} + \frac{1}{6}$ 14. $\frac{1}{4} - \frac{1}{6}$ 15. $\frac{3}{10} + \frac{7}{20}$ 16. $\frac{7}{20} - \frac{3}{10}$

17. $\frac{7}{10} - \frac{1}{4}$ 18. $\frac{1}{4} + \frac{7}{10}$ 19. $\frac{2}{3} - \frac{1}{2}$ 20. $\frac{2}{3} + \frac{1}{2}$ 21. $\frac{3}{8} - \frac{1}{4}$ 22. $\frac{1}{4} + \frac{3}{8}$

23. $\frac{1}{2} + \frac{1}{6}$ 24. $\frac{3}{4} - \frac{1}{8}$ 25. $\frac{7}{10} + \frac{3}{5}$ 26. $\frac{7}{10} - \frac{3}{5}$ 27. $\frac{3}{10} + \frac{41}{100}$ 28. $\frac{8}{10} - \frac{7}{100}$

29. $\frac{1}{3} - \frac{1}{6}$ 31. $\frac{5}{6} + \frac{1}{2}$ 33. $\frac{1}{4} + \frac{7}{8}$ 35. $\frac{1}{3} + \frac{2}{9}$ 37. $\frac{50}{100} - \frac{2}{10}$

30. $\frac{1}{4} + \frac{1}{8}$ 32. $\frac{2}{3} - \frac{2}{5}$ 34. $\frac{1}{4} - \frac{1}{5}$ 36. $\frac{7}{9} + \frac{2}{3}$ 38. $\frac{2}{10} - \frac{18}{100}$

Solve each short story problem.

39. $\frac{1}{3}$ litre milk.
$\frac{1}{2}$ litre water.
How much liquid in all?

40. Had $\frac{3}{4}$ box of cookies. Ate $\frac{3}{8}$ box of cookies. What part of the box is left?

41. John ran $\frac{7}{10}$ kilometre.
Jim ran $\frac{1}{2}$ kilometre.
How much farther did John run?

42. Andy played in $\frac{5}{8}$ of the football game. David played in $\frac{1}{4}$ of the game. How much more did Andy play?

Reflected answers, Set 43: 1. 4, 3. 10, 5. 15, 7. 3, 9. 20, 11. $\frac{7}{20}$, 12. $\frac{3}{20}$, 13. $\frac{5}{12}$, 14. $\frac{1}{12}$, 15. $\frac{13}{20}$, 16. $\frac{1}{20}$, 39. $\frac{5}{6}$ litre, 40. $\frac{3}{8}$ box

Set 44 *For use with page 239*

Change each mixed numeral to an improper fraction.

1. $6\frac{2}{3}$	3. $1\frac{6}{7}$	5. $5\frac{1}{2}$	7. $1\frac{3}{5}$	9. $3\frac{1}{8}$	11. $4\frac{3}{10}$
2. $3\frac{7}{10}$	4. $4\frac{45}{100}$	6. $7\frac{1}{5}$	8. $9\frac{2}{5}$	10. $1\frac{9}{10}$	12. $2\frac{3}{4}$

Change each improper fraction to a mixed numeral.

13. $\frac{36}{5}$	15. $\frac{17}{4}$	17. $\frac{23}{10}$	19. $\frac{143}{100}$	21. $\frac{34}{7}$	23. $\frac{25}{3}$
14. $\frac{20}{7}$	16. $\frac{27}{4}$	18. $\frac{15}{4}$	20. $\frac{59}{9}$	22. $\frac{52}{10}$	24. $\frac{26}{3}$

Solve each short story problem.

25. Bought $\frac{3}{4}$ kg of apples and $\frac{3}{4}$ kg of oranges. Bought how many kilograms of fruit?

26. Exercised $\frac{5}{6}$ of an hour on Monday and $\frac{2}{3}$ of an hour on Tuesday. Exercised for how long on these two days?

Reflected answers, Set 44: 1. $\frac{20}{3}$, 3. $\frac{13}{7}$, 5. $\frac{11}{2}$, 7. $\frac{8}{5}$, 9. $\frac{25}{8}$, 11. $\frac{43}{10}$, 13. $7\frac{1}{5}$, 15. $4\frac{1}{4}$, 17. $2\frac{3}{10}$, 19. $1\frac{43}{100}$, 21. $4\frac{6}{7}$, 23. $8\frac{1}{3}$

Set 45 *For use with page 241*

Find the sums. Give them in lowest terms.

1. $4\frac{1}{4} + 7\frac{1}{2}$	2. $8\frac{1}{6} + 2\frac{1}{3}$	3. $5\frac{3}{4} + 4\frac{1}{8}$	4. $3\frac{1}{2} + 2\frac{1}{3}$	5. $6\frac{1}{3} + 5\frac{1}{6}$	6. $4\frac{3}{10} + 3\frac{1}{5}$
7. $6\frac{7}{10} + 1\frac{1}{2}$	8. $7\frac{1}{2} + 2\frac{1}{4}$	9. $6\frac{3}{10} + 4\frac{1}{5}$	10. $6\frac{1}{6} + 1\frac{3}{4}$	11. $8\frac{1}{5} + 7\frac{1}{4}$	12. $4\frac{2}{3} + 2\frac{1}{5}$

Reflected answers, Set 45: 1. $11\frac{3}{4}$, 2. $10\frac{1}{2}$, 3. $9\frac{7}{8}$, 4. $5\frac{5}{6}$, 5. $11\frac{1}{2}$, 6. $7\frac{1}{2}$

Set 46 *For use with page 243*

Solve each short story problem.

1. $4\frac{1}{3}$ hours going.
   $5\frac{1}{6}$ hours returning.
   How long for entire trip?

2. $2\frac{112}{1000}$ kg of peanuts.
   $1\frac{138}{1000}$ kg of almonds.
   How many kilograms of nuts?

3. $\frac{7}{10}$ cm of rain on Saturday.
   $\frac{3}{5}$ cm of rain on Sunday.
   How much rain in all?

4. Ran $\frac{1}{2}$ kilometre.
   Walked $\frac{9}{10}$ km more.
   How far in all?

5. Lost $1\frac{1}{2}$ kilograms one week
   and $\frac{3}{4}$ kilograms the next week.
   Lost how much in all?

6. Drank $2\frac{1}{2}$ cups of milk
   and $1\frac{3}{4}$ cups of water.
   Drank how much in all?

Reflected answers, Set 46: 1. $9\frac{1}{2}$ hours, 2. $4\frac{1}{4}$

Set 47 *For use with page 245*

Find the differences. Give them in lowest terms.

1. $6\frac{7}{10} - 1\frac{1}{2}$
2. $7\frac{3}{5} - 2\frac{1}{4}$
3. $6\frac{7}{20} - 4\frac{1}{10}$
4. $8\frac{3}{8} - 7\frac{1}{4}$
5. $9\frac{9}{10} - 6\frac{1}{5}$
6. $5\frac{1}{2} - 2\frac{1}{4}$
7. $6\frac{1}{3} - 3\frac{1}{6}$
8. $9\frac{4}{5} - 1\frac{3}{10}$
9. $5\frac{4}{5} - 4\frac{1}{10}$
10. $9\frac{1}{3} - 8\frac{1}{4}$
11. $7\frac{2}{5} - 5\frac{1}{4}$
12. $3\frac{3}{5} - 1\frac{1}{10}$
13. $8\frac{3}{4} - 3\frac{3}{5}$
14. $7\frac{7}{10} - 6\frac{3}{5}$
15. $9\frac{2}{3} - 4\frac{1}{4}$
16. $6\frac{3}{4} - 1\frac{1}{6}$
17. $7\frac{3}{4} - 7\frac{1}{5}$
18. $7\frac{7}{10} - 1\frac{1}{4}$
19. $9\frac{2}{5} - 2\frac{4}{15}$
20. $6\frac{21}{50} - 1\frac{1}{100}$
21. $7\frac{1}{4} - 3\frac{1}{8}$
22. $8\frac{23}{50} - 4\frac{17}{100}$
23. $5\frac{63}{100} - 3\frac{241}{1000}$
24. $9\frac{47}{50} - 8\frac{81}{100}$

Reflected answers, Set 47: 1. $5\frac{1}{5}$, 2. $5\frac{7}{20}$, 3. $2\frac{1}{4}$, 4. $1\frac{1}{8}$, 5. $3\frac{7}{10}$, 6. $3\frac{1}{4}$

Set 48 *For use with page 253*

Give the missing numerator or denominator.

1. $7.2 = 7 + \frac{\square}{10}$	**5.** $2.6 = 2 + \frac{6}{\square}$	**9.** $79.4 = 79 + \frac{\square}{10}$
2. $8.1 = 8 + \frac{1}{\square}$	**6.** $43.3 = 43 + \frac{\square}{10}$	**10.** $9.6 = 9 + \frac{6}{\square}$
3. $28.3 = 28 + \frac{3}{\square}$	**7.** $8.2 = 8 + \frac{2}{\square}$	**11.** $6.2 = 6 + \frac{\square}{10}$
4. $7.4 = 7 + \frac{\square}{10}$	**8.** $3.5 = 3 + \frac{5}{\square}$	**12.** $34.7 = 34 + \frac{7}{\square}$

Give the correct decimal for each sum.

13. $3 + \frac{4}{10}$	**17.** $45 + \frac{1}{10}$	**21.** $17 + \frac{6}{10}$	**25.** $91 + \frac{5}{10}$
14. $26 + \frac{7}{10}$	**18.** $9 + \frac{7}{10}$	**22.** $19 + \frac{4}{10}$	**26.** $36 + \frac{6}{10}$
15. $41 + \frac{3}{10}$	**19.** $3 + \frac{8}{10}$	**23.** $26 + \frac{5}{10}$	**27.** $21 + \frac{4}{10}$
16. $86 + \frac{2}{10}$	**20.** $15 + \frac{2}{10}$	**24.** $42 + \frac{3}{10}$	**28.** $62 + \frac{8}{10}$

Reflected answers, Set 48: 1. 2, 5. 10, 9. 4, 13. 3.4, 17. 45.1, 21. 17.6, 25. 91.5

Set 49 *For use with page 255*

Write each fractional number as in the example.
(Example: $9.362 = 9 + \frac{3}{10} + \frac{6}{100} + \frac{2}{1000}$)

1. 2.61	**4.** 81.6	**7.** 47.63	**10.** 4.956	**13.** 3.057
2. 3.82	**5.** 6.76	**8.** 48.74	**11.** 19.943	**14.** 49.12
3. 6.76	**6.** 62.31	**9.** 3.892	**12.** 4.03	**15.** 16.804

Give the correct decimal for each sum.

16. $8 + \frac{7}{10}$	**20.** $12 + \frac{1}{10} + \frac{3}{100}$	**24.** $8 + \frac{3}{10} + \frac{0}{100} + \frac{4}{1000}$
17. $9 + \frac{4}{10}$	**21.** $15 + \frac{3}{10} + \frac{6}{100}$	**25.** $72 + \frac{4}{10} + \frac{2}{100} + \frac{2}{1000}$
18. $5 + \frac{3}{10} + \frac{2}{100}$	**22.** $24 + \frac{4}{10} + \frac{3}{100}$	**26.** $41 + \frac{7}{10} + \frac{3}{100} + \frac{5}{1000}$
19. $7 + \frac{4}{10} + \frac{8}{100}$	**23.** $16 + \frac{0}{10} + \frac{2}{100}$	**27.** $4 + \frac{0}{10} + \frac{0}{100} + \frac{3}{1000}$

Reflected answers, Set 49: 16. 8.7, 20. 12.13, 24. 8.304

Set 50	*For use with page 257*

Give the correct sign (<, =, or >) for each ●.

1. 83.2 ● 83.6	6. 43.4 ● 44.3	11. 7.634 ● 7.635
2. 42.6 ● 46.2	7. 7.15 ● 7.14	12. 4.158 ● 4.258
3. 16.7 ● 16.7	8. 5.32 ● 5.22	13. 3.749 ● 3.739
4. 65.8 ● 64.4	9. 7.08 ● 7.06	14. 2.584 ● 2.684
5. 41.9 ● 40.9	10. 6.16 ● 6.26	15. 3.629 ● 3.628

Find the sums and differences.

16. $\frac{5}{10} + \frac{4}{10}$	17. $0.5 + 0.4$	18. $\frac{9}{10} - \frac{6}{10}$	19. $0.9 - 0.6$	20. $\frac{53}{100} - \frac{25}{100}$	21. $0.53 - 0.25$

Reflected answers, Set 50: 1. <, 6. <, 11. <, 16. $\frac{9}{10}$, 17. 0.9, 18. $\frac{3}{10}$, 19. 0.3, 20. $\frac{28}{100}$ or $\frac{7}{25}$, 21. 0.28

Set 51	*For use with page 259*

Find the sums and differences.

1. $0.5 + 0.7$	2. $0.8 + 0.6$	3. $1.9 - 0.7$	4. $2.4 - 0.6$	5. $47.6 + 32.9$
6. $0.34 + 0.25$	7. $13.5 - 7.3$	8. $2.5 + 3.8$	9. $29.6 - 19.8$	10. $3.43 + 5.57$
11. $24.39 + 8.7$	12. $6.34 - 4.57$	13. $8.63 + 7.29$	14. $15.28 - 7.63$	15. $70.91 - 12.86$
16. $0.683 + 0.785$	17. $0.902 - 0.610$	18. $8.771 - 5.847$	19. $21.446 - 0.008$	20. $4.844 - 0.369$

Reflected answers, Set 51: 1. 1.2, 2. 1.4, 3. 1.2, 4. 1.8, 5. 80.5

Set 52	*For use with page 261*

Find the total amounts.

1. $8.76 4.32	**2.** $10.39 4.61	**3.** $4.98 3.49	**4.** $36.50 9.98	**5.** $57.86 29.98
6. $6.36 2.83	**7.** $15.49 4.51	**8.** $7.49 8.89	**9.** $49.99 23.46	**10.** $58.67 46.98
11. $104.95 87.75	**12.** $69.99 69.99	**13.** $127.49 98.98	**14.** $89.98 89.98	**15.** $721.11 34.87

Find the differences in the amounts.

16. $5.65 3.27	**17.** $6.57 4.92	**18.** $56.30 27.88	**19.** $64.50 3.98	**20.** $36.50 17.98
21. $29.45 16.59	**22.** $104.25 67.48	**23.** $110.07 39.95	**24.** $365.98 299.99	**25.** $532.48 398.99

Reflected answers, Set 52: 1. $13.08, 2. $15.00, 3. $8.47, 4. $46.48, 5. $87.84, 16. $2.38, 17. $1.65, 18. $28.42, 19. $60.52, 20. $18.52

Set 53	*For use with page 293*

Give the products.

1. $7 \times \frac{1}{5}$	**5.** $\frac{1}{5} \times 4$	**9.** $\frac{1}{6} \times \frac{1}{7}$	**13.** $\frac{2}{2} \times \frac{2}{5}$	**17.** $\frac{5}{6} \times \frac{4}{9}$
2. $8 \times \frac{1}{7}$	**6.** $\frac{1}{8} \times 7$	**10.** $\frac{1}{9} \times \frac{1}{10}$	**14.** $\frac{3}{9} \times \frac{6}{5}$	**18.** $\frac{5}{8} \times \frac{3}{3}$
3. $3 \times \frac{1}{6}$	**7.** $\frac{1}{3} \times \frac{1}{7}$	**11.** $\frac{1}{3} \times \frac{4}{5}$	**15.** $\frac{2}{3} \times \frac{1}{7}$	**19.** $\frac{3}{7} \times \frac{5}{10}$
4. $\frac{1}{3} \times 9$	**8.** $\frac{1}{5} \times \frac{1}{8}$	**12.** $\frac{2}{3} \times \frac{4}{7}$	**16.** $\frac{3}{4} \times \frac{7}{5}$	**20.** $\frac{1}{8} \times \frac{3}{5}$

Reflected answers, Set 53: 1. $\frac{7}{5}$ or $1\frac{2}{5}$, 5. $\frac{4}{5}$, 9. $\frac{1}{42}$, 13. $\frac{4}{10}$ or $\frac{2}{5}$, $\frac{20}{54}$ or $\frac{10}{27}$

Set 54 — *For use with page 295*

Solve the equations.

1. $r \times \frac{1}{3} = \frac{1}{6}$
   $\frac{1}{6} \div \frac{1}{3} = r$

2. $b \times \frac{1}{5} = \frac{1}{10}$
   $\frac{1}{10} \div \frac{1}{5} = b$

3. $n \times \frac{1}{3} = \frac{1}{21}$
   $\frac{1}{21} \div \frac{1}{3} = n$

4. $x \times \frac{1}{3} = \frac{5}{6}$
   $\frac{5}{6} \div \frac{1}{3} = x$

5. $w \times \frac{1}{5} = \frac{3}{10}$
   $\frac{3}{10} \div \frac{1}{5} = w$

6. $t \times \frac{1}{3} = \frac{4}{21}$
   $\frac{4}{21} \div \frac{1}{3} = t$

7. $d \times \frac{2}{3} = \frac{1}{3}$
   $\frac{1}{3} \div \frac{2}{3} = d$

8. $\frac{1}{10} \times h = \frac{3}{40}$
   $\frac{3}{40} \div \frac{1}{10} = h$

9. $q \times \frac{1}{3} = \frac{7}{36}$
   $\frac{7}{36} \div \frac{1}{3} = q$

10. $\frac{3}{10} \times c = \frac{15}{50}$
    $\frac{15}{50} \div \frac{3}{10} = c$

11. $s \times \frac{2}{3} = \frac{6}{30}$
    $\frac{6}{30} \div \frac{2}{3} = s$

12. $i \times \frac{1}{2} = \frac{3}{10}$
    $\frac{3}{10} \div \frac{1}{2} = i$

Reflected answers, Set 54: 1. $r = \frac{1}{2}$, 5. $w = \frac{3}{2}$, 9. $q = \frac{7}{12}$

Set 55 — *For use with page 297*

1. Ate $\frac{1}{2}$ of $\frac{1}{3}$ of a pie. Ate how much pie?

2. $\frac{2}{3}$ of the boys play football. $\frac{1}{3}$ of these boys are on the first team. What part is on the first team?

3. $\frac{3}{4}$ of the students like beans. $\frac{1}{3}$ of these students also like corn. What part likes both vegetables?

4. Travelled $\frac{9}{10}$ of a kilometre. Walked $\frac{1}{2}$ of this distance. Walked how far?

5. Home $\frac{1}{2}$ of the day. Spent $\frac{1}{6}$ of this time eating. Spent how much of the day eating?

6. $\frac{7}{10}$ of the students ride the bus. $\frac{1}{2}$ of these students ride at least 3 kilometres. What part ride at least 3 kilometres?

Reflected answers, Set 55: 1. [illegible] 2. $\frac{2}{9}$

Books to Explore

Adler, Irving. ***The Giant Golden Book of Mathematics.*** New York, Golden Press, 1960. (Available from Whitman Golden Ltd., Cambridge, Ontario)

Have you ever wondered why a tree grows or why a volcano is shaped as it is or what makes a card trick work? This colorful book answers these and many other questions, through exploring the world of mathematics. You will find all kinds of exciting ideas about numbers and what they mean in our daily lives. Here are just a few of the interesting topics:

- The symbol Egyptians used to show a fraction 43
- When the shortest path between two points is a curve 61
- Archimedes' development of giant catapults 85
- How Maxwell's equations led to electronics 90

Bendick, Jeanne, and Levin, Marcia O. ***Take Shapes, Lines and Letters.*** New York, McGraw-Hill Book Co., 1962. (Available from McGraw-Hill Ryerson, Scarborough)

This book sparks your interest about mathematics in art, music, and everyday life; about shapes and curves in nature; and about drawings, graphs, and secret codes. Ideas to explore include:

- What Kepler discovered about planets 25
- What figure is a symbol of the universe 33
- A number system using letters 56
- Chinese tangrams 68

Hogben, Lancelot. ***The Wonderful World of Mathematics.*** New York, Doubleday, 1968. (Available from Doubleday Publishers, Toronto, Ontario)

This book shows that the story of how man became civilized is also the story of how mathematics became a science. You will enjoy going back to the time of the cave man and finding out how man learned to measure and to count, to build and to navigate, to design and to calculate with computers. Some enjoyable things in this book are:

- The Mayan Indian base-twenty number system 11
- Cannonball warfare 55
- Drawing an ellipse with two pegs 59
- Graphs of solid steel 65

Jonas, Arthur. ***New Ways in Math.*** Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1962. (Available from Prentice-Hall of Canada Ltd., Scarborough, Ontario)

Cartoon-style mathematics explains sets, probability, and algebra in a way you'll really enjoy. The chapter "Men in Math" includes both modern giants, like Einstein and Von Neumann, and history's great mathematicians, like Pythagoras and Archimedes. Other chapters you will probably find interesting include:

- The magic of two 24
- When 1001 = 9 31
- What is your hunch? 45
- Doughnuts and pretzels 50

You may also enjoy reading *More New Ways in Math* (1964), by Mr. Jonas.

Other books to explore in your library are listed below:

Adler, Irving. ***Logic for Beginners Through Games, Jokes, and Puzzles.*** New York, John Day, 1964. (Available from Longman Canada Ltd., Don Mills, Ontario)
Good thinking is needed to solve the 285 brain teasers collected here.

Barr, Donald. ***Arithmetic for Billy Goats.*** New York, Harcourt Brace Jovanovich, Inc., 1966. (Available from Longman Canada Ltd., Don Mills, Ontario)
Counting with his two front hoofs, young goat William Gruff invents a two-digit (binary) counting system.

Bendick, Jeanne. ***Take a Number: New Ideas + Imagination = More Fun.*** New York, McGraw-Hill Book Co., 1961. (Available from McGraw-Hill Ryerson, Scarborough)
A cleverly written book about the history of numeration, the binary system and computers, perfect and prime numbers, and fun with numbers.

Bendick, Jeanne, and Levin, Marcia. ***Mathematics Illustrated Dictionary.*** New York, McGraw-Hill Book Co., 1965. (Available from McGraw-Hill Ryerson, Scarborough)
A handy tool for students. If you need to know about ancient or contemporary mathematicians, mathematical terms and definitions or any facts and figures, use this dictionary.

Gardner, Martin. ***Perplexing Puzzles and Tantalizing Teasers.*** New York, Simon and Schuster, 1969. (Available from Musson Book Co., Don Mills, Ontario)
Have fun with riddles, teasers, illusions, tricky questions, word and picture puzzles. All the answers are included. You may enjoy "Sneaky Arithmetic," page 30, "Folding Money Fun," page 54, and "Mother Hubbard's Cupboard," page 67.

Kettlekamp, Larry. ***Puzzle Patterns.*** New York, William Morrow, 1963. (Available from George J. McLeod Ltd., Toronto, Ontario)
Entertaining historical background of puzzles including Sir Isaac Newton's famous problem about trees and the 4000-year-old Chinese tangram.

Leeming, Joseph. ***Fun With Puzzles.*** Philadelphia, J. B. Lippincott Company, 1946. (Available from McClelland and Stewart, Toronto, Ontario)
Also available in paperback from Scholastic Book Service, 1966.
A collection of more than 200 match, coin, paper-and-pencil, cutout, and word puzzles. The answers are all in the back.

Luce, Marnie. ***"Math Concept Books."*** Minneapolis, Lerner Publications Co., 1969. (Available from J. M. Dent & Sons (Canada) Ltd., Don Mills, Ontario)
These books will help you understand the way numbers work. The titles include *Counting Systems, Lines and Planes, Points, Polyhedrons,* and *Primes Are Builders.*

Meadow, Charles. ***The Story of Computers.*** Irvington-on-Hudson, New York, Harvey House, Inc., 1971. (Available from Burns & MacEachern Ltd., Don Mills, Ontario)
An introduction to the fascinating new world of electronic computers—how they are used and how they are programmed to solve problems.

Murray, William, and Rigney, Francis. ***Paper Folding for Beginners.*** New York, Dover, 1960. (Available from General Publishing, Don Mills, Ontario)
See what you can make by folding, tearing, and cutting paper.

Ruchlis, Hy, and Milgrom, Harry. ***Math Projects: Mathematical Shapes.*** Brooklyn, New York, Book Lab, Inc., 1968.
Use straws and pipe cleaners to make cubes, tetrahedrons, octahedrons, prisms, and pyramids.

Sobol, Ken. ***The Clock Museum.*** New York, McGraw-Hill Book Co., 1967. (Available from McGraw-Hill Ryerson, Scarborough, Ontario)
A brief and interesting history of clocks and time-keeping devices from prehistoric times to the present.

Wentworth, D., Stecher, A., Couchman, K., and MacBean, J. ***Mapping Small Places.*** Toronto, Holt, Rinehart and Winston of Canada Ltd., 1972.
This book suggests activities related to maps and map-making. They include measuring distances directly, using map scales, measuring angles and heights, and using the compass.

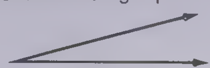
White, Lawrence B., Jr. ***Investigating Science With Paper.*** Reading, Addison-Wesley Publishing Co., 1970.
A chapter on ''paper engineering'' includes bridge building, testing and the like. Math puzzles and a little magic add to the fun.

Glossary

addend Any one of a set of numbers to be added. In the equation $4 + 5 = 9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the one number which is the result of combining the two numbers is called the sum of the addends.

angle Two rays from a single point.



approximation One number is an approximation of another number if the first number is suitably "close" (according to context) to the other number.

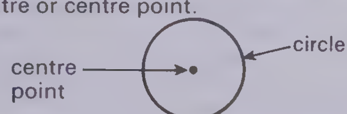
area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

associative principle *See* grouping principle.

average (arithmetic mean) The average of a set of numbers is the quotient resulting when the sum of the numbers in the set is divided by the number of addends.

centimetre A unit of length. One centimetre is $\frac{1}{100}$ metre.

circle The set of all points in a plane which are a specified distance from a given point called the centre or centre point.



clock arithmetic A mathematical system using only the twelve numbers of a clock face. It is also called modular arithmetic or remainder arithmetic.

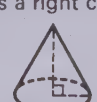
common factor When a number is a factor of two different numbers, it is said to be a common factor of the two numbers.

commutative principle *See* order principle.

compass A device for drawing models of a circle.

composite number Any whole number greater than 1 that is not prime.

cone Generally thought of as a right circular cone, which is illustrated below.



congruent figures Figures that have the same size and shape.

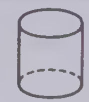
congruent triangles

co-ordinates Number pair used in graphing.

co-ordinate axes Two number lines intersecting at right angles at 0.

cube A rectangular prism (box) such that all faces are squares.

cylinder Generally thought of as a right circular cylinder, which is illustrated below.

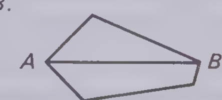


decimal Any base ten numeral that uses place value to represent a fractional number.

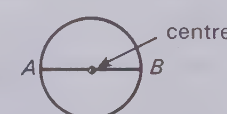
degree An angle unit that is $\frac{1}{90}$ of a right angle.

denominator The number indicated by the numeral below the line in a fraction symbol.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment *AB*.



diameter A chord that passes through the centre point of the circle.



difference The number resulting from the subtraction operation.

digits The basic Hindu-Arabic symbols used to write numerals. In the base-ten system, these are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

distributive principle *See* multiplication-addition principle.

dividend In the problem $33 \div 7$, 33 is called the dividend.

Example: $\begin{array}{r} 4 \\ 7\overline{)33} \\ \underline{28} \\ 5 \end{array}$ ← dividend

division An operation related to multiplication as illustrated:

$$3 \times 4 = 12 \begin{cases} \rightarrow 12 \div 3 = 4 \\ \rightarrow 12 \div 4 = 3 \end{cases}$$

divisor In the problem $33 \div 7$, 7 is called the divisor.

edge An edge of a space figure is one of the segments making up any one of the faces of the space figure.

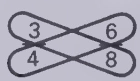
empty set A set that has no objects in it.

equality (equals; or =) A mathematical relation of being exactly the same.

equation A mathematical sentence involving the use of the equality symbol.

Examples: $5 + 4 = 9$; $7 + \square = 8$; $n + 3 = 7$

equivalent fractions Two fractions are equivalent when it can be shown that they each can be used to represent the same amount of a given object. Also, two fractions are equivalent if these two products are the same:



$\rightarrow 4 \times 6 \rightarrow 24$

$\rightarrow 3 \times 8 \rightarrow 24$

equivalent sets Two sets that may be placed in a one-to-one correspondence.

estimate To find an approximation for a given number. (Sometimes a sum, a product, etc.)

even numbers The whole-number multiples of 2 (0, 2, 4, 6, 8, 10, 12, . . .).

face The face of a given space figure is any one of the plane geometric figures (regions) making up the space figure. For example, in a cube each of the square regions is a face of the cube.

factor *See* multiplication. The equation $6 \times 7 = 42$ illustrates that both 6 and 7 are factors of 42.

fraction A symbol for a fractional number, usually written $\frac{2}{3}, \frac{3}{4}, \frac{1}{2}$, and so on.

fractional number The one number we think about for each set of equivalent fractions.

function The set of number pairs (input, output) generated by a function rule applied to a given set of numbers (input numbers).

graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

greater than (>) One of the two basic inequality relations.

Examples: $8 > 5$, $28 > 25$, $80 > 50$

greatest common factor The largest, or greatest, number that is a factor of each of two numbers.

grouping principle (associative principle) When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

Examples: $2 + (8 + 6) = (2 + 8) + 6$
$3 \times (4 \times 2) = (3 \times 4) \times 2$

hexagon A six-sided polygon.

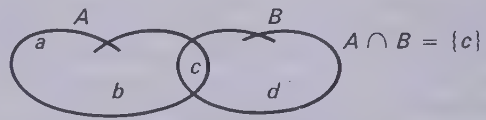
hypotenuse The side opposite the right angle in a right triangle.

improper fraction A fraction in which the numerator is greater than or equal to the denominator.

Examples: $\frac{8}{5}, \frac{9}{6}, \frac{12}{3}, \frac{7}{7}$

inequality ($<, \neq, >$) In arithmetic, a relation indicating that the two numbers are not the same.

intersection of sets The intersection of two sets is the set of elements common to both of the sets. If A and B are sets, $A \cap B$ denotes the intersection of the sets.



least common denominator The least common multiple of two denominators. The least common denominator of $\frac{1}{4}$ and $\frac{5}{6}$ is 12.

least common multiple The smallest non-zero number that is a multiple of each of two given numbers. The least common multiple of 4 and 6 is 12.

length (1) A number indicating the measure of one line segment with respect to another line segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

less than (<) One of the two basic inequality relations. Examples: $5 < 8$, $25 < 28$, $50 < 80$.

line A line is a set of points that "goes on and on" in both directions. There is only one line through any two points.

line segment *See* segment.

lowest terms A fraction is in lowest terms if the numerator and denominator of the fraction have no common factor greater than 1.

measure (1) A number indicating the relation between a given object and a suitable unit. (2) The process of finding the number described in (1).

metre A unit of length in the Metric System. A metre is 100 centimetres.

midpoint A point that divides a line segment into two parts of the same size.

minus (−) Used to indicate the subtraction operation, as in $7 - 3 = 4$ (read, "7 minus 3 equals 4").

mixed numerals Symbols such as $2\frac{1}{2}$ and $3\frac{1}{4}$.

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number. Example: 24 is a multiple of 6 since $4 \times 6 = 24$.

multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the one number which is a result of combining the two numbers is called the product of the two numbers.

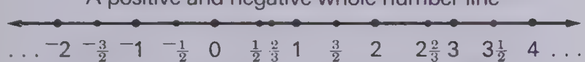
multiplication-addition principle (distributive principle) This principle is sometimes described in terms of "breaking apart" a number before multiplying.

Example: $6 \times (20 + 4) = (6 \times 20) + (6 \times 4)$

number line A line with a subset of its points matched with a subset of the real numbers. We say that the rational number line has "holes" in it because some points are not matched with rational numbers. The real number line is said to be "complete" because each point is matched with some real number.

The number line

A positive and negative whole number line



The rational number line

number pair Any pair of numbers. In this book, usually a pair of whole numbers.

numeral A symbol for a number.

numerator The number indicated by the numeral above the line in a fraction symbol.

odd number Any whole number that is not even.

one principle (for multiplication) Any number multiplied by 1 is that same number.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set and each element of the second set is matched with exactly one element of the first set.

order principle (commutative principle) When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product).

Examples: $4 + 5 = 5 + 4$
$2 \times 3 = 3 \times 2$

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with its opposite sides parallel.

parentheses A pair of curved symbols, ( ), used to indicate grouping or order of performing operations.

Examples: $(5 \times 4) - 2 = 18$
$5 \times (4 - 2) = 10$

pentagon A five-sided polygon.

perimeter The sum of the lengths of the sides of a given polygon.

period In arithmetic, each set of three digits indicated by spacing when writing a numeral is called a period. These periods are called (right to left) units' period, thousands' period, millions' period, and so on.

Example:	3 4 2	6 7 4	2 0 8
	millions' period	thousands' period	units' period

perpendicular lines Two lines that intersect in right angles are perpendicular to each other.

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. In the numeral 3257 the 5 stands for 50; in the numeral 36 289 the 6 stands for 6000.

plus (+) Used to indicate the addition operation, as in $4 + 3 = 7$ (read, "4 plus 3 equals 7").

polygon A closed geometric figure made up of line segments.

prime number A number greater than 1 whose only factors are itself and 1.

product The result of the multiplication operation. In $6 \times 7 = 42$, the product of 6 and 7 is 42.

prism A three-dimensional figure whose bases are congruent polygons in parallel planes and whose faces are parallelograms.

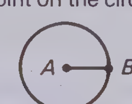
protractor An instrument used for measuring angles.

pyramid A three-dimensional figure with a polygonal base and triangular lateral faces.

quadrilateral A four-sided polygon.

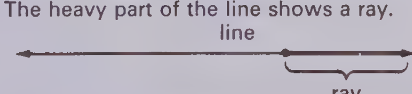
quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.

radius (1) Any segment from the centre point to a point on the circle. (2) The distance from the centre point to any point on the circle.



ratio A pair of numbers used in making certain comparisons. The ratio of 3 to 4 is written $3{:}4$ or $\frac{3}{4}$.

ray The heavy part of the line shows a ray.



rectangle A quadrilateral that has four right angles.

regrouping A method of handling place value symbols in adding or subtracting numbers.

remainder: Example:

$$\begin{array}{r} 6 \\ 7\overline{)47} \\ \underline{42} \\ 5 \end{array} \longleftarrow \text{remainder}$$

repeated addition Finding the sum of a set of numbers, each of which is the same.

Example: $5 + 5 + 5 + 5$

repeated subtraction Starting with a number and repeatedly subtracting the same given number from each difference that is obtained.

rhombus A parallelogram with 4 sides of the same size.

right angle An angle that has the measure of 90 degrees.

right triangle A triangle that has one right angle.

Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.

rotation A motion in which a given figure is turned about a fixed point.

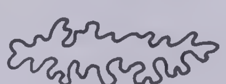
scale drawing A drawing constructed so the ratio of all the dimensions in the drawing to those of the actual object is the same.

segment Two points on a line and all the points on that line that are between the two points.

sequence A collection or set of numbers given in a specific order. Such numbers are commonly given according to some rule or pattern.

set A group or collection of objects.

simple closed curve Can be thought of as a loop of string on a flat surface that does not cross itself.



solution The number or numbers which result from solving an equation or a given problem.

solve To find the number or numbers which, when substituted for the variable or placeholder, make a given equation true.

square A quadrilateral that has four right angles and four sides that are the same length.

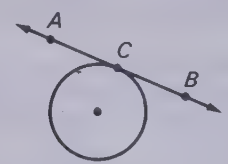
subtraction An operation related to addition as illustrated:

$$7 + 8 = 15 \begin{cases} \nearrow 15 - 8 = 7 \\ \searrow 15 - 7 = 8 \end{cases}$$

sum The result obtained by adding any set of numbers.

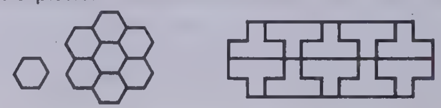
symmetric figure A figure that can be folded in half so the two halves match.

tangent A line is tangent to a circle if the two figures are in one plane and have exactly one point in common.



Line AB is tangent to the circle at point C.

tessellation A repeated pattern of regions that can cover a plane.



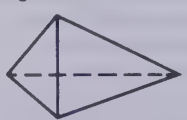
times (×) Used to indicate the multiplication operation, as in $3 \times 4 = 12$ (read, "3 times 4 equals 12").

translation A motion in which each point of a figure is moved the same distance and the same direction.

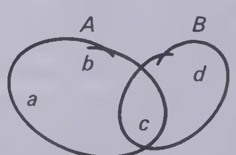
trapezoid A quadrilateral with at least one pair of parallel sides.

triangle A three-sided polygon.

triangular pyramid A 4-sided space figure that has triangular regions for all faces.



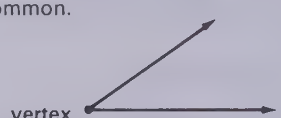
union of sets If A and B are sets, then $A \cup B$ (the union of A and B) is the set consisting of all elements that belong to at least one of the two sets.



$A \cup B = \{a, b, c, d\}$

unit An amount or quantity adopted as a standard of measurement.

vertex The point that the two rays of an angle have in common.



volume The measure, obtained by using an appropriate unit (usually a cube), of the interior region of a space figure.

whole number Any number in the set $\{\ldots, {}^-3, {}^-2, {}^-1, 0, 1, 2, 3, \ldots\}$

zero principle (for addition) Any number added to zero is that same number.

Tables of Measures

LENGTH	
10 millimetres (mm) = 1 centimetre (cm)	1000 millimetres = 1 metre
10 centimetres = 1 decimetre (dm)	100 centimetres = 1 metre
10 decimetres = 1 metre (m)	10 decimetres = 1 metre
1000 metres = 1 kilometre (km)	1/1000 kilometres = 1 metre

TIME	
60 seconds (s) = 1 minute (min)	52 weeks = 1 year
60 minutes = 1 hour (h)	12 months (mo) = 1 year
24 hours = 1 day	365 days = 1 year
7 days = 1 week (wk)	366 days = 1 leap year

CAPACITY
10 millilitres (ml) = 1 centilitre (cl)
10 centilitres = 1 decilitre (dl)
10 decilitres = 1 litre (*l*)
1000 litres = 1 kilolitre (kl)

WEIGHT
1000 grams (g) = 1 kilogram (kg)
1000 kilograms = 1 tonne (t)

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